PERFORMANCE MANAGEMENT AND MEASUREMENT WITH DATA ENVELOPMENT ANALYSIS

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Ali Emrouznejad, Ibrahim H. Osman, Abdel L. Anouze
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PREFACE

On behalf of the members of the 8th International Data Envelopment Analysis Conference (DEA2010) Committees, I would like to extend a warm welcome to all of you to Lebanon—Known as the “Switzerland of the East”—and to its capital Beirut—widely referred to as the “Paris of the East”—and rated in 2009 by New York Times as “Number One City to Visit in the World.”

History of education in the land of Cedars goes back thousands of years when our ancestors created the alphabet in Byblos (Lebanon) and spread it to the world. Hence, it is not a surprise for Lebanon to host this International Conference at the Olayan School of Business (OSB) of the American University of Beirut. After last year’s completion by OSB of a number of milestones including the inauguration of our new magnificent four-story building in December 2009 and the achievement of the AACSB accreditation and securing a formal ranking as first in the Middle East, I aspired to host this prestigious International Data Envelopment Analysis Conference in celebration and crowning of such successes with world-class colleagues and friends.

The DEA community has established a reputation within the management science for rigor and relevance to professional practice. These features are demonstrated again in the DEA2010, which attracted more than 80 academic participants from 29 countries around the world. The presentations include two tutorials to provide transfer of knowledge to students and four keynotes on recent developments and advances in the theory and applications. The remaining presentations are grouped into parallel special focused sessions on performance measurement, and management in areas such as agriculture, banking, education, environment, human development, electronic government, manufacturing, policy analysis, project and service management, process negotiation, risk and transport. We hope that DEA2010 will contribute to the advancement of the field, create an atmosphere for the diffusion of insightful research ideas, foster new collaborations, simulate creative discussions and provide up-to-date knowledge and learning tools on performance management and measurement for the benefits of Mankind.

DEA2010 would not have been possible without the support of many people, including patronage by our Trustee and Prime Minister Mr. Saad Hariri, who is also a Member of Parliament, and a successful businessman; sponsorship by the Center National Council for Scientific Research of Lebanon, the HSBC Bank, Performance Improvement Management Software Company, the International Data Envelopment Society; as well as our University authorities; in particular President Peter Dorman, Provost Ahmad Dallal and Dean George Najjar; Associate Dean Khalil Hindi. We are also grateful to the DEA2010 International Advisory Committee, Organizing Committee and Scientific Committee for the support they provided to ensure a high quality selection of refereed papers to include in the conference program and to organize an excellent social program; and finally to our School research and staff in particular, Ms Hala Azar, Ms Maya Helou, Ms Nada Khalidi, and Ms Mayssa Saffar.

Again, welcome to DEA2010. For many of you, this may be the first visit to Lebanon and I trust you will enjoy your stay and will have a pleasant time while enjoying the traditional hospitality of the Lebanese People. I hope you will treat yourselves to some magnificent touristic attractions in Lebanon, in addition visiting our AUB beach if possible.

With my warmest welcome and best regards,

Ibrahim H. Osman,
Beirut, June 10, 2010
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Higher Education Applications Stream
Resource allocation with DEA ranking: evidence from Iran medical universities

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ABSTRACT
This study proposes the Data Envelopment Analysis (DEA) ranking for resource allocation between Iran's Medical Universities based on a five years observation data (2004-2008). We compared the DEA ranking results with the traditional ranking approach applied by the Ministry of Health and Medical Education of Iran. Based on the DEA score the amount of budget is proposed for each university for the incoming financial year. The findings confirm that Competitive ranking can provide an appropriate direction for policy-makers to allocate resources between universities.

Keywords: Data Envelopment Analysis, Resource Allocation, Medical Universities

INTRODUCTION
Universities play an important role in the social and economical development of a country. Therefore, governments usually provide the financial resources universities need.

On the other hand, universities should be efficient in satisfying the government's conditions of functional resources. Performance improvement is entering a new era, raising important questions for universities and research centers, which is required to be efficient for the absorbing the shareholder's investment and share.

Finding a transparent and systematic way to distributing the funds to each university is a major challenge for government.

There are some 50 medical sciences universities (MSUs) in Iran which are governed by the Ministry of Health and Medical Education.

In today's increasingly highly competitive and globalized environment, these universities as any other organizations are forced to operate with a higher degree of efficiency relative to the competitors, especially in order to receiving financial resources from stakeholders, such as the governments.

Teaching, researching and transferring of the new knowledge have been agreed upon as the three most important activities of a university. There are many studies that have attempted to measure performance of universities based on input and output indicators, to demonstrate whether or not universities are efficient in their functions.

In this context, the universities are considered as productive units which use resources (inputs) to carry out their scientific and technological activities (outputs). The problem which the Ministry of Health and Medical Education is faced is decision making about how to allocate the budget for each university.

This paper concentrates on developing a resource allocation method of annual financing to the medical universities by the Ministry of Health and Medical Education. In this context, we have chosen "University" as a Decision Making Unit (DMU) for analysis.

The main purpose of this method is to identify the maximum annual budget which a university is able to receive from the governmental agency based on it performance.

The proposed approach constructs the competitive ranking of universities by Data Envelopment Analysis (DEA) scores for allocating the financial resources to each MSU. Competitive ranking model make it possible to evaluate the different MSUs performance according to efficiency criteria.
This mechanism will improve the resource allocation process between the universities by the government.

Based on this model, each university has to improve relative rank position in order to receive more the annual required budget from the government.

This paper is set out as follows.

Section 2, refers to the background of evaluation and ranking of medical universities based on partial efficiency measures by the Iranian Ministry of Health and Education (MHE).

The methodology and observations is outlined in section 3.

Input-output indicators of the university system are discussed in next section.

In section 5, the results of efficiency estimations are presented. Finally, implications for the future research are made in Section 6.

BACKGROUND

Dating back nearly a century and a half, university rankings are not a new phenomenon (Tyler, ND). Rankings become a highly significant process for universities with many stakeholders. Currently, there are roughly 50 major university ranking models in use around the world which use hundreds of different indicators on global, regional or national level (Engr, 2009).

Since 2002, evaluation of research activities in MSUs has been implemented in some 47 universities by department of research and technology of the Iranian Ministry of Health and Education (MHE).

This process has been repeated each year based on guidelines issued by the authorities to enhance the governance and supervisory Leadership, empowerment and knowledge production in universities and faculties of medical sciences.

Specific objectives:

Monitoring the research activities of the MSUs and faculties;

To determine the rank of MSUs and colleges by their performance on research activities scores level;

Identification of capacities of medical sciences universities and faculties on different axes;

To determine the implication policies based on the results of the evaluation.

For the governments it is necessary to monitor the activity of universities and to assess the results, in order to modulate the allocation of resources.

For each university it is necessary to understand and evaluate its own activity, in order to bring its performance to meet the criteria and parameters indicated by the government, and thus maximize the amount of resources which could be obtained.

The present study aims to draw up a method for resources allocation for research activities between universities by government. A new model is presented which incorporates allocative efficiency criteria in allocation process between MSUs.

The eventual purpose of this research is to construct a method suitable for resources allocation between universities by the government.

METHODOLOGY AND OBSERVATIONS

The underlying assumption is that a similar group of universities usually have similar technical process, so we can assume them as homogeneous decision making units (DMUs). There is a dispersion of efficiency across the universities, i.e., some universities having greater efficiency than the average and some lower than this.

Then, raising this question that does applying DEA will lead to better resource allocation between universities?

In this paper, a new procedure for resource allocation across medical universities using DEA is introduced. To apply relative ranking for resources allocation between medical universities, we used concept of efficiency frontier (see Coelli et al., 2005) to identify the best practice performance in the use of resources and Malmquist index (Fare et al., 1994), calculated based on Data Envelopment Analysis (DEA) for trend analysis. Data are available in panel form with several observations for each medical university in the sample over 5 years; then we can analyze the efficiency trends over time as well as across universities with malmquist index.

Malmquist index provides a useful way of distinguishing between changes in technical efficiency, pure technical efficiency, scale, total factor productivity (TFP) and shifts in the efficiency frontier (technological change) over time.

This index is the geometric mean of two TFP indices, one evaluated with respect to the technology (efficiency frontier) in the current period t and the other with respect to the technology in the base period s (Coelli et al., 1998).

Resource allocation is the process of distribution of resources, usually financial, among the competing groups of entities or
institutes. When we talk about allocation of funds for universities, we need to consider three distinct levels of decision-making:
Allocating resources to universities versus other social needs;
Allocating resources among the different universities; and
Allocating resources among individual departments of a university.
This study concerned the second level decision-making, i.e. allocating the resources among the different medical universities.

The level of technology influences on the utilization of resources in each university. Therefore, our method for resource allocation is constructed based on technical efficiency scores. According to this procedure, the financial recourses will be allocated to each university with regard to a variety between zero and one according to the output-orientated TE scores.

Johnes (2003) has categorized the studies on measuring the efficiency of universities into three main groups:
University level approach, based on the observations concerning each of the higher education institutions;
Department level approach, based on the observations concerning one department within the university; and
Individual level approach, based on the observations concerning the students individually.

In present study, the University level approach has chosen which a University is considered as a Decision Making Unit (DMU) for analysis.

DEA as an analytical technique is used to identify the best practice performance in the use of resources among the 41 Iranian medical universities as governmental organizations.

In DEA, each university is assigned the highest possible efficiency score by optimally weighing the inputs and outputs.

Technical efficiency is a valid performance measure for the scientific and technological activities of universities at a given level of quality with regard of resource constraints.

Some authors argue that, next to the delivery of quality education, technical efficiency is probably the only valid measure of performance of tertiary institutions (see Pestiœau and Tulkens, 1993).

In case of governmental universities, a fixed quantity of resources are given to DMUs and asked them to produce as much output as possible. In this case, as the objective of universities lies in increasing outputs rather than decreasing inputs, an output orientation would be more appropriate.

The data used in this analysis is obtained from annual evaluation of research activities in MSUs implemented by MHE. We have data for 41 medical universities each using 2 inputs and producing 3 outputs. The output-oriented technical efficiency of each university is then calculated by solving the

\[
\max_{\lambda, \theta} \theta
\]

Subject to:

\[-\theta y_n + Y \lambda \geq 0 \]

\[x_n - X \lambda \geq 0 \]

\[\lambda \geq 0 \]

Where \(1 \leq \theta \leq \infty\) is a scalar and \(\lambda\) is a \(41 \times 1\) vector of constants. \(X\) is a \(2 \times 41\) matrix of inputs, \(Y\) is a \(3 \times 41\) matrix of outputs, and let \(x_n\) denote the inputs used by university \(n\). The technical efficiency score is then defined as \(\theta^{-1}\), which by definition is the maximum proportional increase in outputs possible for a given set of inputs.

The following formula is used to distribute the research-related funds between medical universities:

Suppose that the government has a sum of RLS \(B\) billion to allocate between \(U\) universities. University \(u\), \(u=1, 2, \ldots, U\), consists of 41 medical universities.

At period of \(t\) the proposed resources allocate (\(pra_u\)) for a MSU is calculated according to the following formula:

\[pra_u = ae_{ui, t} \times \left( (pra_{ui, t})^{* PPI_{t, i}} \times (pra_{ui, t})^{* (PPI_{t, i} - PPI_i)} \right) \]

Subject to:

\[\Sigma_{pra_u} \leq B_i,\]

\[B_i = B_{t, i} \times \Sigma f ch_{ui, t, i} \times (B_{t, i}) \]

\[ch_{ui} \% = ch \% \Sigma_{pra_u} \]

\[pra_u > 0\]

Which:

\[pra_u = \text{the proposed resources allocate for research activities of a university at } t \text{ period},\]

\[PPI = \text{Producer Price Index (PPI)}^1,\]

\[B = \text{the research-related funds}\]

\[ae_{ui, t} = \text{allocative efficiency score at previous year}\]

---

1 Producer Price Index (PPI) measures average changes in prices received by domestic producers for their output.
Finally, the proposed resources allocate \((pr_{au})\) is adjusted with the real Producer Price Index.

To expand research capacity of universities, finance is needed which may be raised either internally or externally by institutions such as governmental institutions. In both situations, the access to finance depends on the university performance.

The efficient universities will be able to raise finance for research capacity expansion. This means that the rate of growth of university capacity expansion has a positive relationship with the efficiency level.

**INPUT-OUTPUT INDICATORS**

The first step in the ranking relates to indicator selection. Competitive ranking of universities can be based on subjectively perceived "quality," on some combination of empirical statistics, or on surveys of educators, scholars, students, prospective students or others.

In order to applying efficiency scores based on data envelopment analysis, we need the input-output indicators.

To do ranking, we need some suitable indicators to demonstrate the relative efficiency of universities.

The mission of a University generally is to create and diffuse the new knowledge to society. In this study, three and two preferred indicators have been considered for outputs and inputs of university respectively (Table 1).

The following input indicators are used in the efficiency analysis:

- The first input is the number of full-time equivalent of academic staff, which is a proxy for the university's human capital. The second input is the amount research budget. The latter is a proxy for the university’s capital investment.

We follow the literature in assuming inputs are homogenous across universities.

As the objective of universities lies in increasing outputs rather than decreasing inputs, the output orientation is considered.

**EFFICIENCY OF MEDICAL UNIVERSITIES**

Next step in the relative efficiency measuring relates to indicators data need to be collected. The analysis contains data for 41 MSUs each using 2 inputs and producing 3 outputs over a five year period (2004-8).

The DEA scores are undertaken using DEAP software (Coelli, 1996).

The calculated scores under condition VRS output orientated and CRS cost efficiency are presented in Table 2.

As can be seen from the table 4, the average VRS TE for the universities is 0.80, 0.88, 0.83, 0.88 and 0.80 for 2004 to 2008. If the universities were efficient, they could have reduced their inputs by 20%, 12%, 17%, 12% and 20%, respectively, for the given level of output.

In 2004, the mean efficiency under input orientated DEA model with CRS scale assumption was 0.24 and 0.801 in the VRS output orientated model. Only three MSUs, Gonabad, Shahroud and Sabzevar were full efficient in the first model, however, in the later DEA model, 12 (29% of 41) universities are full efficient, which include Tehran, Shiraz, Mazandaran, Gilan, Qazvin, Golestan, Shahrekord, Lorestan, Gonabad, Shahroud, Sabzevar and Zabol universities.

In 2005, the number of full efficient universities increased to 7 under input orientated DEA model with CRS scale assumption which are Birjand, Gonabad, Shahroud, Sabzevar, Qom, Zabol and Jahrom universities.

In this year, 19 (46% of 41) universities are full efficient in the VRS output orientated model, which include Tehran, Isfahan, Shiraz, Bagiyat Allah, Mazandaran, Qazvin, Kashan, Golestan, Birjand, Shahrekord, Gonabad, Shahroud, Bushehr, Yasuj, Sabzevar, Qom, Zabol, Jahrom and Shahid Beheshti universities.

With malmquist DEA, which is an application of DEA to a panel data, five indices are calculated for each university in each year (Table 4). These are:

- Technical efficiency change (eff ch);
- Technological change (tech ch);
- Pure technical efficiency change (pe ch);
- Scale efficiency change (se ch); and
- Total factor productivity change (tpf ch).

It is a geometric average of efficiency index and technical progress index.

Based on Malmquist productivity index, efficiency change (CRS TE) increased by 3.2%, pure efficiency change by 11.1% (VRS TE) and scale efficiency has decreased 6.1% over the 2005 year.

The technological change of 1.390 indicates that during the 2005 year, there has been a 39% technological progress. Over this year, the average total factor productivity of the 41 medical universities has enhanced by 43.5%.

Efficiency change (CRS TE) and scale efficiency has increased 5.1% and 12.8% respectively during the year 2006. While, pure efficiency, technological and total factor
productivity change decreased 6.8%, 54.9% and 52.6% respectively.

In 2007, under input orientated DEA model with CRS scale assumption five universities include Bagiyat Allah, Gonabad, Shahroud, Qom and Zabol were the most efficient university. While, in the VRS DEA model, 15 (37% of 41) universities are full efficient, which include Tehran, Isfahan, Shiraz, Bagiyat Allah, Mazandaran, Zahedan, Golestan, Zanjan, Shahrekord, Gonabad, Shahroud, Sabzevar, Qom, Zabol and Shahid Beheshti.

In this year, under CRS cost efficiency DEA model, the Bagiyat Allah, Gonabad and Qom universities were full efficient.

In 2007, despite of positive change of efficiency (18.7%), pure efficiency (6.1%) and scale efficiency (11.9%), the technological change and total factor productivity (TFP) of the sample medical universities have dropped strongly, 68% and 62% respectively.

In 2008, the mean efficiency under input orientated DEA model with CRS scale assumption was 0.496 and 0.802 in the VRS output orientated model.

In this year, the most efficient university in the CRS Input orientated DEA model were the Babel, Shahrekord, Gonabad, Qom and Shahroud Universities (100%), which comprise 12 % from 41 universities.

In the VRS Output orientated DEA model 12 (29% of 41) universities are full efficient, which include Tehran, Shiraz, Mazandaran, Gilan, Qazvin, Golestan, Shahrekord, Lorestan, Gonabad, Shahroud, Sabzevar and Zabol universities.

Only three universities comprise Shahrekord, Gonabad and Shahroud were full efficient in both CRS and VRS input- output orientation. Therefore, the results imply that in the VRS model more DMUs are full efficient.

In cost efficiency DEA model with CRS scale assumption, the three universities, Babel, Shahrekord and Qom were full efficient.

These universities were full efficient in the input orientated DEA model with CRS scale assumption.

The finding reveals that the medical universities were not operating full efficient in the years 2004 to 2008 and so were using resources that they did not actually need. Thus, these universities needed to reduce their inputs to attain a given level of outputs to become efficient.

For Malmquist productivity index, the first year of the study period, viz. 2004, has been taken as the technology reference period. Over the period 2005 to 2008, the average total factor productivity of the sample medical universities has dropped by 30% (Table 5). While efficiency change (CRS TE) increased by 2.7%, pure efficiency change by 2.6% (VRS TE) and scale efficiency has remained at the same level over the same period. Efficiency change (1.027) is a product of the pure efficiency change (1.026) and scale efficiency change (1.001). The technological change of 0.680 indicates that during the period under study, there has been a 32% technological change decrease (Table 3).

Finally, the proposed resource to allocate (PRA) for research activities of medical universities are calculated each year for the next financial year based on allocative efficiency score (Table 4). The proposed research budget is adjusted by the real Producer Price Index (PPI Ch %).

This paper has attempted to demonstrate the capability of the ranking of medical universities to apply annual resources allocation by government.

The findings of this study are expected to provide directions for policy-makers to allocate resources.

Based on the proposed approach, each medical university will react to the relative rank situation. Since it is a matter of survival for them, they will activate themselves to make improvement in their efficiency. The inefficient universities has to direct its fund-raising efforts toward the fulfillment of organizational purposes and conducts them in accordance with the ministry policies by benchmarking the best practice performance in the use of resources amongst peer universities and maximizes its relative rank. Thus, universities are required to establish appropriate mechanisms to analyze its performance with peers one and use the results of these analysis for its performance improvement.

The proposed method for resource allocation may enhance the efficiency and effectiveness of universities resources.

IMPLICATIONS FOR THE FUTURE

The nature of inefficiency is important while designing policies to improve resource allocation. Many factors can influence the relative rankings of such a wide variety of medical universities.

Our aim for the future is to complete the methodology proposed to take into account the factors affecting on inefficiency of the universities.
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<th>Indicator</th>
<th>measure</th>
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<td>Governance and leadership</td>
<td>1. ratio of the plans referred to the Ethics Committee to the total approved plan</td>
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<tr>
<td></td>
<td>2. fulfilment of universities on research priorities</td>
</tr>
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<td></td>
<td>3. existence of an active science and technology system</td>
</tr>
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<td></td>
<td>4. Performance of University publications Commission</td>
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<td>5. quality of cooperate in implementing the annual evaluation of research activities</td>
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<td></td>
<td>6. Performance of Central Laboratory</td>
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<td>7. Cooperation and interaction with the Department of Research and Technology</td>
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<tr>
<td>Empowerment</td>
<td>1. international or regional conferences</td>
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<td>2. National Seminars</td>
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<td>3. obtain of Top Rank from Kharazmi and Razi Festival</td>
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<tr>
<td></td>
<td>4. held workshops for researchers and other stakeholders</td>
</tr>
<tr>
<td>Knowledge production</td>
<td>1. Compiled Books</td>
</tr>
<tr>
<td></td>
<td>2. referred to articles of university in International reference books</td>
</tr>
<tr>
<td></td>
<td>3. innovation, exploration, patents and technology localization</td>
</tr>
<tr>
<td></td>
<td>4. papers presented as lectures or posters in internal conferences</td>
</tr>
<tr>
<td></td>
<td>5. papers presented as lectures or posters in foreign and international conferences</td>
</tr>
<tr>
<td></td>
<td>6. articles published in external and internal research journals not indexed</td>
</tr>
<tr>
<td></td>
<td>7. articles published in scientific journals by index</td>
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Table 2: Efficiency scores of 41 medical universities

<table>
<thead>
<tr>
<th>Year</th>
<th>CRS</th>
<th>VRS</th>
<th>Scale</th>
<th>TE</th>
<th>AE</th>
<th>CE</th>
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<tr>
<td>2004</td>
<td>0.452</td>
<td>0.801</td>
<td>0.551</td>
<td>0.452</td>
<td>0.947</td>
<td>0.424</td>
</tr>
<tr>
<td>2005</td>
<td>0.499</td>
<td>0.881</td>
<td>0.55</td>
<td>0.499</td>
<td>0.944</td>
<td>0.476</td>
</tr>
<tr>
<td>2006</td>
<td>0.49</td>
<td>0.829</td>
<td>0.582</td>
<td>0.49</td>
<td>0.827</td>
<td>0.401</td>
</tr>
<tr>
<td>2007</td>
<td>0.553</td>
<td>0.875</td>
<td>0.628</td>
<td>0.553</td>
<td>0.829</td>
<td>0.453</td>
</tr>
<tr>
<td>2008</td>
<td>0.453</td>
<td>0.802</td>
<td>0.551</td>
<td>0.496</td>
<td>0.931</td>
<td>0.467</td>
</tr>
<tr>
<td>Mean</td>
<td>0.489</td>
<td>0.838</td>
<td>0.572</td>
<td>0.498</td>
<td>0.8956</td>
<td>0.444</td>
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Table 3: Malmquist index of annual means

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<tr>
<th>Year</th>
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<th>tp</th>
<th>pe</th>
<th>se</th>
<th>tfp</th>
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<td>2005</td>
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<td>1.390</td>
<td>1.111</td>
<td>0.929</td>
<td>1.435</td>
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<td>2006</td>
<td>1.051</td>
<td>0.451</td>
<td>0.932</td>
<td>1.128</td>
<td>0.474</td>
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<td>2007</td>
<td>1.187</td>
<td>0.321</td>
<td>1.061</td>
<td>1.119</td>
<td>0.381</td>
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<tr>
<td>2008</td>
<td>0.864</td>
<td>1.064</td>
<td>1.010</td>
<td>0.856</td>
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<tr>
<td>Mean</td>
<td>1.027</td>
<td>0.680</td>
<td>1.026</td>
<td>1.001</td>
<td>0.699</td>
</tr>
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</table>

Table 4: Proposed Resources to allocate (PRA)

<table>
<thead>
<tr>
<th>Year</th>
<th>Bi= Σpra&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>ae&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>PPI Ch %</th>
<th>pra&lt;sub&gt;diff&lt;/sub&gt;</th>
<th>ch% to Bi</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>212681</td>
<td>0.944</td>
<td>9.5</td>
<td>220680</td>
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</tr>
<tr>
<td>2006</td>
<td>395422</td>
<td>0.827</td>
<td>12.2</td>
<td>192844</td>
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</tr>
<tr>
<td>2007</td>
<td>434250</td>
<td>0.829</td>
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<td>298265</td>
<td>-31.3</td>
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<td>2008</td>
<td>404886</td>
<td>0.931</td>
<td>21.8</td>
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<td>Mean</td>
<td>361810</td>
<td>0.8956</td>
<td>15.6</td>
<td>271113</td>
<td>-25.1</td>
</tr>
</tbody>
</table>
An efficiency analysis in higher education system in Turkey

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ABSTRACT
Efficiency in higher education is a significant issue in Turkey as the other countries. The focus of this application is a case study of efficiency within higher education system in Turkey. The study method was developed using data envelopment analysis (DEA) decision theory framework. We considered the problem of determining research, teaching and budgeting efficiencies for state universities. Utilizing data from 47 Turkish state universities, several alternative models were developed using different variables in the identification of top-performing universities.

Keywords: Data Envelopment Analysis, High Education, Turkey

INTRODUCTION
Education is one of the main factors that transform the society to a nation in this informatics era. All values that constitute a nation are the consequences of education in all levels. A nation where to place in a merciless global competition and power area cannot stand without a robust and developed education system. Regarding to sustain our development, restructuring of our education system is crucially essential. Demanding for higher education and university education has been increasing every year. This dynamism is the indication of people’s desire to transform and fit them in the world’s changing conditions. Higher education is a basic need for all levels of society which integrates to participate to a modern life.

Higher education, university education has a key role and vital importance for individuals all over the world. It means a good job with high salary, social esteem, enlightenment and personal satisfaction concisely with a better standard of living for an individual. It means simultaneously increasing productivity, high technology, high competitiveness and the economic growth for a society (Casterona, 2001).

The qualities of higher education outcomes are quite a controversial subject in Turkey. Well education is important for an individual who desires to achieve success. It is relevant to developed universities with graduates of high qualified professionals.

The main function of the university is to contribute to scientific progress. When it is studied on developed country’s higher education systems, two main functions of universities are education and research. They practiced their own models based on the balance of these two functions (Pehlivanoğlu, 2005).

This study is an efficiency measurement of universities in Turkey. Well known technique in this field, Data Envelopment Analysis (DEA) was used for measurement. We considered the problem of determining research, teaching and budgeting efficiencies for state universities. Utilizing data from 47 Turkish state universities, several alternative models were developed using different variables in the identification of top-performing universities.

DEA IN EDUCATION
There have been numerous DEA works in literature on education. Rhodes study in 1978 was also on education, since then many researchers studied this topic. Based on Gattoufi’s et al. (2004) survey there are nearly 100 articles appeared on education and 15 of them on higher education. It was only mentioned here about universities (Gattouifi, Oral and Reisman, 2004).

It is considered that Arrow’s work in 1973 was first studied on efficiency of universities. His work was discussed in a conference in Mexico before it was published. Although this was not related to DEA, it can be seen as an effort on analytical manner (Arrow, 1973). White studied on the management of higher education in 1987 and placed a survey including 146 articles (White, 1987).
Johnes studied on research efficiencies of economics departments in UK in 1988. In this study publications in 20 journals were considered as outputs. It was shown that the differences among departments were related inputs. Obtained results were compared to the results of national ranking institution (Johnes, 1988). Johnes and Johnes obtained efficiencies of departments of economics by using DEA in 1995 (Johnes and Johnes, 1995). Johnes showed performance criteria that were taken as inputs and outputs for universities in 1996 (Johnes, 1996).

Tomkins and Green used DEA for comparing accounting departments in 1988. They defined 6 different DEA models. As a result of their study they mentioned the advantages of DEA on other methods for measuring efficiency (Tomkins and Green, 1998).

Similar study was applied by Kwimbere in the departments of chemistry, physics and mathematics in 1987 (Kwimbere, 1987). Cave et al. conducted a work on performance criteria in higher education in 1988 (Cave, Hanney, Kogan and Trevett, 1988). Rhodes and Southwick applied DEA on 96 state and 54 private universities in 1986. They used 5 inputs and 6 outputs as variables. They found that the average efficiency of private universities is higher than state universities (Rhodes and Southwick, 1986).

Sinuany-Stern et al. measured the efficiency of 21 departments in Ben-Gurion using DEA in 1994 (Sinuany-Stern, Mehrez and Barboy, 1994). Operation expenses and faculty member salaries were inputs; funds, the number of publications, the number of alumni and the number of credits hours were outputs. Seven departments were efficient. Additionally, they tested the effect of variations on inputs and outputs to efficiency scores. It was shown that the decrease of inputs-outputs caused the decrease of efficiency scores. Ahn and Seiford measured with DEA 153 higher education institute which gives PhD degree in 1993 (Ahn and Seiford, 1993). The domain has had 104 state universities and 49 private universities. The main purpose of this study was to determine the effect of different set of variables on efficiency.

Breu and Raab applied the DEA on first 25 national universities which took place in the ranking of US News and World report in 1994 (Breu and Raab, 1994). US News and World Report used 12 performance criteria for ranking such as reputation, the selection of student, funds, and financial resources. Breu and Raab used 4 out of 12 as inputs. The percent of faculties that have PhD, the ratio of faculties to students, the ratio that the expenses for education purposes to students and the median value of SAT/ACT were used as outputs.

Beasley compared departments of physics and chemistry in UK universities in 1990 (Beasley, 1990). Similar study was done by Beasley in 1995 for measuring research and education efficiencies of the same departments (Beasley, 1995).

These studies are some remarkable works as published articles. But there are numerous master thesis and dissertations on higher education also.

AN APPLICATION OF DEA ON STATE UNIVERSITIES IN TURKEY

The benefits hoped from this study can be summarized as the following:

Central Authority (Higher Education Institute, HEI and Government) can use the results for evaluation of their policies on Higher Education.

Universities can use the results for determining their positions relatively.

Society needs can be taken into account for better services.

This study can be improved and used as a sample study for similar countries.

This method can be used as a decision support system.

Inputs and Outputs

14 different variables were used in this study and as showed in Table 1. These variables were used in four different scenarios and three different efficiency scores.

Monetary value consisted of some detail items were shown in Table 2.

Decisions Making Units (DMU)

There are two groups of universities in the analysis as DMU. 1992 was a milestone for universities in Turkey, due to a significant policy changing in that year. Private university was not established then in Turkey and many state universities were divided into new universities. Based on this distinction universities were grouped before and after 1992 (Baysal, 2005).

Based on BCC model four different set of variables, two groups of universities and two different years were selected and was applied DEA. The results of analysis were shown in Table 3 and Table 4.

CONCLUSION

It was aimed to measure efficiencies of universities and to rank the universities with respect to obtain scores. Students, faculties,
institutions, governments and public in general have different reasons to deal with the ranking of universities, but multiple criteria ranking procedure is unavailable for this purpose. So, it was thought that this study helps to define the universities in Turkey based on academic and research efficiency scores.

When it was discussed about the objectivity of ranking it cannot be said that certain judgments are only based on quantitative variable. So, any ranking can be conflicted. Researchers should pay attention to the results of such studies.

It is too difficult to rank the universities with respect to efficiency scores of academic, research and budget. Two years was used as period of time, 2003 and 2004. Universities were divided into two groups; established before and after 1992. 47 state universities handled in the study. There are some exceptions such as medical science, social science faculties and the biggest distant learning university.

**IMPLICATIONS FOR THE FUTURE**

After the verification and validation by experts, the method can be improved and ranking can be re-done. Different distinction can be done such as engineering, medical, social science etc. Method could be used as a decision support system. Uncertainty can be included in the study.

**REFERENCES**


**ACKNOWLEDGMENTS**

This study is partially based on corresponding author’s dissertation. This dissertation was supported by Republic of Turkish, Prime Ministry State Planning Organization (DPT). It was granted as the number of 2003K120470-21.
Table 1: Inputs and outputs

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<th>2006</th>
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</thead>
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<td>STAFF EXPENSES</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>OTHER CURRENT EXPENDITURE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>TRANSFERS</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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Table 2: Monetary value details

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<tr>
<td>03</td>
<td>TRANSFERS</td>
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Table 3: The efficiency scores for the years 2003-2004 and the universities established before 1992

<table>
<thead>
<tr>
<th>DMU</th>
<th>M1 Efficiency Scores</th>
<th>M2 Efficiency Scores</th>
<th>M3.1 Efficiency Scores</th>
<th>M3.2 Efficiency Scores</th>
<th>M1 Efficiency Scores</th>
<th>M2 Efficiency Scores</th>
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<td>Akdeniz</td>
<td>39.83</td>
<td>100</td>
<td>77.78</td>
<td>72.87</td>
<td>41.98</td>
<td>97.28</td>
<td>96.89</td>
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<td>Ankara</td>
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<td>81.7</td>
<td>78.73</td>
<td>100</td>
<td>100</td>
<td>73.19</td>
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<td>Atatürk</td>
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<td>93.6</td>
<td>84.92</td>
<td>84.35</td>
<td>72.42</td>
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<td>72.24</td>
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<td>100</td>
<td>97.09</td>
<td>78.31</td>
<td>100</td>
<td>100</td>
<td>85.92</td>
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<tr>
<td>Cumhuriyet</td>
<td>74.3</td>
<td>85.08</td>
<td>55.32</td>
<td>41.09</td>
<td>55.2</td>
<td>83.14</td>
<td>59.11</td>
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<td>Dicle</td>
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<td>43.84</td>
<td>100</td>
<td>64.29</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>İnönü</td>
<td>81.43</td>
<td>95.75</td>
<td>92.11</td>
<td>66.54</td>
<td>73.69</td>
<td>96.25</td>
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### Table 4: The efficiency scores for the years 2003-2004 and the universities established after 1992

<table>
<thead>
<tr>
<th>DMU</th>
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<tr>
<td></td>
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<td>M2</td>
</tr>
<tr>
<td></td>
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<td>Efficiency Scores</td>
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<tr>
<td>İstanbul</td>
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<td>100</td>
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<td>İstanbul Teknik</td>
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<td>78,83</td>
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<td>Karadeniz Teknik</td>
<td>100</td>
<td>84,48</td>
</tr>
<tr>
<td>Marmara</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Ondokuz Mayıs</td>
<td>74,94</td>
<td>91,7</td>
</tr>
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<td>Orta Doğu Teknik</td>
<td>98,29</td>
<td>85,32</td>
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<tr>
<td>Selçuk</td>
<td>97,44</td>
<td>100</td>
</tr>
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<td>Trakya</td>
<td>56,38</td>
<td>46,8</td>
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<td>Uluadag</td>
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<td>43,82</td>
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<tr>
<td>Yüzüncü Yıl</td>
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An algorithm for evaluating the performance of higher education organizations in Egypt using a stochastic DEA

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ABSTRACT

Data Envelopment Analysis (DEA) is a powerful quantitative tool that provides a means to obtain useful information about efficiency and performance of firms, organizations, and all sorts of functionally similar, relatively autonomous operating units which named Decision Making Units (DMU). Usually the investigated DMUs are characterized by a vector of multiple inputs and multiple outputs. Education was one of the earliest applications of DEA; it served to test and validate it as a tool for analysis and decision support. Education was an ideal introduction for DEA because it typically deals with comparisons of many similar and autonomous ‘non-profit’ entities described by inputs and outputs. The main concern in this paper is to develop an algorithm to help any educational organization for evaluating their performance, the developed algorithm based on the DEA model and working in a stochastic environment.  

Keywords: Stochastic DEA, High Education Performance

INTRODUCTION

Measuring the efficiency of educational institutions has become an important and appealing research area in recent years. Some Higher Education (HE) sectors in many countries around the world obtain at least some of their income from public funds, thus making it essential, in the interest of accountability, to measure the efficiency of the institutions which comprise these sectors. Those public Higher Education Institutions (HEIs) are “non-profit” making organizations; thus there is an absence of output and input prices while producing multiple outputs from multiple inputs. This poses a challenge in measuring their efficiencies.

An assortment of methodological approaches have been employed in an effort to resolve the problem of efficiency measurement in this context, from early studies which use ordinary least-squares (OLS) regression methods, to more recent studies which use frontier methods such as DEA. DEA, as originally proposed by Charnes et al. in 1978, is a non parametric frontier estimation methodology based on linear programming for measuring relative efficiencies and performance of a collection of related comparable entities called Decision Making Units (DMUs) which transform multiple inputs into multiple outputs. It is a powerful quantitative tool that provides a means to obtain useful information about efficiency and performance of firms, organizations, and all sorts of functionally similar, relatively autonomous operating units. DEA’s domain can be any group of many entities characterized by the same set of attributes. The objective of a DEA study is to assess the efficiency of each DMU in relation to its peers. The result of a DEA study is a classification of all DMUs as either “efficient” or “inefficient.”

In Egypt, as for the rest of the world, there has been an interest in assuring the quality of HE and increasing the efficiency of the HEI. Most of the HE institutions are public, “non-profit” organizations. These organizations have multiple inputs and outputs; therefore the most appropriate technique for studying the efficiencies of these organizations is through DEA. Most of the previous DEA applications in the HE DMUs were deterministic in their inputs and outputs although some of the variables may have a random nature. Therefore in this study, we are looking forward to apply a Stochastic DEA since some of our input parameters have a random nature.
Accordingly, we are proposing a modification to the existing stochastic DEA model which is chance constrained output oriented in order for the DEA model to be chance constrained input oriented.

The rest of the paper is organized as follows. The coming section investigates some DEA applications in education. The third section discusses the methodology of the general DEA model and that for the stochastic DEA model. The fourth section includes the proposed stochastic chance constrained input oriented DEA model. We will then briefly introduce a framework for applying the model to Egyptian public HEI. The paper will end with the customary conclusions and implications for the future.

**DEA APPLICATIONS IN EDUCATION**

In recent years, several studies have undertaken analysis of efficiency in educational facilities using the DEA methodology. In the coming few paragraphs we will discuss a few these DEA applications. Each study differs in its scope; meaning the definitions of DMUs being subject to analysis. Some defined the DMU as being the universities, or specifically confined it to public universities, while others referred to departments within HE faculties, or to different educational school levels.

Among those who attempted to measure the efficiency among universities were Abbott and Doucouliagos (2001) and Johnes and Yu (2008). Abbott and Doucouliagos (2001) used DEA to estimate technical and scale efficiency of individual Australian universities. Various measures of output and inputs were used. The results showed that regardless of the output–input mix, Australian universities as a whole recorded high level of efficiency relative to each other. In this study four inputs were used which were the total number of academic staff (full-time equivalent), the number of non-academic staff (full-time equivalent), expenditure on all other inputs other than labor inputs and the value of non-current assets. This is a rough proxy for the university’s capital stock. The output included the number of equivalent full-time students (EFTS), the number of post-graduate and under-graduate degrees enrolled, as well as the number of post-graduate degrees conferred and the number of under-graduate degrees conferred. EFTS is arguably the better measure.

On the other hand, Johnes and Yu (2008) applied the DEA on 109 Chinese public universities in 2003 and 2004. The authors in this study focused on examining the relative efficiency in the production of research among these universities. Input variables reflected staff, students, capital and resources; while the output variables measured the impact and productivity of research. The study showed that the mean efficiency is just over 90% when all input and output variables are included in the model, and this falls to just over 80% when student-related input variables are excluded from the model. The rankings of the universities across models and time periods were highly significantly correlated. Further investigation suggested that mean research efficiency was higher in comprehensive universities when compared to specialist universities, and in universities located in the coastal regions compared to those in the western regions of China. The former result offered support for the recent merger activity which has been taking place in Chinese higher education.

At the departmental level, Martín (2006) applied the DEA methodology to assess the performance of departments at the University of Zaragoza (Spain). The selected indicators were related to both the teaching and the research activity of these departments. The input indicators included units of measurement that represent those factors employed in delivery of the teaching and research services (human resources, financial resources, and material resources). Output indicators measured the yield or the level of activity of programs and services (teaching indicators, and research indicators). The results revealed that departments that carry out these activities more efficiently according to the variables included in each analysis. Finally, he discussed the existence of differences in the strengths and weaknesses among departments covering different areas, suggesting several initiatives aimed at improving their performance in the light of current Spanish higher education reform.

Two recent studies attempting to investigate efficiencies at pre-university levels included the study by Conroy and Arguea (2007) applied to elementary schools while Kantabutra (2009) addressed public upper-secondary school. Conroy and Arguea (2007) used DEA to analyze whether elementary schools in Florida are operating at an efficient level, and to attempt to explain any inefficiency. The motivation for this analysis came from the recent state and federal level educational initiatives designed to improve school accountability and reduce class sizes. The input data used in the analysis was pupil–teacher ratios, free-lunch eligibility and the number of students. The output variable is a proxy for school performance approximated by the mean Florida Comprehensive Assessment
Test (FCAT) score for the school (as it is standard in studies of this type). The study indicated that while Florida elementary schools were not operating at efficient levels (with regional mean inefficiency), they compared favorably to published results for other states.

Kantabutra (2009) examined urban-rural effects on public upper-secondary school efficiency in northern Thailand using the DEA. Urban-rural effects were examined through a Mann-Whitney nonparametric statistical test. Results indicated that urban schools appear to have access to and practice different production technologies than rural schools, and rural institutions appeared to operate less efficiently than their urban counterparts. In addition, a sensitivity analysis, conducted to ascertain the robustness of the analytical framework, revealed the stability of urban-rural effects on school efficiency. The author recommended that any policy initiative to improve school efficiency should thus take varying geographical area differences into account, viewing rural and urban schools as different from one another. Moreover, policymakers might consider shifting existing resources from urban schools to rural schools, provided that the increase in overall rural efficiency would be greater than the decrease, if any, in the city. Data used in the analysis came from the Ministry of Education (MOE), Student Loans Fund, National Statistical Office (NSO), and the schools themselves (455 public general upper-secondary schools in northern Thailand in 2003). The inputs were teacher-student ratio, proportion of students not from low-income families and teaching aide–student ratio. While the outputs were passing, graduates and national test score (SAT).

In all the previous applications, and to our knowledge, in most of the DEA applications related to education, the DEA applied in measuring the efficiency was deterministic, i.e. the input and output variables are deterministic in nature. This is not necessarily true for all variables; some of these variables could be random by nature. Examples of variables that could be stochastic include human resources, impact and productivity of research, number of graduates per term, and the number of projects acquired by the DMU from industry. In this research, we are looking forward applying the stochastic DEA in the education sector.

**GENERAL MODEL OF DEA**

The basic DEA model for n DMUs with m inputs and (s) outputs was first proposed by Charnes, Cooper and Rhodes (CCR) (1978). The model determines the relative efficiency score for the different DMUs. The model depends on maximizing a production function estimated by DEA. This function is a deterministic frontier. For any inputs, the value of the DEA estimate defines the maximum output producible from inputs under all circumstances. On the other hand, for any outputs, the value of the DEA estimate defines the minimum input producing a given output under all circumstances. In this sense, it is comparable to the parametric frontier with one-sided deviations estimated using mathematical programming methods. Therefore, the relative efficiency for the \((p^*)\) DMU can be obtained according to the following:

\[
\begin{align*}
\text{Max } Z_p &= \sum_{k=1}^{s} \frac{\sum_{j=1}^{m} v_k y_{kp}}{\sum_{j=1}^{m} u_j x_{jp}} \\
\text{s.t. } & \sum_{k=1}^{s} v_k y_{ki} \leq 1 \quad \forall i \\
& v_k, u_j \geq 0 \quad \forall k, j
\end{align*}
\]

where:

- \(k = 1\) to \(s\) (no. of outputs);
- \(j = 1\) to \(m\) (no. of inputs);
- \(i = 1\) to \(n\) (no. of DMUs);
- \(y_{ki}\) = amount of output \(k\) produced by DMU \(i\);
- \(x_{ji}\) = amount of input \(j\) utilized by DMU \(i\);
- \(v_k\) = weight given to output \(k\);
- \(u_j\) = weight given to input \(j\).

The fractional programming shown in Model (1) can be reduced to LPP as follows:

\[
\begin{align*}
\text{Max } Z_p &= \sum_{k=1}^{s} v_k y_{kp} \\
\text{s.t. } & \sum_{j=1}^{m} u_j x_{jp} = 1 \\
& \sum_{k=1}^{s} v_k y_{ki} - \sum_{j=1}^{m} u_j x_{ji} \leq 0 \quad \forall i \\
& v_k, u_j \geq 0 \quad \forall k, j
\end{align*}
\]

This latter model is known as the output oriented maximization DEA model. The main idea of the model is to maintain the same levels for the inputs while trying to maximize the outputs given operating under the same environment. The efficiency score of \(n\) DMUs is obtained by running the above LPP \(n\) times.

According to the assumptions relating the change in outputs as a result of the change in inputs, the DEA model can be classified as having either constant returns to scale (CRS) or variable returns to scale (VRS). The resulting models will be the subject of the coming few paragraphs.
THE CONSTANT RETURNS TO SCALE (CRS) MODEL:

Output oriented:
Under this model the outputs are not affected by the size of the DMU; rather they change in direct proportion to the change in inputs assuming that the scale of operation does not influence efficiency. The model is as follows:

\[
\max \phi \\
\text{s.t. } \sum_{i=1}^{n} \lambda_i y_i \geq \phi y_p \\
\sum_{i=1}^{n} \lambda_i x_i \leq x_p \\
\lambda_i \geq 0, (i = 1, 2, ..., n)
\]

where:
\( \phi \) = efficiency score;
\( k \) = 1 to \( s \) (no. of outputs);
\( j \) = 1 to \( m \) (no. of inputs);
\( i \) = 1 to \( n \) (no. of DMUs);
\( y_i \) = amount of output produced by DMU\(_i\);
\( x_i \) = amount of input utilized by DMU\(_i\);
\( \lambda_i \) = weight given to DMU\(_i\).

Input oriented:
\[
\min \theta \\
\text{s.t. } \sum_{i=1}^{n} \lambda_i x_i \leq \theta x_p \\
\sum_{i=1}^{n} \lambda_i y_i \geq y_p \\
\lambda_i \geq 0, (i = 1, 2, ..., n)
\]

where:
\( \theta \) = efficiency score;

Therefore, in the CRS model the output and input oriented measures of efficiency are equal.

THE VARIABLE RETURNS TO SCALE (VRS) MODEL:

Output oriented:
Under this model, changes in outputs are not necessarily proportional to the changes in the inputs. This is a more preferable alternative to the CRS model. Therefore, the only modification done to the above model is adding a constraint to ensure the total weights (\( \lambda_i \)) add up to 1. Accordingly, the model will be as presented below.

\[
\max \phi \\
\text{s.t. } \sum_{i=1}^{n} \lambda_i y_i \geq \phi y_p \\
\sum_{i=1}^{n} \lambda_i x_i \leq x_p \\
\lambda_i \geq 0, (i = 1, 2, ..., n)
\]

\[
\sum_{i=1}^{n} \lambda_i \leq 1
\]

\[
\lambda_i \geq 0, (i = 1, 2, ..., n)
\]

Input oriented:
\[
\min \theta \\
\text{s.t. } \sum_{i=1}^{n} \lambda_i x_i \leq \theta x_p \\
\sum_{i=1}^{n} \lambda_i y_i \geq y_p \\
\sum_{i=1}^{n} \lambda_i = 1 \\
\lambda_i \geq 0, (i = 1, 2, ..., n)
\]

In the VRS model the output and input oriented measures of efficiency scores are not equal for inefficient units.

STOCHASTIC DEA MODEL

Land et al. (1993) modified the standard DEA model to measure technical efficiency in the presence of random variation in the outputs produced from given inputs. Their chance-constrained DEA model builds on the method of chance-constrained programming (CCP) and is as follows:

The chance constrained output oriented VRS DEA model for DMU\(_p\):

\[
\max \phi \\
\text{s.t. } pr\left( \sum_{i=1}^{n} \lambda_i y_i \geq \phi y_p \right) \geq (1 - \alpha) \\
\sum_{i=1}^{n} \lambda_i x_i \leq x_p \\
\sum_{i=1}^{n} \lambda_i = 1 \\
\lambda_i \geq 0, (i = 1, 2, ..., n)
\]

Assume that each output \( y_k \) is normally distributed with mean \( \mu_p \) and variance \( \sigma^2_p \), and also assume \( \text{cov}(y_k, y_l) = 0 \). Accordingly, we can define the random variable:

\[
u = \sum_{i=1}^{n} \lambda_i y_i - \phi y_p
\]

with mean:

\[
E(u) = \sum_{i=1}^{n} \lambda_i \mu_i - \phi \mu_p \equiv \mu_u
\]

and variance:
\[ \text{var}(u) = \sum_{i=1}^{n} \lambda_i^2 \sigma_i^2 + (\lambda_p - \phi)^2 \sigma_p^2 = \sigma_u^2 \]

Since \( y_k \)'s are normally distributed, so is variable \( u \),

\[ z = \frac{u - \mu_u}{\sigma_u} \]

Hence,

\[ \text{pr} \left\{ \sum_{i=1}^{n} \lambda_i y_i \geq \Phi y_p \right\} = \text{pr}(u \geq 0) = \text{pr} \left\{ z \geq -\frac{-\mu_u}{\sigma_u} \right\} \]

Given the symmetry property of normal distribution,

\[ \text{pr} \left\{ z \geq -\frac{-\mu_u}{\sigma_u} \right\} = \text{pr} \left\{ z \leq \frac{-\mu_u}{\sigma_u} \right\} = \Phi \left( \frac{-\mu_u}{\sigma_u} \right) \]

where \( \Phi(\cdot) \) is the cumulative standard distribution function.

The random inequality restriction in the chance constrained DEA problem (CCDEAP) can be replaced by the equivalent restriction.

\[ \Phi \left( \frac{\mu_u}{\sigma_u} \right) \geq (1 - \alpha) \]

\[ \Phi \left( \frac{\mu_u}{\sigma_u} \right) \geq \Phi(e) \]

\( \Phi(e) \rightarrow \) from the table of standard normal distribution.

\[ \mu_u = e \sigma_u \]

That is,

\[ \sum_{i=1}^{n} \lambda_i \mu_i - \phi \mu_p \geq \sum_{i=1}^{n} \lambda_i^2 \sigma_i^2 + (\lambda_p - \phi)^2 \sigma_p^2 \]

The revised DEA problem can be specified as:

\[ \text{Max } \Phi \]

\[ s.t. \sum_{i=1}^{n} \lambda_i \mu_i - \phi \mu_p \geq \sum_{i=1}^{n} \lambda_i^2 \sigma_i^2 + (\lambda_p - \phi)^2 \sigma_p^2 \]

\[ \sum_{i=1}^{n} \lambda_i x_i \leq x_p \]

\[ \sum_{i=1}^{n} \lambda_i = 1 \]

\[ \lambda_i \geq 0, (i = 1, 2, \ldots, n) \]

**MODIFIED INPUT ORIENTED STOCHASTIC DEA MODEL**

In this section we present a modification to the standard DEA model (Deterministic DEA) in order to measure technical efficiency in the presence of random variation in the inputs. Our chance-constrained DEA model which is also based on the method of chance-constrained programming (CCP) is provided below.

The chance constrained input oriented VRS DEA model for DMU p:

\[ \text{Min } \theta \]

\[ \text{s.t. } \text{pr} \left\{ \sum_{j=1}^{m} \lambda_j x_j \leq \theta x_p \right\} \geq (1 - \alpha) \]

\[ \sum_{k=1}^{s} \lambda_k y_k \geq y_p \]

\[ \sum_{i=1}^{n} \lambda_i = 1 \]

\[ \lambda_i \geq 0, (i = 1, 2, \ldots, n) \]

Assume that each input \( x_j \) is normally distributed with mean \( \mu_{p_j} \) and variance \( \sigma_{p_j}^2 \). Also, assume that \( \text{cov}(x_j, x_p) = 0 \). We can then define the random variable \( u \):

\[ u = \sum_{j=1}^{m} \lambda_j x_j - \theta x_p \]

with mean:

\[ E(u) = \sum_{j=1}^{m} \lambda_j \mu_j - \theta \mu_p \equiv \mu_u \]

and with variance:

\[ \text{var}(u) = \sum_{j=1}^{m} \lambda_j^2 \sigma_j^2 + (\lambda_p - \theta)^2 \sigma_p^2 \equiv \sigma_u^2 \]

Since the \( x_j \)'s are normally distribution, so does variable \( u \).

\[ z = \frac{u - \mu_u}{\sigma_u} \]

which has standard normal distribution. Hence

\[ \text{pr} \left\{ \sum_{j=1}^{m} \lambda_j x_j \leq \theta x_p \right\} = \text{pr}(u \leq 0) = \text{pr} \left\{ z \leq -\frac{-\mu_u}{\sigma_u} \right\} \]

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Given the symmetric property of the normal distribution, then:

\[ \Pr \left\{ x \leq \frac{-\mu_u}{\sigma_u} \right\} = \Pr \left\{ x \geq \frac{-\mu_u}{\sigma_u} \right\} = 1 - \varphi \left( \frac{\mu_u}{\sigma_u} \right) \]

where \( \varphi (\cdot) \) is the cumulative standard distribution function.

The random inequality restriction in the chance constrained DEA problem (CCDEAP) can be replaced by the equivalent restriction:

\[
1 - \varphi \left( \frac{\mu_u}{\sigma_u} \right) \geq (1-\alpha) \\
-\varphi \left( \frac{\mu_u}{\sigma_u} \right) \geq -\alpha \\
\varphi \left( \frac{\mu_u}{\sigma_u} \right) \leq \alpha \\
\varphi \left( \frac{\mu_u}{\sigma_u} \right) \leq \varphi (\epsilon)
\]

\( \varphi (\epsilon) \) is obtained from the table of standard normal distribution. Hence,

\[ \mu_u \leq \sigma_u \]

That is,

\[ \sum_{j=1}^{m} \lambda_j \mu_j - \theta \mu_p \leq \epsilon \sum_{j=1}^{m} \lambda_j^2 \sigma_j^2 + (\lambda_p - \theta) \sigma_p^2 \]

Therefore, the revised DEA problem can be specified as:

\[
\begin{align*}
\text{Min } \theta \\
\text{s. t. } & \sum_{j=1}^{m} \lambda_j \mu_j - \theta \mu_p \\
& \leq \epsilon \sum_{j=1}^{m} \lambda_j^2 \sigma_j^2 + (\lambda_p - \theta) \sigma_p^2 \\
& \sum_{k=1}^{s} \lambda_k y_k \geq y_p \\
& \sum_{i=1}^{n} \lambda_i = 1 \\
& \lambda_i \geq 0, (i = 1, 2, \ldots, n)
\end{align*}
\]

**FRAMEWORK FOR APPLICATION**

The main aim of this study was to measure and rank the efficiency of some public Egyptian HEIs. The scope is to perform a comparative study in three main directions:

- Among faculties within Cairo University,
- Among Faculties of Computers and Information across Egypt, and
- Between a number of leading public universities in Egypt.

In order to proceed with this study, we had to identify the main input and output variables on which the analysis will be based and that are of relevance for the Egyptian HEIs. Some of the input variables that have been identified relate to the human resources at the different HEIs, number of programs offered, scientific and industrial research, material resources, space utilization and capacities, among others. With respect to the output variables, these include the number of graduates, research productivity, number of finished postgraduates, and number of publications. Unfortunately, this data is not readily available. Therefore, the research team has designed a questionnaire in order to collect the data for the current academic year (2009/2010).

Given this database, we will analyze the data, and apply the DEA model. At this stage, the intention of the research team is to apply the deterministic DEA model and then the stochastic DEA model taking into consideration those input variables with random nature. Accordingly, a comparison for the efficiency ranking will be performed. The results will be published in upcoming papers.

**CONCLUSION**

Data Envelopment Analysis is an excellent tool for the evaluation of performance. It has the advantage over alternative methods that it can be applied in a multiple inputs and outputs production context.

A review of DEA applications in the HE sector revealed that most (if not all) has used the standard deterministic DEA model. They based their selection on the fact that the input and output variables are deterministic by nature, although some might be stochastic in nature. A new model from the standard DEA model to handle random input variables was developed. This model will be applied to HEIs in Egypt.

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DEA method in efficiency assessment of public higher education institutions

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ABSTRACT

Public higher education sector in Poland is under a growing pressure to increase efficiency and to improve the quality of its activities. Limited financial resources as well as the detailed regulations and supervision of their spending are the important features of the public higher education sector. Another important and debated issue is the division of public money among higher education institutions (HEI). It is therefore crucial to create stimuli for the rational management of public funds by HEI and for the quality improvement of HEI services. One of the proposed ways to achieve that is the comparative efficiency assessment of the HEI activities. Such an assessment may be treated as a substitute of market competition by setting clear reference points for HEI.

This paper describes a comparative efficiency study of 19 Polish technical universities. Detailed analysis of potential input, output and environmental variables describing the HEI efficiency model has been carried out. The study uses the CCR-CRS output-oriented DEA model. It was assumed that HEI have more influence on their achieved results than on the amount of their resources. The economies of scale have also been studied in relation to the efficiency achieved. Sensitivity of the model to data errors has been tested.

Keywords: Data Envelopment Analysis (DEA), higher education, efficiency

INTRODUCTION

Public higher education sector is under the growing pressure to increase efficiency and to improve the quality of its activities. Expectations of the state, society, media and other stakeholders stimulate universities to manage their resources more effectively and also cause increased transparency in state funding of the higher education sector. Another factor contributing to that phenomenon is the necessity to conform to the European Union standards in the Polish higher education system.

It is more and more frequent in the public sector to apply the corporate standards and models of management. However, the specificity of the public sector often makes it impossible to copy those patterns directly. Public sector is characterized, among others, by the complexity of the sector’s environment and its instability (frequent political and legal changes), by the multitude and ambiguity of goals and by the variety of stakeholders with contradicting expectations. Another factor is the limited amount of public funds which are distributed and supervised according to detailed regulations. Furthermore, activities of public sector institutions are not subjects to high competitive pressure and are not profit-oriented as is the case with their private counterparts. Additionally, there is a lack of objective criteria for the assessment of the sector. This leads to the problem of state money distribution that has nothing to do with efficiency of its management by the public institutions.

It is therefore crucial to create stimuli for rational management of public funds and for the improvement in the quality of services offered by the public sector academic institutions. One of the well-tried ways to do that is the systematic comparative study of the efficiency of public sector units (Nazarko et al., 2008, 2009). Such an assessment defines reference points (benchmarks) for studied activities. It may be therefore treated as a substitute for competition and it may contribute to the more efficient allocation of public funds, to greater care for the efficiency of conducted processes, to the higher quality of the offered services and to the improvements in public institutions management.

Data Envelopment Analysis (DEA) method occupies an important place in the comparative efficiency studies in the public sector worldwide (Chalos and Cherian, 1995; Odeck, 2005). It is also applied in the higher education sector because outcomes of DEA analysis may provide valuable information supporting HEI management. DEA does not just enable the identification of areas requiring improvement but also describes the development possibilities in those areas. Moreover, it allows answering questions concerning HEI strengths and weaknesses, the mode of fund allocation among HEI organizational units, or the optimal size of these units.

Examples of DEA application in the area of higher education has been described in works such as Leitner, 2007; Taylor and Harris, 2004;
In Austria (Leitner, 2007), studies with the use of DEA allowed to assess the efficiency of natural sciences and engineering departments in HEI. Models developed there consisted of two input variables (number of academic teachers and floor area of the department) and 12 output variables (extramural grants, ratio of completed projects to the total academic staff, number of projects completed by the department, number of exams, diploma students, monographs, reports, presentations and other publications, number of patents obtained, and graduated PhD students). According to the researchers, it has been demonstrated that DEA method surpasses traditional approaches based on simple calculation of indicators. Application of DEA method does not only allow determining department’s efficiency but also helps specify improvement possibilities of each one.

In South Africa, 10 out of 21 public HEI were studied from the perspective of their efficiency during a period of 4 years (Taylor and Harris, 2004). Taking into account the limitations of the method, seven models were tested. In each model the output variables consisted of the number of graduates and the indicators characterizing HEI engagement in research. Input variables varied in each model and included: total costs, financial resources, number of students and employees. Demonstrated efficiency differences between HEI have allowed specifying four main factors determining HEI efficiency: increase in the number of students, quality of recruited student, quality of academic staff and the level of fixed costs.

In Canada, efficiency of 45 HEI has been studied (McMillan and Datta, 1998). Three types of Canadian HEI were specified: comprehensive with medical school, comprehensive without medical school and primarily undergraduate. 9 different models were used in the analysis. Output variables included among others: number of students sorted by the field of studies, number of sponsored research grants, etc. Input variables consisted of the number of academic staff with the division between exact science and humanities, number of employees obtaining research grants, etc. Authors stress the utility of DEA method as a benchmarking tool applied by HEI. They recommend that DEA is used to study more homogenous administrative units such as departments.

Another instance of DEA application in the higher education sector are the efficiency and productivity studies of more than 500 English in-service training institutions during the period of 5 years (Bradley et al., 2006). Five main types of studied units were specified: general/tertiary colleges, Sixth Form Colleges, Specialist Colleges, Specialist Designated Institutions and External Institutions. Variables describing the number and the quality of students and teachers were used as input variables for the DEA model. Student achievements measured as the number of students continuing their education and the number of attained qualifications was treated as output variables. An environmental variable describing the socio-economic situation of students was also taken into account.

This paper describes the application of DEA method in the comparative efficiency study of 19 Polish technical universities.

**CHARACTERISTICS OF THE SECTOR**

Higher education in Poland is divided into two sectors: public and private. There are approx. 500 HEI in total functioning in these two sectors, 130 of them are public institutions. Almost all PhD granting HEI (approx. 100), including all of the 19 technical universities, are public.

Government budget subsidies are the primary funding source for the public HEI. Subsidies are assigned for education of full time undergraduate and masters’ students, education of full time PhD students, salaries of academic staff and facility maintenance. The size of subsidy depends on: (i) number of students (including different weights given to various fields of study); (ii) number of PhD students (with different weights assigned to various academic specialties); (iii) number of teaching and research staff (with different weights assigned to their seniority and formal qualifications); (iv) number of research grants obtained in a given year; (v) number of licenses to award PhD and higher doctorate degrees; (vi) student exchanges with foreign universities.

There are about 1,930,000 students (year 2008) in different types of HEI in Poland. 1,270,000 in public HEI and 660,000 in private HEI. About 930,000 are full time students (public: 810,000, private: 120,000) and about 1,000,000 are part-time students (public: 460,000, private: 540,000). Technical universities provide education for 320,000 students (full-time programs: about 220,000, part-time programs: about 100,000). All HEI are the primary workplace for more than 62,000 academic teachers, including 13,000 tenured professors. Technical universities employ 15,500 academic teachers, including 2,900 tenured professors.

In 2008 government budget subsidy for public HEI amounted to about US$ 3 billion, out of which US$ 730 million went to technical universities. There is a general consensus among scientists and politicians that the current level of financing is far from sufficient. However, the costs of
maintaining the public higher education sector are increasingly difficult to bear even for rich countries’ budgets (Johnes, 2006, Ønsel et al., 2008). Similarly to other public institutions, HEI are under the growing pressure to increase the efficiency in spending of public resources, to actively search for alternative funding sources and to compete for a good position in the educational market (Higher Education..., 2009).

According to DEA methodology, in order to analyze the efficiency of Polish technical universities it was assumed that each university (DMU – Decision Making Unit) may be characterized by its initial assets (system input), effects (results, system output) and transformation processes which convert assets into effects (taking into account the impact of the environment which remains outside of university’s control). Hence, in case of DMUs characterized by a certain amount of input and results, efficiency may be defined as the relation between the weighted sum of results to the weighted sum of inputs, taking into account the impact of the environment (Fig. 1).

Figure 1: DEA method concept

Source: Derived by author, based on Thanassoulis, 2003.

DATA ANALYSIS AND SELECTION FOR THE MODEL

Comparison of teaching and scholarly achievements of universities is complex and evokes considerable amount of controversy. It is often argued that such a comparison is subjective and lacks a clear framework. DEA has its limitation too and cannot pretend to be a universal and fully objective method. However, its conscious use may prove to be a source of valuable information on the HEI performance. The possibility to measure and compare values expressed in different units is an important advantage of DEA method. Variables selection is the primary and often the most difficult aspect of DEA application in DMUs comparative analysis. This paper presents two essential stages in the variables selection process: the merit-related and the statistics-related stage.

15 variables concerning the financial, staff, organizational and qualitative aspects of university performance were analyzed. The merit-related analysis resulted in the selection of 5 input variables, 8 output variables and 2 environmental variables. Table 1 presents the set of analyzed variables with their description.

Table 1: Model variables

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Output variables</th>
<th>Environment variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>I₁ Government budget subsidy [PLN]</td>
<td>O₁ Weighted No of full-time students</td>
<td>E₁ Population size of the city where the university is located</td>
</tr>
<tr>
<td>I₂ No of academic teachers</td>
<td>O₂ Weighted No of full-time PhD students</td>
<td>E₂ Percentage of students with need-based financial aid</td>
</tr>
<tr>
<td>I₃ No of other employees</td>
<td>O₃ Percentage of students studying abroad</td>
<td></td>
</tr>
<tr>
<td>I₄ No of licenses to award PhD degrees</td>
<td>O₄ Percentage of international students</td>
<td></td>
</tr>
<tr>
<td>I₅ No of licenses to award doctorate degrees</td>
<td>O₅ Percentage of students with university scholarships</td>
<td></td>
</tr>
<tr>
<td>I₆ Government ministry scholarships</td>
<td>O₆ Percentage of students with government ministry scholarships</td>
<td></td>
</tr>
<tr>
<td>I₇ Employers’ preference for hiring alumni</td>
<td>O₇ Parametric assessment of scholarly achievements of faculty</td>
<td></td>
</tr>
<tr>
<td>I₈ Percentage of students with university scholarships</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I₉ Student satisfaction with quality of teaching</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I₁₀ Percentage of students studying abroad</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I₁₁ Percentage of students with government ministry scholarships</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I₁₂ No of full-time students</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I₁₃ Percentage of students with need-based financial aid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s own elaboration.

In order to detect relations between the variables, a correlation analysis was carried out in each group of variables.

All input variables are strongly and significantly correlated with each other (Table 2). The strongest correlation of all input variables may be observed with the variable I₁ (government budget subsidy obtained by a university). Thus, this variable is a very good representative of all input variables analyzed initially. It is therefore accepted in the model as a variable representing input.

Table 2: Correlation coefficient of input variables

<table>
<thead>
<tr>
<th>I₁</th>
<th>I₂</th>
<th>I₃</th>
<th>I₄</th>
<th>I₅</th>
<th>I₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.984</td>
<td>0.982</td>
<td>0.953</td>
<td>0.988</td>
<td></td>
</tr>
<tr>
<td>0.984</td>
<td>1.000</td>
<td>0.968</td>
<td>0.958</td>
<td>0.968</td>
<td></td>
</tr>
<tr>
<td>0.982</td>
<td>0.968</td>
<td>1.000</td>
<td>0.942</td>
<td>0.953</td>
<td></td>
</tr>
<tr>
<td>0.953</td>
<td>0.958</td>
<td>0.942</td>
<td>1.000</td>
<td>0.944</td>
<td></td>
</tr>
<tr>
<td>0.988</td>
<td>0.968</td>
<td>0.953</td>
<td>0.944</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Results of a university’s performance should be related to the input variable. It orders to determine the strength of that relation,
correlation between the input variable and the output variables was calculated (Table 3).

Table 3: Pearson correlation coefficient of input and output variables

<table>
<thead>
<tr>
<th></th>
<th>O₁</th>
<th>O₂</th>
<th>O₃</th>
<th>O₄</th>
<th>O₅</th>
<th>O₆</th>
<th>O₇</th>
<th>O₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>I₁</td>
<td>0.97</td>
<td>0.96</td>
<td>0.22</td>
<td>0.15</td>
<td>0.18</td>
<td>0.43</td>
<td>0.93</td>
<td>0.96</td>
</tr>
<tr>
<td>p</td>
<td>0.00</td>
<td>0.00</td>
<td>0.36</td>
<td>0.53</td>
<td>0.46</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Only four out of eight output variables are strongly and significantly correlated with the input variable: O₁ – weighted number of full-time students based on their field of study; O₂ – weighted number of full-time PhD students calculated on the basis of their scholarly disciplines; O₃ – employers preferences determined through survey research and O₅ – parametric assessment of scholarly achievements of universities carried out by the Ministry of Science and Higher Education. Correlation of the remaining output variables with the input variable is insignificant. Thus, these variables were excluded from further analysis.

In order to examine the impact of the environmental variables on the achieved results the correlation between the environmental variables E₁ (population size of the city where the university is located), E₂ (percentage of students with need-based financial aid) and the output variables was calculated. It was established that the two environmental variables are characterized by a strong and significant correlation with output variables (Table 4). Variable E₂ shows negative correlation with the output variables. The obtained results indicate the need to include the environmental variables in the model.

Table 4: Pearson correlation coefficient of output and environmental variables

<table>
<thead>
<tr>
<th></th>
<th>O₁</th>
<th>O₂</th>
<th>O₃</th>
<th>O₄</th>
<th>O₅</th>
<th>O₆</th>
<th>E₁</th>
<th>E₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₁</td>
<td>0.7186</td>
<td>0.8391</td>
<td>0.8314</td>
<td>0.8563</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E₂</td>
<td>-0.5496</td>
<td>-0.5803</td>
<td>-0.5079</td>
<td>-0.6368</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Variables selected for the model should be characterized by a high level of variation, which enables clear diversification of HEI in respect to their input and achieved effects. All variables present in the model are characterized by a sufficiently high level of variation (CV > 50%) (Table 5).

Table 5: Coefficient of variation of model variables

<table>
<thead>
<tr>
<th></th>
<th>I₁</th>
<th>O₁</th>
<th>O₂</th>
<th>O₃</th>
<th>O₄</th>
<th>O₅</th>
<th>O₆</th>
<th>E₁</th>
<th>E₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV</td>
<td>0.59</td>
<td>0.65</td>
<td>1.10</td>
<td>1.19</td>
<td>0.82</td>
<td>0.78</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s own calculations.

Ultimately variables I₁, O₁, O₂, O₅, and O₈ were selected for the comparative efficiency calculations with the use of DEA method (Table 6).

Table 6: Variables selected for DEA model

<table>
<thead>
<tr>
<th>Input variable</th>
<th>O₁</th>
<th>Government budget subsidy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output variables</td>
<td>O₁</td>
<td>Weighted number of full-time students</td>
</tr>
</tbody>
</table>

COMPARATIVE ANALYSIS OF THE UNIVERSITIES EFFICIENCY

Due to the character of the task a CCR-CRS output-oriented model was chosen for the calculations. That model was considered suitable because universities have no direct influence on the size of the government budget subsidy. As a result of the very strong linear correlation of output variables with the input variable and the impossibility to rapidly increase the effects, a CSR (constant returns to scale) model was selected. Calculations were carried out with the use of the Frontier Analyst v. 4.1.0, Statistica 9 and Microsoft Office Excel 2007 software.

In the first stage of calculations the efficiency of the universities was determined excluding environmental variables. On the basis of the results it was found that the O₇ variable (employers’ hiring preferences) has a low share in the DMU’s efficiency assessment. As a consequence, the calculations were repeated excluding this variable. The obtained results turned out to be practically identical with the previous ones (Table 7).

Table 7: Efficiency scores for 19 universities

<table>
<thead>
<tr>
<th>No</th>
<th>Univ.</th>
<th>Score</th>
<th>No</th>
<th>Univ.</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>U₁</td>
<td>100.00%</td>
<td>11</td>
<td>U₁₁</td>
<td>91.10%</td>
</tr>
<tr>
<td>2</td>
<td>U₂</td>
<td>100.00%</td>
<td>12</td>
<td>U₁₂</td>
<td>86.50%</td>
</tr>
<tr>
<td>3</td>
<td>U₃</td>
<td>100.00%</td>
<td>13</td>
<td>U₁₃</td>
<td>84.10%</td>
</tr>
<tr>
<td>4</td>
<td>U₄</td>
<td>100.00%</td>
<td>14</td>
<td>U₁₄</td>
<td>83.30%</td>
</tr>
<tr>
<td>5</td>
<td>U₅</td>
<td>100.00%</td>
<td>15</td>
<td>U₁₅</td>
<td>83.10%</td>
</tr>
<tr>
<td>6</td>
<td>U₆</td>
<td>100.00%</td>
<td>16</td>
<td>U₁₆</td>
<td>82.80%</td>
</tr>
<tr>
<td>7</td>
<td>U₇</td>
<td>97.30%</td>
<td>17</td>
<td>U₁₇</td>
<td>81.20%</td>
</tr>
<tr>
<td>8</td>
<td>U₈</td>
<td>96.60%</td>
<td>18</td>
<td>U₁₈</td>
<td>79.80%</td>
</tr>
<tr>
<td>9</td>
<td>U₉</td>
<td>95.70%</td>
<td>19</td>
<td>U₁₉</td>
<td>75.00%</td>
</tr>
<tr>
<td>10</td>
<td>U₁₀</td>
<td>93.90%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore the O₇ variable was excluded from the further calculations.

Since in several cases the DEA algorithm omitted some output variables (e.g. number of students) the author decided to impose constraints on the weighs ascribed to the output variables. It is also justified by the fact that the government budget subsidy to the Polish HEI is mainly spent on educating students and that the technical universities are required to carry out research and PhD-level education. On these premises it was assumed that the share of O₁, O₂ and O₈ variables may not be lower than 30%, 10% and 20%, respectively. Calculations
conducted with these assumptions slightly changed the results of particular universities but five out of six universities considered efficient beforehand kept their status. In turn, relative efficiency of some universities fell drastically (U12, U16, U13, U2), which indicates that their research strength and PhD-level education are relatively very weak in comparison to other universities (Fig. 2).

Next the $E_1$ and $E_2$ environmental variables were introduced to the model by including them in the Frontier Analyst software as uncontrolled inputs. Due to the software requirements the $E_2$ variable was replaced by the $1/E_2$ variable in order to obtain the positive correlation between that variable and the outputs. During the process of calculation it was observed that the introduction of $E_1$ and $E_2$ variables resulted in assigning a zero weight to the $I_1$ variable by the DEA algorithm. Since the utilization of the government budget subsidy is the basis for the relative efficiency analysis of the universities the author decided to impose additional constraint on the variable weighs. It was assumed that the share of $I_1$ variable may not be lower than 70% and the share of $E_1$ and $E_2$ variables may not be higher than 30%. Calculations carried out with such assumptions hardly changed the results of the analysis (except single cases – U15) (Fig. 3). It is an indicator that environment in which a university functions has no significant influence on its efficiency.

In order to study the sensitivity of the calculations to data error simulations were carried out where output variable were distorted with ±3%, ±5% and ±10% distortions. Input variables remained unchanged since they are determined with high accuracy. The simulation demonstrated that the calculation results remain stable with the distortion level of ±3%. Distortion of ±5% causes significant shifts but the general picture of the ranking is sustained. Distortion of ±10% causes the instability of the results. Simulation results lead to the conclusion that since the weighted number of students (including PhD students) and the number of points in the parametric assessment of research achievements carried out by the Ministry of Science and Higher Education are based on the factors and indicators which are set arbitrarily, one should exhibit far reaching caution in interpreting the results of the university efficiency calculations. These results may to a large extent be determined by some arbitrary assumptions. This problem may be a premise for further detailed studies in this area.

The last analysis aimed at studying the influence of a university size on its relative efficiency. University size (measured by the size of the government budget subsidy) shows high correlation ($r = 0.53$) with relative efficiency. It may lead to the conclusion that larger universities on average achieve higher efficiency. This conclusion is supported by the visual analysis of the efficiency graph in the university size function (Fig. 4).

CONCLUSIONS

The paper presented an example of DEA method implementation in the efficiency assessment of the Polish technical universities. This example shows the usefulness and rationality of DEA application in the higher education sector. Systematic and multi-criteria assessment of public sector institutions may bring many benefits not only to the authorities that operate with limited public funds but first of all to the assessed units. DEA results carry significant information on the efficiency of HEI functioning in relation to other institutions with a similar scope of activity. They point at the attainable results and at the factors which influence most the efficiency of a unit. Author
is convinced that the comparative efficiency analysis may be one of the important stimuli to increase the quality of education and research, to improve the efficiency of public funds spending and their allocation as well as to perfect the HEI management.

The study presented in the paper – though limited in scope – shows that Polish technical universities are diversified in respect to the efficiency of their performance. It is demonstrated that there are considerable reserves for efficiency improvement in particular schools. At the same time one should warn against too hasty and straightforward reading of the calculation results obtained from DEA method. Proper interpretation of these results requires deep knowledge of the studied area and a high degree of caution when formulating radical conclusions.

REFERENCES


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A Knowledge Value Chain Management framework for the Relative Performance Evaluation of Academics staff in Higher Education

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ABSTRACT

Knowledge value chain management in higher education (HE) involves external and internal stakeholders in the effective creation and efficient provision of high quality learning, research and service outputs and outcomes. Academic staff is one of the main stakeholders responsible for the value chain quality of co-creating, co-providing and co-delivering of such outputs and outcomes. The design of a proper framework to evaluate, motivate and align Academic staff towards common institutional mission, goals and values becomes very important. In this paper, we propose a knowledge value chain management framework for the Relative Performance Evaluation of HE academics. The framework uses Data Envelopment Analysis (DEA) model with tangible and intangible indicators related to the HE organization's mission and description of academics' job. An Analytical Hierarchy Process (AHP) measurement is integrated into DEA as a reduction technique to aggregate the more inputs/outputs criteria into fewer measures to establish a better DEA discrimination. The proposed DEA-AHP approach is validated on a real-data collected on 35 faculty members at the Olayan School of Business. Suggestions on how to implement the Relative Performance Evaluation management system are discussed. Guidelines for improving the quality of assessment are provided with highlights on further research directions.

Keywords: Data Envelopment Analysis, Analytical Hierarchy Process, Knowledge Value Chain, Academic Staff

INTRODUCTION

In the present digital era of knowledge economy, the role of higher education (HE) becomes crucial in the overall socio-economic and technological-environmental development of any region or country. Globalization brings global but competitive opportunities for a HE institution to become an exporter of knowledge and educational services. Such global competitiveness requires a HE institution to design an appropriate knowledge value chain framework to sustain its continuous development, creation and provision of knowledge services to meet dynamically changing society needs. The framework should operate using an efficient and effective system-wide approach incorporate all stakeholders. In the literature there exist only few attempts on the transformation and integration of Supply chain Management (SCM), Quality Management (QM), Service Management (SM), Business Process Re-Engineering (BPR) concepts and principles in an efficient and effective system-wide approach to the knowledge value chain management in public and private HE sectors. For instance, Voss et al. (2005) compared the service profit chain in further education colleges in public sector to that of organizations in private sector. They recommended that if the college managers are to improve their service levels, they should adapt the service profit chain practices intelligently to their context. Sasse et al. (2008) proposed a value chain model in which the university brings knowledge from faculty to students leveraging the linkages of knowledge creating activities with distribution of the final product. Their value chain goes sequentially through stages starting from: 1) creating research knowledge; 2) applying knowledge; 3) producing learning curriculum; 4) design and delivery; to 5) student learning (final product). They distinguished between the upstream value chain (1 to 3 stages calling it discipline-based faculty focus) and the recent right-side shift downstream (4-5 stages calling it student learning based faculty focus). The downstream shift was justified by the much developed efforts expected of faculty in the
areas of applying knowledge to course design and student learning, grading and assessment functions and the more focus on learning from the community practices outside the discipline and academia. Makkar et al. (2008) developed another conceptualized value chain model for the service sector with a special reference to colleges in HE sectors in India and Tanzania. The authors presented a good discussion on the value management, co-creation and co-delivery of values of HE services. Barut et al. (2006) designed, implemented and evaluated a global learning environment in teaching a logistic and supply chain course where students would be required to team up with other students from overseas and utilize the World Wide Web, and video conferencing technologies to solve real-life global logistic problems. They discussed institutional units involved, course materials, team formations, and infrastructures to deliver a global value to “world citizens” who have to operate at all levels. They found unless students are involved in such global experiences, they are reluctant to use such sophisticated technologies. Gonzalez et al. (2008) utilized quality function deployment and benchmarking to develop a voice of customers matrix to design a customer-centered SCM undergraduate program. The most important need for the program was the identification of the program’s main needs and expectations of customers. Robinson & Malhorta (2005) integrated quality and supply chain concepts into a quality SC framework for the automotive industry sector. Moreover, Spence and Kale (2008) quoted Herb Kelleher (former CEO, Southwest Airlines, 1996) saying that “it used to be a business conundrum: who comes first; the employees, customers, or shareholders? That’s never been an issue for me. The employees come first. If they are happy, satisfied, dedicated, and energetic, they will take real good care of the customers. When customers are happy, they come back. And that makes the shareholders happy”. Similarly academic staff should come first in the academic value chain. They play central and important roles in the co-creation of scholarly knowledge, the co-delivery teaching and learning, and the co-provision of other related academic services. They need a proper Relative Performance Evaluation Management (RPEM) approach which can be used for various purposes including short-listing candidates, planning faculty development, mentoring progress, compensating and rewarding performance, promoting and retaining faculty, motivating and aligning them towards common institutional desired academic goals. However, existing performance evaluation methods may have pros and cons, often subjective and incomplete with no proper alignment to HE institutions. Moreover, RPEM methods use ratios of single output to single input performance indicators such as number of published research papers to the number academic years per faculty, or compare individual performances to an average performance value of peers rather than comparing them to the best-peer performers. Therefore, after the short background, we shall propose a Knowledge Value Chain Management (KVCM) framework to generate various academic knowledge value chain models for HE institutions that integrate SCM concepts, QM initiatives, BPR and information system best practices into the management of people, processes, policies, and supporting activities to provide high quality values to stakeholders, and sustain competitive leaderships in HE sectors based on the vision and mission of the HE organization. Hence, the following sections are organized as follow: section 2 introduces the new KVCM framework and discusses its concepts and components. An associated KVCM performance measurement and evaluation approach is also proposed in Section 3. It is based on Data Envelopment Analysis (DEA) approach which uses multiple inputs and multiple outputs measurable indicators for teaching, research and service activities. Furthermore, the Analytical Hierarchy Process (AHP) approach, which sets weight priorities to inputs/ outputs by stakeholders’ judgments, is integrated into the DEA approach as a reduction technique to aggregate the more inputs/outputs into fewer ones to establish a better discrimination and rankings of faculty members. Section 4 presents an application of the suggested framework of OSB faculty staff in 2008. Finally, section 5 concludes with future research directions.

KNOWLEDGE VALUE CHAIN MANAGEMENT FRAMEWORK IN HE

To propose a new Knowledge Value Chain Management (KVCM) framework its concepts and main components artifacts approach where a clearly defined set of models and concrete implementation methods are defined. The proposed framework builds on exiting literature and includes additional components to provide an effective and efficient system-wide approach to find a KVCM value taking into accounts the views of all stakeholders. The high-level structured framework focuses on implementations of specific academic knowledge components involving people, service activities, and supporting processes
following the principles of a business process re-engineering (BPR) approach. BPR provides both managers and academics with a knowledge base to improve business process change practice of organizational structures from planning, organizing to controlling with associated BPR efforts. The proposed KVCM framework (Figure 1) follows a top-down strategic directive and bottom-up operational feedback-cycle approach. The aim is to achieve an optimal knowledge chain value (KCV) which is a combination of perceived and expected satisfactions of stakeholders resulting from performing and conducting various scholarly research works, teaching and education learning, and providing external and internal service activities related to the mission of the HE organization.

Figure 1: Knowledge Value Chain Management Framework

The framework is formed of a network of links connecting six separate components, namely: Mission of the HE organization (C0); external stakeholders (C1); internal stakeholders (C2); Research, teaching and service activities (C3); Operating support processes (C4) and decision making support systems and information technology systems infrastructure (C5). The first one-directional arrows are originating from the mission (C0) to provide guidance on strategic directives of the mission, vision and desired goals to all functional and operating bodies of the HE organization. The second dashed single-direction feedback arrow originates from C5 to provide measurements and impact assessments of various directives on the achieved KCVs and provide insightful recommendations to close gaps using internal benchmarking and best practices. The bi-directional arrows represent interdependencies. The KVCM framework provides a mechanism to generate various closed-looped chains that each chain starts with C0 involves at least one additional component, goes through the performance measurement component in C5 and loops back to C0. Hence, it is very flexible to cater for different organizations. It provides a mechanism to re-design, re-assess and measure success factors in a continuous improvement process philosophy for a better alignment with an organization’s mission and achievement of desired goals.

Although, the framework is simple by design, it requires clear descriptions of specific knowledge activities supporting and evaluating processes to deliver a complete and successful implementation. An implementation involves strategic, tactical and operational decisions that require proper plan, design, operate, govern and control interdependencies of processes for the purpose of conforming to the organization’s goals and desired values. Stockholders often have conflicting objectives unless a system-wide integrative approach to follow to efficiently minimizing the system-wide total cost conflicts and effectively maximize the system-wide total values. This efficient and effective system-wide system can be designed if a proper sharing-benefit strategy is established among stakeholders to coordinate better around common shared values implementation efforts. The absence of no-coordination and no-sharing strategies would always lead conflicts, inferior system-wide values and lack of common goals, Jaber and Osman (2006), and Jaber et al. (2006).

Furthermore, stakeholders attach a great value to the characteristics of the HE institution provider. They have pride of being associated with good institutions to establish life-long relationships. However, if there is a mismatch in the expected values among internal and external stakeholders, there will be perception gaps. The HE service provider should deliver academic services in an optimal way to minimize cost and maximize values with no gap to manage. Therefore the main spirit of our framework is to have all stakeholders commit, integrate, and coordinate efforts using a properly designed integrated information technology for fast sharing and communication information on workflow status in order to pursue coherent and innovative practices. The outcome would be an achievable of a mutual desired satisfaction representing the whole desired values of all stakeholders in the chain. From the stakeholders’ point of views, satisfaction is always and will continue to be subjective; hence adopting a system-wide optimization approach is very essential to find the right balance among conflicts of individual desires. Finally, co-creation and co-delivery of value chains will always lead to a mutual satisfaction, Makkar et al. (2008).

To summarize, the proposed framework has many different characteristics and features than exiting frameworks. First, it is a closed-loop feedback network of looped cycles from the mission forward through activities/processes that would allow the assessment of
The mission is normally translated into strategic initiatives to guide the design and delivery of knowledge activities, control and management of operating support processes, integrate information technologies structure, coordinate effort among stakeholders and to achieve one globally institutional academic knowledge value chain rather than many local stakeholders’ values. It indicates whether an institution is a consultancy and training company, a teaching college, a research institution, or a university involved in high learning research, teaching and service (called academic knowledge activities). The academic desired value is a combination of sub-values related to the various academic knowledge activities.

Stakeholders Roles
Stakeholders consist of internal and external groups in the provision of high quality academic knowledge activities. The internal group includes academic faculty, non-academic supporting staff; management offices, and governance boards. The external group includes students; employers, corporate sponsors, partners of compatible intuitions, and independent professional advisory boards. The students start as external stakeholders, they are admitted and enriched by the academic knowledge values. If satisfied, they would become external loyal stakeholders of the HE organization, contributing to its continuous development and improvement, and its academic value by either being potential employees, corporate sponsors, or internal stakeholders if they accomplished further knowledge. Therefore, in our model, the students are not just customers; they are also partners and co-creators of the academic knowledge value. Moreover, students cannot buy at any cost the academic knowledge value even if they can afford it only if they meet the admission requirements which access their intellectual abilities and compatibility with the desired academic knowledge value. Therefore, stakeholders are contributors who link their efforts together aiming at a common objective, which ought to be maximization of their overall satisfaction through co-creation, co-production and co-provision. Sponsors and governing boards inject capital to conduct research activities, build appropriate facilities, and establish information technology infrastructures and other operating supporting processes. Academic managers play a significant role in the governance, recruitment of faculty and their development. Faculties engage to the moment of truth in the academic knowledge value delivery processes. The supporting staff performs various activities to

THE FRAMEWORK COMPONENTS
Mission of a HE Learning Organization
The mission defines the set of objectives, education activities, and scope of operations of a HE learning organization. For instance, the American University of Beirut (AUB) Mission is stated as follows: “AUB is an institution of higher learning founded to provide excellence in education, to participate in the advancement of knowledge through research and to serve the peoples of Middle East and beyond”.

independent and combined components on the desired values. Its fundamental principle is the separation between the research, teaching and service activities and the operating support processes. This separation would lead to flexible design of academic activities, independently of the underlying operating support infrastructure responsible for the delivery of activities. However, the separation approach should be coordinated and supported by joint governing teams who enforce quality procedures, communicate and share information to achieve system-wide minimal cost and maximized desired quality value. The separation is motivated by the availability of advanced and innovative technology support at a reasonable cost and by the short supply of expensive academics. If the scarce academics are given more time using some appropriate operating technology and nontechnology support, then they would provide more added academic values than being constraint by doing low-level activities which can be done more efficiently by other non-academic and new advanced technological support. In this separation approach, optimization of system-wide total cost and values can be optimized to allow agreeable sustainable working environment. A similar approach was recommended by the International Telecommunication Union for the design and implementation of new generation network architecture for the successful telecommunication service sector, Kingston et al. (2005) and Choi and Won-Ki-Hong (2007). Second, the students in our value chain model are not raw materials that undergo different transformation to reach end customers and bought at different prices for consumption with reduced values over time. On the contrary, they start as internal stakeholders may become external stakeholders with increased values over time. Finally, the services are either tangible or intangible, and need to be created and may be provided simultaneously but cannot be stored or transported. The value of people in the KVCM service delivery becomes extremely important to the co-creation of value unlike manufacturing and other services.
make sure that the various systems are operating within desired parameters. Students need to effectively responsible for their acts and playing their role in the learning process.

**Academic Knowledge Activities and Supporting Processes**

The proposed framework suggests a separation between academic knowledge activities with external outcomes and operating support processes within the organization. Knowledge activities include research publications, teaching and learning offerings, external consultancies and community services. Supporting services may include research, teaching and learning support centers, non-academic staff, research and teaching assistants, stakeholder’s management centers, computer and technology support centers, and organizations offices (admission, registrar, finance, auditing, quality, improvement and development efficiency, external relationships and student offices). The implementation of an AKVM influences both internal and external groups. Normally both groups are already in the same place within an existing HE organization. The implementation of the framework may lead to a redesign of existing processes. From an overall viewpoint, all activities should be focused and linked to stakeholders’ requirements according to the mission desired values. This can be a correction step in line with Grapentine (1998) who said: “too much” academics and practitioners focus on measuring customer satisfaction (or expectations) and “too little” on connecting customer needs to business processes.

**Decision Making Support System and IT infrastructure**

In this framework, a decision support system is defined as “the development of approaches for applying information technology systems to increase the effectiveness of decision makers in situations where the computer system can support and enhance human judgment in the performance appraisal tasks”, Holsapple and Whinston (2000). The information technology (IT) systems infrastructure is regarded as one of the main driver for the delivery of research, teaching and service activities and can be one the competitive advantage of an academic institution. They include the online teaching support systems, the library databases and all other organization databases from payrolls to stakeholders’ records. To harness their benefits, IT systems should have harmonized interactions and have an integrated data model which is a solution for the integration of the overall IT infrastructure from academic knowledge activities, to operating support resources to provide information to decision maker with reliable connectivity access anytime, anywhere, anyplace. The lack of effective and integrative IT infrastructure support would lead to stakeholders’ dissatisfaction and a reduction of the organization chain value due to various reasons including the need to repeat efforts in data entry and transformation from different formats, to even loss of data. However in our view, an informed decision making (IDM) support system would provide a knowledge-based approach to enable decision makers to make sound strategic, tactical or operational decisions in a systematic way. An IDM support system for a given academic knowledge activity (for instance the evaluation and appraising process) would involve the following steps: 1) data collection, entry, storage and retrieval from various database sources; 2) data analysis using management science (data envelopment analysis, analytic hierarchy process, statistic) or other relationship models to identify and select the best alternative scenario; 3) a communication and reporting strategy available to managers to get feedback on decisions on implementation. All these steps have to be efficiently processed using information systems, tools and effective models, and implemented by appropriately trained people for a competitive advantage and high value.

**Implementations and Governance Issues**

Since the framework is a mission-process driven approach and adaptive to basic structure of the organization’s needs, it will be attractive to top management and secure their support for actual implementation. The framework has arrows reflecting the interdependencies between academic activities and operating support processes. A successful implementation requires a strategic action plan with clear governance policies for implementation of strategic, tactical to operational decisions concerning stakeholders involvement, external use of consultancy and training services on both technical aspects and on activity educational aspects, effective change management support from senior management during the execution of reform policy, project phasing and follow up, proper resource plan, appropriate contingency plan, and quality control and accountability management plan. In addition, top management should provide Infrastructure Support, Standardized IT infrastructure, Suitable operating IT legacy systems. All activities and supporting processes must be aligned and in conformity with the organization’s mission and desired values. Further, ACKVM strategy
should be put into implementation action focusing on success factors, and key performance criteria and indicator for each activity/process. Consequently, implementing each success factors should be measured to ensure successful implementation. The final step of the framework implementation focuses on measuring the impact of AKVCM strategy on overall financial budget cost minimization, the maximization of the academic knowledge value, the realization of institutional objectives and the assessments of the effect of other possible external factors.

One of main framework features is interdependencies between academic knowledge activities and the operational support processes for which we propose an end-to-end implementation process model based on a stakeholder-focus approach. The end-to-end process model covers all interdependencies from an end-to-end perspective. For instance, a student contact management office may have end of end processes for receiving order requests to change courses/programs, receive answers for questions and complaints; for processing, responding and analyzing requested orders, and finally automated transaction automation IT support that starts at the enquiry stage.

A detailed example of an end-to-end student-focus for admission may start from downloading the application, providing the necessary documents, evaluating the candidates, sending the acceptance letter and ending with the acceptance of the offer. The upstream faculty-focus, the downstream student-focus of Sasse et al. (2008) previously mentioned in section 1, are also examples of such process models for academic knowledge activities. Examples of the operating support processes include technical support, readiness and preventive maintenance, handling of complaints, billing, order fulfillments, teaching and learning and technical support. Since each of academic activity involves a sequence of interdependencies, effective management control tools become essential to manage conflicts and complexities with an overall aim to achieve an effective system-wide of highest benefits with efficient operating performance. Examples of operational support external or internal inefficiencies include: an external case involves the students’ lack of proper communication and understanding of the admission application requirements may lead to delay in submitting supporting documents, delay in making payment of application fee, and unaware of the acknowledgement letter communication means by post an e-mail. An internal resources support involves the lack of expertise of new staffs who have little experience of the admission system. A lack of proper training support would contribute to slow processing of applications; unsystematic filing of applications and operational inefficiency of various committees. Any internal or external inefficiencies, would lead to low stakeholders value regardless how excellent the design of admission academic activity and its support processes. Hence high synchronization among interdependent academic activities and their operating supports are highly recommended to achieve efficient and highly effective framework implementations.

Finally, according to United Nations Economic and Social Commission for Asia and the Pacific, governance can be defined as a process of decision-making by which decisions are implemented (or not implemented). Governance and accountability can be problematic; if a strategic or tactical decision is taken by the HE management by considering only the interest of one of its stakeholder at the expense of other stakeholders. This would lead to undesirable consequences from the reduction of the value chain, the creation of environment of distrust, and the de-alignment from common goals.

**Assessment and Measurement of the Quality of AKVCM**

AKVCM involves a number of complex interlinked academic knowledge activities and operating support processes involving different stakeholders with different desired values. All of them make the assessment and measurement a difficult task. However, the main aims remain the creation, dissemination and transfer of basic and applied knowledge to prepare a future knowledgeable generation, to provide various futuristic consultancy services to the profession and society. The quality AKVCM value could be defined as the stakeholders’ perception/satisfaction about the whole bundle of benefits, either tangible or intangible, to satisfy their needs timely, effectively and efficiently. Therefore, an incremental assessment and measurement approach may conduct from individual stakeholders, intuitional, and a program focuses. Such independent assessment and measurements may be combined to provide an estimate of a holistic evaluation. This step by step approach does not provide a system-wide efficient and effective, since the satisfaction is in the eyes of beholders, it will always be subjective and more works would be needed to achieve a more objective holistic evaluation.

We shall present briefly the different internal or external available approaches used for the measurement and certification of AKVCM
values. The types of recognition and certification can be classified into institutional processes accreditations by peers and/or governmental agencies; professional accreditations of programs by professional associations, award of degrees to stakeholders. Since the quality of entering students have impact on the final quality of their knowledge enrichment and transformation, each HE institution set its own evaluation criteria for the admission aligned with the desired quality of its expected outputs and learning outcomes. For instance, students upon completion of their degrees are given certificates in recognition for their success as an internal stakeholder. A HE organization may seek recognition from an association of peers institutions by obtaining an institutional accreditation to ascertain the claim made in its mission statement. The Quality Assurance Agency (QAA) for Higher Education checks how universities in the United Kingdom maintain their own academic standards and quality. QAA reviews and reports on how they meet their responsibilities identify good practice and make recommendation for improvement. QAA also publishes guidelines to help institutions develop effective systems to ensure students have the best learning experience (QAA, 2009). Similarly, The Middle States Commission on Higher Education (MSCHE, 2009) is the unit of the Middle States Association of Colleges and Schools that accredits degree-granting colleges and universities in the Middle States region. MSCHE is a voluntary, non-governmental, membership association that defines, maintains, and promotes educational excellence across institutions with diverse missions, student populations, and resources. It examines each institution as a whole, rather than specific programs within institutions. The European Foundation for Management Development (EFMD, 2009) is recognized globally as an accreditation body of quality in management education. EFMD has established three accreditation programs. The European Quality Improvement System (EQUIS) which is one of the leading international accreditation for business schools, and business school programs. The programme accreditation for technology-Enhanced Learning (CEL), which aims to raise the standards of technology-enhanced learning programmes worldwide and finally the Corporate Learning Improvement Process (CLIP) which is a unique certification for corporations that focuses on identifying the key factors that determine quality in the design and functioning of internal corporate universities and learning organisations. Similarly, the Association to Advance Collegiate Schools of Business, (AACSB, 2009); AACSB international mission is to advance quality management education worldwide through accreditation and leadership. Other professional disciplines such as engineering and nursing have their own similar accreditation bodies. It should be noted that the accreditation process is a continuous process repeated periodically every five year period with annual progress reports submitted to accreditation agencies. Another different type of assessment conducted by jointly Higher Education Funding Councils in the United Kingdom, called Research Assessment Exercise (RAE, 2008). The primary purpose of the RAE was to produce quality profiles for each submission of research activity made by institutions. The quality profiles are used by the councils to determine their grant for research to the institutions. Since the framework is a new education model necessary to measure an organization’s success, everyone in the organization needs to be involved to assure a high quality of attainment. It calls for proper manage of people and integrating technology, connecting with stakeholders, collaborating and partnering with other organizations, governing and measuring performance to achieve maximum academic knowledge values according to the mission of the HE organization. However, the purpose of measuring performance is also to appreciate and reward employees to increase their commitment to the organization, and to motivate them to constantly. In this regards, Heck et al. (2000) developed an evaluation of assessing and mentoring the effectiveness of academic deans and directors. The assessment approach was based on the faculty and staff assessments of their Deans’ leadership effectiveness. Goodall (2009) investigated how different types of leader affect a university’s later performance. Based on a blend of qualitative and quantitative evidence, it was found that on the average the research quality of a university improves some years after it appoints a president who is an accomplished scholar.

RELATIVE APPRAISAL DECISION MAKING SUPPORT SYSTEM

Performance appraisal system is the most influent management process over employees’ careers (Richard and Grote, 2002). When used well, it gathers the energy of all employees toward achieving the strategic goals of the institute on daily basis to focus on the mission, vision and values. Richard and Grote (2002) asked a very crucial question: how many of you can see the same words in both hands if we put the corporate strategy on the right hand
and the performance appraisal system on the left hand? If employees don’t see any inter-relation between what the organizations is trying to convey in its mission statement and what “they are held accountable for in their performance system, they will become cynical about the importance of the stated mission”. Also, existing evaluation practices may not link properly all inputs, outputs and outcomes from various academic activities and their operating supports in a systematic way to generate a fair and transparent evaluation and appraising decision making system. If employees are happy, satisfied, dedicated, and energetic, they will take real good care of the other stakeholders. When other stakeholders are happy, they will come back, and that makes the whole shareholders happy leading to increased mission’s values. In this section, we propose a customized version of the AKVCM framework to develop a Relative Appraising System (RAS) of academics. Following the guideline of Section 2, our framework requires clear descriptions of specific knowledge activities and their supporting processes in order to have a complete implementation. Our framework is presented in Figure 2, while a tentative illustration of its set of academic activities, operating supports and associated decisions are presented in Table 1 (in the appendix). Moreover, some expected tangible quantitative inputs/outputs and intangible qualitative outcomes are also presented in Table 1.

The main objective of the framework is to present a guide to the RAS end-to-end chain process according with the mission of the HE organization’s mission. The chain of the end-to-end process consists itself from the followings sub-processes: i) the mentoring process, ii) the data collection process, iii) the data analysis process, and iv) the appraising and managerial implication. In the next subsection, we shall elaborate the RAS implementation process for the relative evaluation and appraising performance of academics at the Olayan School of Business, OSB. We shall discuss RAS chain processes, while noting that each sub-process may have its own end-to-end sub-processes. We shall start first by explaining our methodologies for the data analysis as they would help in setting the requirement for data collection.

Figure 2: The Relative Appraising System (RAS) Framework

DEA AND ANALYTIC HIERARCHY PROCESS METHODOLOGIES

Data Envelopment Analysis

DEA was first proposed by Charnes et al. (1978), and is a non-parametric method of efficiency analysis for comparing units relative to their best peers (efficient frontier) rather than average performers, and to identify benchmarks for inefficient units. It does not require any assumption on the shape of the DMUs frontier surface and it makes simultaneous use of multiple inputs and multiple outputs. DEA defines the relative efficiency for each DMU (faculty, departments, schools, banks, services) by comparing the DMU’s inputs and outputs to other DMUs data in the same “cultural or working” environment. The outcomes of a DEA analysis includes: i) A piecewise linear empirical envelopment frontier surface of the best practice, consisting of DMUs exhibiting the highest attainable outputs for their given level of inputs; ii) An efficiency metric (score) that represents the maximal performance measure for each DMU measured by its distance to the frontier surface; iii) Efficient projections onto the efficient frontier with identification of an efficient reference set consisting of the “close” efficient DMUs for benchmarking and improving each inefficient unit; iv) a ranking of units from best (highest score) to worst (lowest score).

There are basically two types of DEA models: Charnes et al. (1978) introduced the constant returns-to-scale (CRS) and later on Banker et al. (1984) introduced the variable returns-to-scale (VRS) model. Both models are also classified as input-oriented, output-oriented, or radial-additive based on the direction of the projection of the inefficient unit onto the efficient frontier surface. In the present study, DEA output-oriented model is chosen because we want the outputs of a DMU (an academic staff) to be maximized to increase the Academic Value Chain from a given HE organization’s operating supports and perspectives.
To introduce the DEA concept; consider a set N= \{1, 2, ..., n\} DMUs for evaluations. Let denote by \(X_j\) and \(Y_j\) the column-vectors of its \(m\) inputs and \(s\) outputs values for each \(j\) in \(N\) respectively, let also \(u\) be the \(m\)-dimension column-vector of input weights (resources), and \(v\) be the \(s\)-dimension column-vector of output weights (values). The DEA-CRS standard model of Charnes et al. (1978) measures the productive efficiency of \(DMU_0\) to be maximized as the ratio of its total virtual outputs over its total virtual inputs, i.e.

\[
\Theta^* = \text{Max} \ \frac{u^T \times Y_0}{v^T \times X_0} 
\]  

Subject to the condition that the virtual ratio \(\Theta\) of any unit \(j\) does not exceed 1 and all the weights are strictly positive. The non-linear model in (1) can be transferred into a linear programming model in the following way:

\[
\Theta^* = \text{Max} \ \frac{u^T \times Y_0}{v^T \times X_0} \quad (2)
\]

Subject to

\[
u^T \times X_0 = 1 
\] \( (3)\)

\[
u^T \times Y_j - v^T \times X_j \leq 1 \quad (4)
\]

\(u \ and \ v \geq 0\) \( (5)\)

The model in (2) maximizes the objective value (total virtual outputs) while keeping constant set to one theominator (virtual total inputs) of the unit under evaluation. Constraint (3) is called a normalization vector, while constraint (4) leads to the relativity of DEA evaluation concept since the optimal weights for the \(DMU_0\) inputs and outputs that maximized \(\Theta^*\) are used in the ratios of other units. \(DMU_0\) is said to be \emph{efficient if and only if} \(\Theta^*= 1\). Otherwise, when \(\Theta^*< 1\), \(DMU_0\) is \emph{inefficient} and is dominated by the frontier of best performing \(DMU\) units, i.e. it can either increase its output level or decrease its input levels by projection onto the efficient surface which identifies corresponding benchmarks. \(\Theta^*\) represents the output-oriented efficiency score of \(DMU_0\). The mathematical formulation in (2) to (5) represents the output-oriented model similarly one can generate the input-oriented model by fixing the numerator to one. Other Envelopment models and formulations can be found in Cooper et al. (2007).

**Analytic Hierarchy Process Approach (AHP)**

AHP is a measurement theory that prioritizes the hierarchy and consistency of judgmental data provided by a group of decision makers. AHP incorporates the evaluations of all decision makers into a final decision, without having to elicit their utility functions on subjective and objective criteria, by pair-wise comparisons of the alternatives (Saaty, 1990).

Based on the literature review, the AHP and the DEA-analyses have earlier been used together: Shang and Sueyoshi (1995); Yang and Kuo (2003); Yoo (2003); Korpea et al. (2007); and Meng et al. (2008) among others used the AHP to generate inputs or outputs for the DEA-model. Meng et al. (2008) used the integrated DEA and AHP approach to evaluate the fifteen research institutes in the Chinese Academy of Sciences. The AHP was used to group the five sub-research indicators into one indicator. Therefore, we feel that combining the objective power of DEA with methods that are very good at handling subjective values – such as AHP – could potentially enhance the usefulness of DEA.

**RESULTS AND DISCUSSION**

The proposed framework targets an academic institution of higher learning that offers programs leading to bachelor, Master, and Doctoral degrees. The institution consists of multi-disciplinary departments. Every department has its distinguished faculty members of all ranks: Full Professor, Associate Professor, Assistant Professor, Lecturer and Instructor. Although the results can be used to implement a promotion plan or merit increase strategy, it can also be used as a mentoring tool to promote academic excellence, leadership development and career exploration for faculty members.

**Input and output variables**

Based on the reviewed literature a total of 9 inputs and 16 outputs have been selected. These input and output variables covered all HE system aspects: Research; Teaching; Academic Support; Service to Community; and Service to Profession. However, the data availability is limited to 35 faculty members. A large number of inputs and outputs with respect to the number of faculty members lead to less discrimination between the best and worst performance. Hence, to have a robust system of evaluation, aggregation of inputs and outputs is needed. In order to reduce the number of inputs and outputs AHP technique is adopted. For example the 3 outputs that measured teaching dimension are aggregated into one output based on the weights computed from the AHP model. Followings are the attributes of each dimension with their relative’s weights calculated based on AHP analysis using the view of academic staff;

**Table 1: General Support from the University (Input)**

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support from the Institution (SFL)</td>
<td>0.20%</td>
</tr>
<tr>
<td>Support from the University (SU)</td>
<td>0.09%</td>
</tr>
<tr>
<td>Morale Support (MS)</td>
<td>0.21%</td>
</tr>
<tr>
<td>Research Leave (RL)</td>
<td>0.27%</td>
</tr>
<tr>
<td>Research Grant (RG)</td>
<td>0.23%</td>
</tr>
</tbody>
</table>
Based on table 1, the aggregated General Support variable is calculated as follows:

\[ \text{General Support} = 0.20 \times SPL + 0.09 \times SFU + 0.21 \times MS + 0.27 \times RL + 0.23 \times RG \]

The same is calculated for the other dimensions. Table 2 illustrates the resulted input and outputs

Table 2: DEA Model Inputs and outputs

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of courses exempted</td>
<td>Teaching evaluation by student</td>
</tr>
<tr>
<td>Last merit increase</td>
<td>Number of publications</td>
</tr>
<tr>
<td>Year of graduation</td>
<td>Number of publications completed but not yet submitted</td>
</tr>
<tr>
<td>Year within the institution with the same rank</td>
<td>Service to Community</td>
</tr>
<tr>
<td>Support received from the university or institution</td>
<td>Service to Profession</td>
</tr>
</tbody>
</table>

DEA-Analysis

The DEA model (2-5) is computed using the aggregated inputs and outputs. Table 3 and table 5 in the appendix illustrate the results. The relative faculty efficiency is varying between 53% - 100%. Out of 35 faculty members only 20 are fully efficient. The overall average efficiency score is 87.8% i.e. the faculty members should produce an extra 12.2% outputs from the given inputs to be fully efficient.

Table 3: Summary of DEA model results

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average scores</td>
<td>87.8%</td>
</tr>
<tr>
<td>No. of efficient DMUs</td>
<td>20</td>
</tr>
<tr>
<td>No. of inefficient DMUs</td>
<td>15</td>
</tr>
</tbody>
</table>

DEA has other advantages such as the improvement level. Here DEA can provide management with clear direction on how to improve the performance of inefficient members, table 4 shows the improvement level of faculty member 8 (DMU8) to be fully efficient. DMU8 obtained an efficiency score of (79 %) - rank 24. To be fully efficient DMU8 should achieve the following:

- Increase the teaching quality by 27%;
- Increase number of publications by 225%;
- Increase research score by 122%;
- Increase his services to community by 639%; and
- Increase the hours spent servicing the profession by 84%.

This means that faculty member number 8 needs to develop his skills in teaching; increase the number of publication; and spending more time in servicing the community and the profession.

CONCLUSION

It is very important to have a reliable and accurate Performance Appraisal System that leads to improvement, better communication, and innovation. However the effective implementation is the key for success. Moreover, the evaluation of a faculty member should be based on the environment enrolled in and the comparison should be done among all other faculty members that have the same input and output. The proposed framework using DEA and AHP approach identifies the best performers as well as the efficient and inefficient faculty members. It highlights the main areas of weaknesses and suggests ways for improvement to reach high efficiency level. Using the proposed framework, a poor performance faculty could be transformed to an outstanding one provided the existence of proper management supported and a transparent feedback system. Furthermore, the DEA model can be easily integrated into other existing Human Resources data-base system where data information on appraisal forms can be stored, linked to the payroll pay and compensation system. Once access to the original evaluation data is assured, the DEA excel-based system will compute automatically evaluation reports on efficiency scores, projections, rankings and correlations among input and output variables as opposed to the manual calculation in the current performance which is still in use at OSB. However, using the proposed DEA model entails the automation of the whole appraisal and evaluation process. It is easy to use, provides a better documentation and above all it avoids manual human errors. Also, it provides a measure of impact of new policy taken by management periodically. Finally, this paper provides a new application in the performance appraisal of faculty staff and can be recommended to the evaluation of employee in similar sectors.
REFERENCES


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QAA (2009). http://www.qaa.ac.uk/

RAE (2008). http://www.rae.ac.uk/


### APPENDICES

#### Table 5: Faculty members overall performance (DEA efficiency score)

<table>
<thead>
<tr>
<th>DMU</th>
<th>Eff. Score</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.65</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>0.45</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.96</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.79</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>0.53</td>
<td>34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DMU</th>
<th>Eff. Score</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0.76</td>
<td>25</td>
</tr>
<tr>
<td>14</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>0.69</td>
<td>30</td>
</tr>
<tr>
<td>19</td>
<td>0.76</td>
<td>26</td>
</tr>
<tr>
<td>20</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>0.69</td>
<td>29</td>
</tr>
<tr>
<td>24</td>
<td>0.70</td>
<td>28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DMU</th>
<th>Eff. Score</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>28</td>
<td>0.94</td>
<td>22</td>
</tr>
<tr>
<td>29</td>
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<tr>
<td>30</td>
<td>0.83</td>
<td>23</td>
</tr>
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<td>0.63</td>
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<td>31</td>
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<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>1.00</td>
<td>1</td>
</tr>
</tbody>
</table>

#### Table 6: REAPS Framework's academic activities, operating supports and associated decisions

<table>
<thead>
<tr>
<th>AUB Mission</th>
<th>Decisions</th>
<th>Academic Activities</th>
<th>Operating Support (inputs)</th>
<th>Infrastructures (inputs)</th>
<th>Various Outputs/Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic</td>
<td>• Identify and Design of teaching / research programs to offer;</td>
<td>• Design criteria for assessing and measuring performance (AMP) in conformity with mission‘ goals for learning &amp; teaching (LT); • Identify Stakeholders to involve AMP of LT; • Establish E-learning and Teaching support Centers • Establish institutional data center</td>
<td>• Design IT network (inter-net) to access the operating support from anytime and anyplace.</td>
<td>• Teaching books produced, • New program/course designed.</td>
<td></td>
</tr>
<tr>
<td>Tactical</td>
<td>• Develop contents of program;</td>
<td>• Define AMP indicators for assessing and measuring of LT criteria. • Provide LT assistant support</td>
<td>• Prove general of E-learning and teaching systems</td>
<td>• Number of different courses taught and their number of students.</td>
<td></td>
</tr>
<tr>
<td>Operational</td>
<td>• Participate in learning and teaching of courses.</td>
<td>• Collect data on AMP criteria. • Amount of provision of teaching assistant support</td>
<td>• Provide specific LT systems for specific courses/ programs. • Provide online help –desk support</td>
<td>• Numbers of taught or supervised students.</td>
<td></td>
</tr>
<tr>
<td>Strategic</td>
<td>• Generate research fund; • Identify research themes</td>
<td>• Define criteria and type research activities that can be funded internally; • Reward scheme for getting external grant.</td>
<td>• Provide research books/journals and databases</td>
<td>• Number of research obtained grants.</td>
<td></td>
</tr>
<tr>
<td>Tactical</td>
<td>• Join professional &amp; community committees/ boards; • Produce scholarly research works; • Identify Impact criteria</td>
<td>• How much fund Allocation of Fund • Criteria for obtaining internal research grant • Amount of institutional support.</td>
<td>• Connect such databases in a one point-system access network</td>
<td>• Number of papers in international refereed journals with impact factors. • Number of proceedings published/ presented. • Number of author` citations.</td>
<td></td>
</tr>
<tr>
<td>Operational</td>
<td>• Disseminate research works</td>
<td>• Provide technical help – desk support</td>
<td>• Provide online research help– desk support</td>
<td>• Number of research grant applied/ working papers/report produced.</td>
<td></td>
</tr>
<tr>
<td>Strategic</td>
<td>• Identify strategic academic, educational and service stakeholders to be partners in each country of working region.</td>
<td>• Establish Institutional Office to engage faculty. • Strategy reward stakeholders • Promote the services.</td>
<td>• Design location to store mirror to easy access and communication with stakeholders. • Provide video / telephone conferencing IT infrastructures.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tactical</td>
<td>• Appropriate compensation scheme types to engage people</td>
<td>• Collection of data on involvement types and measures</td>
<td>• Provision of decision making support systems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operational</td>
<td>• Operate the services • Count the level and amount of service engagements. • Number of active service committees</td>
<td>• Effective communication tools.</td>
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</tr>
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</table>
Measuring productivity of Polish research units: A DEA approach

Joanna Urban
Bialystok University of Technology, Faculty of Management, Wiejska Street 45A, 15-351 Bialystok, Poland, j.urban@pb.edu.pl

ABSTRACT

The objective of this article is to present the results of measuring and comparing productivity in Polish research units through data envelopment analysis (DEA). It describes the development of the methodology and the instrument to support a research funding allocation decision by the Polish government. The assessment was done on the basis of the information from the questionnaires that are submitted by all scientific units in Poland to the Ministry of Science and Higher Education (MNiSW). DEA method is presented as an alternative tool to the present parametric assessment of scientific units in Poland conducted by the Ministry of Science and Higher Education. The method applies a multiple input and output variables approach. To deliver reasonable results, suitable input and output variables have been determined in a previous step using statistical analysis.

Keywords: data envelopment analysis (DEA), research productivity, parametric assessment.

INTRODUCTION

Scientific achievements and prestige of scientists, scientific teams and institutions are subject to a more scrutinised assessment process by different bodies. It is largely due to the growing role of science in the development of the knowledge-based economy as well as due to the common knowledge that the economic and social growth depend on the level of the research advancement and on the application of its results. High level and innovative research can become a driving force in the process of the country modernisation [0, 0, 0].

The evaluation of the research activity results is an important instrument in the organisational and financial management of research institutions. It becomes a crucial element in shaping the research policy and in implementing the research activities in different units of the research sector [0, 0, 0]. Evaluation and comparison of the research results improves the quality of science and creates a more competitive environment leading to wider openness of the research institutions to market needs and their greater flexibility in the co-operation with other research entities or units [0].

Given the complexity of the research activity, the evaluation of research units on the national level is a difficult and complicated task. Scientific achievements or prestige of either individual researchers or research teams is subject to diversified assessment methods.

A traditional approach towards the evaluation of the level and the impact of the research achievements is expressed in a qualitative way and it is largely based on the opinions of other specialists in the relevant field of study.

Currently, a range of criteria and methods for the research activity evaluation has increased due to the external pressure on the practical application of the research results [0, 0, 0].

Since the early 70's in the research results management, more importance has been given to the questions of the research productivity and its evaluation. A research unit can be characterised by means of inputs and outputs combined with the transformation processes which turn the resources into the results. Consequently, research productivity is understood as a result of the research activity in relation to the resources used for a given activity in a given period of time [0, 0, 0, 0]. The main objective of the research productivity analysis is the evaluation of the research activity and the quality assessment.

RESEARCH ASSESSMENT IN POLAND

The present paper uses data from the questionnaires submitted by all public scientific units in Poland to the Ministry of Science and Higher Education (MNiSW) covering the period between 2001-2004 [0]. On the basis of the submitted data, the MNiSW conducted a parametric assessment of the public scientific units. Public research units are composed of basic organisational units of higher education institutions (faculties), research institutions of the Polish Academy of Sciences, and non-university public R&D units. The research institutes are mainly public funded with the aim to produce scientific research according to the general guidelines set by MNiSW.

The parametric assessment and the categorisation resulting from this method
constitute a comparative system of the ‘research production’ of individual units. The system is based on the multiple outputs and as an input it considers a number of staff employed in a given unit to perform the scientific research and R&D activities.

Research outcomes of the parametric assessment are measured with regards to the three aspects: (1) activity of research units, (2) results of research performance and (3) practical implementations. Each field is represented by detailed variables (Appendix 1). In general, 36 quantitative variables are considered to which numerical values were assigned, which determine the weight of the features.

For every research unit \( j \) partial index of efficiency \( E_{j,k} \) is calculated as a weighted sum of variables in each aspect \( k \) as follows:

\[
E_{j,k} = \frac{\sum_{i=1}^{n_k} x_{j,i} v_i}{N_j}
\]

\( E_{j,k} \) – partial index of efficiency for \( j \)-unit in the \( k \)-field

\( x_{j,i} \) – value of \( i \)-variable for \( j \)-unit

\( v_i \) – weight of \( i \)-variable

\( n_k \) – number of variables in the \( k \)-field

\( N_j \) – number of R&D employees for \( j \)-unit

The Ministry classified all the research units into 20 homogenous groups according to the type of research activity they specialise in.

The partial index of each unit is divided by the highest index in the homogenous group. The same procedure is conducted in second and third field so in each of the homogenous groups relative effectiveness indexes \( (E_{w,j,k}) \) for each \( k \) field are determined.

\[
E_{w,j,k} = \frac{E_{j,k}}{\max(E_{j,k})}
\]

\( E_{w,j,k} \) – relative partial index of efficiency for \( j \)-unit in the \( k \)-field within homogenous group

The final index of efficiency is a weighted sum of the relative partial indexes. Every homogenous group has different weights.

\[
E_j = \sum_{k=1}^{3} E_{w,j,k} w_{k,g}
\]

\( w_{k,g} \) – weight of partial index of efficiency for \( k \) field in the homogenous group \( g \)

The obtained efficiency indicators form a ranking basis within the homogenous groups and determine the assignment of an adequate research category. The range is based on a scale from 1 to 5, which determines the distribution of financial resources for a research activity.

**DEA ANALYSIS**

For the evaluation of decision making units (DMUs) with multiple-inputs and multiple-outputs in a public sector, data envelopment analysis (DEA) is now one of the most widely accepted methods to measure relative productivity of research institutions \([0, 0, 0, 0, 0, 0] \).

The reliability of DEA models decreases if too many inputs or outputs are used. The selection of comprehensive indicators becomes difficult if stakeholders aim to achieve a relatively holistic evaluation as too many variables disturb differentiation of DMUs.

This is especially the case in the evaluation of public research units in Poland, where many different outputs are measured in the evaluation in order to produce relatively comprehensive performance profiles of these institutes.

Yet a different problem is observed in the case of research units which want to apply the system of research unit evaluation to create their research policy or to mark out new fields of the research activity. At the beginning of the evaluation, many research units prove no output values.

The rules of parametric assessment that have been used so far have raised many doubts in the academic environment, concerning their rationale, usefulness and the criteria of the assessment. Different examples of DEA method implementation lead to the conclusion that its application for the evaluation of Polish research units productivity is well justified, which is a purpose of the present paper.

**RESEARCH METHODOLOGY**

The study of the methodology of parametric assessment was initiated with the sampling of the variables. The analysis covered 65 units of the homogenous group G1 – units representing mechanics, materials, chemical and processing engineering. As the input variable the number of R&D staff was taken into consideration. The coefficients of variation were checked – all variables significantly differentiate the objects. Next, Spearman’s correlation analysis disclosed that the part of the variables is highly correlated and these variables convey similar information. It means that the number of variables can be limited, because its excess set only seemingly improves diversification of the assessed units.
Variables highly and insignificantly correlated with the input variable were excluded from the analysis. After the correlation analysis was carried out, the number of output variables was reduced to 5 variables: X1, X3, X8, X19, and X23.

As a subsequent step the mechanism of factor analysis was applied. The factor analysis comprises a set of methods and procedures used to replace a big number of variables with a few insignificantly correlated factors.

Figure 1: Illustration of the CCR model results
Source: Author’s elaboration

Figure 2: Results of DEA analysis (BCC output oriented model)

They keep relatively much information conveyed in the initial variables and at the same time each of them is a means of different merit content.

As a result of the analysis two factors were obtained. The separated main components describe over 81% of variation of the data (Table 1). The variables inside the factors are highly correlated, the variables were therefore isolated which are attributed by the highest factor weight. Later, while assigning them to the factors, two fields were determined, which should be taken into consideration in assessment of the research units.

For the sake of the further analysis a model with one input variable – number of R&D employees (Input1) – and two output variables: professor promotions (Output1), non-SCI publications (Output2) were taken into consideration. This set of variables enables a clear graphic interpretation of the results in a two-dimensional coordinate system (Picture 1).

The CCR and BCC output oriented models were used. In the appendice 2, the result of DEA analysis is presented.

Out of 65 units the 100% productivity indicator appeared in 9 units with BCC model (figure 2).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Factor 1</th>
<th>Factor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.864</td>
<td>0.044</td>
</tr>
<tr>
<td>X3</td>
<td>0.879</td>
<td>0.132</td>
</tr>
<tr>
<td>X8</td>
<td>0.861</td>
<td>0.239</td>
</tr>
<tr>
<td>X19</td>
<td>0.085</td>
<td>0.927</td>
</tr>
<tr>
<td>X23</td>
<td>0.174</td>
<td>0.906</td>
</tr>
</tbody>
</table>

The level of productivity is quite diversified. The mediane shows that productivity of the half of the units is below 66.10%. The scientific units demonstrate also changeable economies of scale.

On this basis setting the research categories could be more objective and clarified.

CONCLUSION

The author claims that the comparative analysis of productivity of research units with the DEA method can be a source of valuable managerial information, which could be the basis for scientific assessment in Poland. The application of the method may contribute to the increase of the objectivity of the weights which are currently arbitrarily assigned. It can also assess the level of the conducted scientific research and identify the units of a model efficiency level in national circumstances. A separate optimization for each unit exposes strengths of the unit determined by the resources and the
environment. Furthermore it allows also determining the so called “dead” resources, which do not influence significantly the results achieved by the unit.

The example used in the analysis has an illustrative character and requires further studies.

REFERENCES


APPENDICES

Appendix 1: Variables of the parametric assessment

<table>
<thead>
<tr>
<th>Group of variables</th>
<th>Name of variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Activity of research units

- X1 Doctoral degrees
- X2 Postdoctoral degrees
- X3 Professor promotions
- X4 Entitlement to grant a PhD
- X5 Entitlement to grant a PhD, DSc
- X6 Implementation of a procedure of an international quality assurance system
- X7 Laboratories with the accreditation the Polish Centre of Accreditation or other equal
- X8 Own scientific grants
- X9 Ordered grants
- X10 Direct grants with the input of the applicant of over 500 thousand zlotys
- X11 Direct grants with the input of the applicant of between 200 - 500 thousand zlotys
- X12 Direct grants with the input of the applicant of below 200 thousand zlotys
- X13 Completed international contract financed from international sources
- X14 International award or honourable mention
- X15 National award for scientific activity
- X16 Reward or honourable mention for practical implementation of the results of the R&D

### Results of research performance

- X17 SCI publications
- X18 International or domestic non-SCI publications in ethnic language
- X19 International or domestic non-SCI publications
- X20 Publication in a reviewed journal of a local scope
- X21 Authorship of monographs or academic textbooks in English
- X22 Authorship of chapters in monographs or academic textbooks in English
- X23 Authorship of monographs or academic textbooks in Polish or any other language despite English
- X24 Authorship of chapters in monographs or academic textbooks in Polish or any other language despite English
- X25 Editorship of monographs or academic textbooks

### Practical implementations

- X26 Contacts with another subject for R&D of the value of over 50 thousand zlotys, with the aim fulfilled
- X27 Permanent or long-lasting contracts with another subject for scientific research
- X28 National patents
- X29 International patents
- X30 Utility model protection rights
- X31 Financial income deriving from technological transfer activities of over 10 million zlotys
- X32 Financial income deriving from technological transfer activities of 5-10 million zlotys
- X33 Financial income deriving from technological transfer activities of 1-5 million zlotys
- X34 Licences with the licence fee of over 1 million zlotys
- X35 Licences with the licence fee of from 500 thousand zlotys to 1 million zlotys
- X36 Licence with the licence fee of below 500 thousand zlotys

### Appendix 2: Scores of DEA analysis: Output Oriented Model

<table>
<thead>
<tr>
<th>Activity of research units</th>
<th>Results of research performance</th>
<th>Practical implementations</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1 Doctoral degrees</td>
<td>X17 SCI publications</td>
<td>X26 Contacts with another subject for R&amp;D of the value of over 50 thousand zlotys, with the aim fulfilled</td>
</tr>
<tr>
<td>X2 Postdoctoral degrees</td>
<td>X18 International or domestic non-SCI publications in ethnic language</td>
<td>X27 Permanent or long-lasting contracts with another subject for scientific research</td>
</tr>
<tr>
<td>X3 Professor promotions</td>
<td>X19 International or domestic non-SCI publications</td>
<td>X28 National patents</td>
</tr>
<tr>
<td>X4 Entitlement to grant a PhD</td>
<td>X20 Publication in a reviewed journal of a local scope</td>
<td>X29 International patents</td>
</tr>
<tr>
<td>X5 Entitlement to grant a PhD, DSc</td>
<td>X21 Authorship of monographs or academic textbooks in English</td>
<td>X30 Utility model protection rights</td>
</tr>
<tr>
<td>X6 Implementation of a procedure of an international quality assurance system</td>
<td>X22 Authorship of chapters in monographs or academic textbooks in English</td>
<td>X31 Financial income deriving from technological transfer activities of over 10 million zlotys</td>
</tr>
<tr>
<td>X7 Laboratories with the accreditation the Polish Centre of Accreditation or other equal</td>
<td>X23 Authorship of monographs or academic textbooks in Polish or any other language despite English</td>
<td>X32 Financial income deriving from technological transfer activities of 5-10 million zlotys</td>
</tr>
<tr>
<td>X8 Own scientific grants</td>
<td>X24 Authorship of chapters in monographs or academic textbooks in Polish or any other language despite English</td>
<td>X33 Financial income deriving from technological transfer activities of 1-5 million zlotys</td>
</tr>
<tr>
<td>X9 Ordered grants</td>
<td>X25 Editorship of monographs or academic textbooks</td>
<td>X34 Licences with the licence fee of over 1 million zlotys</td>
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<tr>
<td>X10 Direct grants with the input of the applicant of over 500 thousand zlotys</td>
<td>X26 Contacts with another subject for R&amp;D of the value of over 50 thousand zlotys, with the aim fulfilled</td>
<td>X35 Licences with the licence fee of from 500 thousand zlotys to 1 million zlotys</td>
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<tr>
<td>X11 Direct grants with the input of the applicant of between 200 - 500 thousand zlotys</td>
<td>X27 Permanent or long-lasting contracts with another subject for scientific research</td>
<td>X36 Licence with the licence fee of below 500 thousand zlotys</td>
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<tr>
<td>X12 Direct grants with the input of the applicant of below 200 thousand zlotys</td>
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<tr>
<td>X13 Completed international contract financed from international sources</td>
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<td>X14 International award or honourable mention</td>
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<td>X15 National award for scientific activity</td>
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<td>X16 Reward or honourable mention for practical implementation of the results of the R&amp;D</td>
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<tr>
<td>X17 SCI publications</td>
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<td>X18 International or domestic non-SCI publications in ethnic language</td>
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<td>X19 International or domestic non-SCI publications</td>
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<tr>
<td>X20 Publication in a reviewed journal of a local scope</td>
<td>X36 Licence with the licence fee of below 500 thousand zlotys</td>
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</tbody>
</table>

### Efficiency Score

#### Unit

<table>
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<tr>
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<th>Unit</th>
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</thead>
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<tr>
<td>CCR</td>
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#### Efficiency Score

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</tr>
<tr>
<td>DMU56</td>
<td>55.90%</td>
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</table>
Health Care Applications Stream
Efficiency of Turkish provincial general hospitals with mortality as undesirable output

Nurhan Davutyan  
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Murat Bilsel  
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ABSTRACT

We present a directional distance model where quality of care is brought in by treating mortality in each hospital as a strongly disposable ‘bad output’. After deriving pure technical and scale inefficiencies under strong disposability, we derive “congestion” inefficiencies via allowing weak disposability. A second stage ‘seemingly unrelated’ regression of these inefficiencies against hospital level variables like spare capacity, inpatient to outpatient ratio and bed turnover rate, allows pinpointing the critical areas for hospital performance improvement.

Keywords: Data Envelopment Analysis, Turkish Hospital Efficiency, Undesirable output,

LITERATURE REVIEW

Nunamaker and Lewin (1983) provided the first application of Data Envelopment Analysis to health care. Sherman (1984) was the first author to use DEA to evaluate overall hospital efficiency. This literature is competently surveyed by O’Neill et al (2008), Ozcan (2008) and Hollingsworth (2008). The first paper emphasizes national differences in hospital efficiency research. The second has a broader scope: it encompasses every aspect of health care delivery, as well as providing an overview of existing techniques. The last author classifies 317 published papers into various subcategories and offers comments as to their practical usefulness.

The works dealing with the Turkish health care system comprise Ersoy et al (1997), Sahin and Ozcan (2000) and Sahin (2009). The first study computes CRS efficiency scores for 573 acute general hospitals and identifies 90.6% of them as inefficient. The second paper treats the public hospital system in each one of Turkey’s 80 provinces as a DMU. The average VRS efficiency score turns out to be 0.88, the standard deviation being 0.15. This paper carefully analyzes the structure of input excesses. The authors conclude excess nurses and other personnel are 13% and 12% whereas excess specialists and general practitioners are 6.5% and 5% respectively. The final paper by Sahin (2009) contains useful institutional information about the Turkish health care system and its evolution over time.

MODELING

As stressed by Jacobs et al (2006), efficiency analysis should be based on outcomes of care. However researchers are often constrained to examine efficiency on the basis of measured activities like patients treated or surgeries performed. When there is room to suspect that the effectiveness of such measured activities differ between institutions, it is imperative to augment activity counts with indicators of quality of outcome. Although direct measures of health gain are best, the few analysts addressing the issue are typically forced to use proxies like mortality or readmission rates. For instance of the 317 studies surveyed by Hollingsworth (2008) only 9% use outcome measures like change in health status, mortality or quality of care.

In this study we use mortality figures of each hospital as a quality indicator. Clearly, mortality is a ‘bad’ or ‘undesirable’ output. However researchers are often constrained to examine efficiency on the basis of measured activities like patients treated or surgeries performed. When there is room to suspect that the effectiveness of such measured activities differ between institutions, it is imperative to augment activity counts with indicators of quality of outcome. Although direct measures of health gain are best, the few analysts addressing the issue are typically forced to use proxies like mortality or readmission rates. For instance of the 317 studies surveyed by Hollingsworth (2008) only 9% use outcome measures like change in health status, mortality or quality of care.

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NON ORIENTED DIRECTIONAL DISTANCE

The model we use has its origins in the environmental efficiency literature, Fare and Grosskopf (2004), where undesirable by products like sulfur emissions are of interest. The directional distance approach allows good...
output expansion and input as well as bad output contraction simultaneously.

O’Neill et al (2008) point out the hospital efficiency literature prefers the input orientation since in most countries, but particularly the US, cost containment has been and is the order of the day. But even in that litigation prone country, despite spiraling costs, a substantial portion of the population lacks meaningful access to health care, as it became painfully clear during President Obama’s first year in office. Thus as pointed out by Hollingsworth and Spinks (2009) the public wants both good health outcomes and cost containment. In the Turkish case, OECD (2008) finds its bed occupancy rate of 69% below the OECD average of 75% and calls for its increase. This implies output expansion, since to increase the occupancy rate via input contraction would be tantamount to saying there are too many hospital beds in Turkey! However since most health care in Turkey is publicly provided and prior studies find considerable “input waste” we adopt a non-oriented approach.

The CRS version of the directional distance model consists of

\[
\max \quad \beta_0, \lambda
\]

\[
\text{ST: } \sum_{j=1}^{N} \lambda_j g_{rj} \geq (1 + \beta_0)g_{rv0}, r = 1 \to R \quad \text{(good outputs)}
\]

\[
\sum_{j=1}^{N} \lambda_j b_{kj} \leq (1 - \beta_0)b_{kv0}, k = 1 \to K \quad \text{(bad outputs)}
\]

\[
\sum_{j=1}^{N} \lambda_j x_{ij} \leq (1 - \beta_0)x_{iv0}, i = 1 \to I \quad \text{(inputs)}
\]

Where \( r \) indexes the \( R \) good outputs to be expanded, \( k \) indexes the \( K \) bad outputs to be contracted and \( i \) indexes the \( I \) inputs to be shrunk. The “\( \leq \) or LTE” symbol imposes strong disposability. It means technology is such that reducing mortality does not necessarily require reducing other useful hospital activities. The VRS model is obtained by appending the convexity constraint.

Weak disposability involves replacing the inequality of the bad output(s) equation(s) with equality. This implies reducing “bads” is costly; it necessitates the reduction of the good outputs as well. In addition weak disposability under VRS involves modifying the output equations (both good and bad) as follows:

\[
\sum_{j=1}^{N} \rho \lambda_j g_{rj} \geq (1 + \beta_0)g_{rv0} r = 1 \to R \quad \text{(good outputs)}
\]

\[
\sum_{j=1}^{N} \rho \lambda_j b_{kj} = (1 - \beta_0)b_{kv0}, k = 1 \to K \quad \text{(bad outputs)}
\]

where \( 0 \leq \rho \leq 1 \). Strictly speaking this makes the model non-linear. However a grid search for the value of \( \rho \) over \([0, 1]\) is sufficient to solve it as an LP problem\(^3\). See Picazo-Tadeo and Prior (2005).

Data-Inputs and Outputs

In Turkey there are 1205 hospitals, 42 military and 1163 civilian. The Ministry of Health is in charge of civilian hospitals. It directly owns and operates 769 hospitals and oversees the rest (394). Out of this total 332 are private, 56 are university hospitals and the remaining 6 are operated by municipalities. However since private hospitals are smaller, the share of the public/semi-public sector is larger than the ownership figures suggest. For instance the public sector accounts for 92% of overall bed capacity. The functional breakup of these 831 public/semi-public hospitals is as follows: 562 general, 107 specialties, 56 universities and 52 teaching. Out of these 562 general hospitals 64 are located in metropolitan areas, 92 are located in provincial centers and 406 are located in rural areas or small towns. In order to compare “like with like” we focus on 402 rural and small town general hospitals since 4 were dropped due to data irregularities.\(^4\)

Table 1 displays the summary statistics for these 402 hospitals. They represent 18.7% of overall Turkish bed capacity. By contrast, according to the 2008 electronic population registry figures, about 39 % of Turkish people live in rural areas and small towns.

Our data is from the Ministry of Health’s website. We use 3 inputs: bed capacity, specialists and general practitioners. Figures

\(^2\) According to OECD (2009), Turkey has 2.7 acute hospital beds per 1000 population, substantially less than the OECD average of 3.8

\(^3\) We thank Diego Prior for clarifying this point.

\(^4\) For instance, one hospital reported 2865 deaths, more than six times the next highest number and ten times the average mortality rate.
for nurses and other staff and general expenses are not available. Since our data set is “sufficiently homogenous” we hope their variation among hospitals is not excessive. We have 1 bad, 4 good outputs. The good ones are: Inpatient Discharges, Outpatient Visits, Surgeries and Births. There is no diagnostic related groupings index in Turkey. Therefore outputs were not weighted on a DRG basis. However major, medium and minor surgeries were converted into a major surgery equivalent. The weights were major=1, medium=1/3 and minor 1/7, see Buyukkayikci and Sahin (2000).

Table 2 presents the break-up of these 402 hospitals according to capacity and their average input-output levels. As can be seen the majority are small (0 to 50 beds). Average input usage and average production rises with size.

INEFFICIENCY ESTIMATES

We computed CRS, VRS, Scale and Output Congestion inefficiencies using our directional distance model. The CRS and VRS figures are obtained directly, assuming strong disposability. Subtracting the VRS estimate from the CRS one, yields the scale inefficiency estimate. The difference between the VRS figures obtained under strong vs. weak disposability respectively, gives the “congestion” inefficiency. This figure shows whether and to what extent reducing “deaths” competes with other desirable hospital activities. Thus an output congestion inefficiency of zero implies no such “competition”, whereas a positive figure means reducing deaths involves giving up some desirable activities. In other words congested hospitals need more resources. Table 3 displays the average inefficiencies by hospital size as well as the overall figures.

According to our estimates overall CRS inefficiency is around 23% which breaks up as 13% VRS and 10% scale. The overall congestion inefficiency is 20 per 10,000. Taken literally this suggests out of about 7,700 deaths that occurred, around 15 (7700 x 0.002) were due to “output congestion” or “resource shortage”. Our regression results presented below indicate the critical element was a lack of specialists. Further analysis of our estimates indicates congestion inefficiency is particularly critical in 31-50 bed hospitals, followed by the 76-100 range. The 31-50 bed hospitals also have the highest pure technical inefficiency or “waste”. This suggests careful analysis by the Health Ministry is warranted.

Small hospitals with 0-30 beds exhibit high levels of scale inefficiency. This suggests that economically, operating such small hospitals is not the best use of available resources. As policy makers have to balance the conflicting objectives of maximizing access to health care and minimizing cost, except in remote and isolated locations shifting resources from these hospitals to primary health care should be considered to provide “more equitable, inclusive and fair” health care service.

REGRESSION

We use seemingly unrelated regression (SUR) to identify factors that influence the VRS, scale and congestion inefficiencies for each hospital. The method exploits the fact that all 3 dependent variables pertain to the same hospital and therefore the disturbances of each equation may be correlated. Thus the 3 equations are treated as a system. The greater the correlation across the disturbances, the larger is the efficiency gain. On the other hand the less correlation there is across the explanatory variables of each equation, the greater is the gain from adopting a system approach. Within the DEA tradition the method was first used by Fried et al (1996). Following McDonald (2009) we used least squares estimation with heteroskedasticity consistent standard errors to allow for the high proportion of zeroes-corresponding to efficient hospitals.

Our regression results suggest pure technical inefficiency rises with SPARE CAPACITY (Bed Capacity minus actual usage6), inpatient to outpatient RATIO and number of SPECIALISTS. This latter finding lends credence to the common perception that some specialists use their public hospital jobs as a means of attracting patients to their private practices. The impact of the in-to-outpatient RATIO merits further consideration. Because the lack of a DRG based case mix weighting tends to “show” large hospitals with many specialists as inefficient. To some extent the same applies to our finding about SPECIALISTS. On the other hand VRS inefficiency falls with bed TURNOVER rate and number of SURGERIES performed. As for Scale inefficiency, it falls with bed TURNOVER rate and BED CAPACITY and rises with number of SURGERIES. We believe the first two indicate the presence of increasing returns to scale in small hospitals and the third

---

6 This is remarkably close to Sahin and Ozcan (2000)’s finding.

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6 From Director-General’s message, World Health Report 2008, WHO

7 Actual bed usage figures as well as bed turnover rates and inpatient to outpatient ratios are from the Health Ministry’s website.
points out to decreasing returns in the relatively few large hospitals performing the bulk of surgeries (out of 402 hospitals, the 21 (48) largest perform 48% (65%) of the surgeries). Finally, in the Congestion inefficiency regression, the positive SPARE CAPACITY and the negative SPECIALIST coefficients indicate the following: where it occurs, “competition” between ‘death reduction’ and other hospital activities is mainly due to a shortage of specialists rather than availability of beds.

**SUMMARY**

Using a directional distance model, we calculated CRS, VRS, Scale and Output Congestion inefficiencies for 402 provincial general hospitals in Turkey. Our main findings include: a) scale inefficiency is concentrated in the 107 smallest hospitals, b) there is some evidence of decreasing returns to scale wrt surgeries in the largest hospitals, c) congestion inefficiency is related to a lack of specialists in the (31-50) and (76-100) bed hospitals, d) the observed pure technical inefficiency in public hospitals might be linked to some specialists’ using their public jobs to attract patients for their private practices and e) more precise analysis requires introducing DRG based case mix weighting.

**REFERENCES**


Sahin, I (2009). “Total factor productivity analysis of Turkish Social Security hospitals that were transferred to the Health Ministry” (in Turkish), Iktisat Isletme ve Finans, 24: 9-40.


### Table 1: Summary Statistics of Inputs and Outputs

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>StDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bed Capacity</td>
<td>25</td>
<td>550</td>
<td>82</td>
<td>75</td>
</tr>
<tr>
<td>Specialists</td>
<td>0</td>
<td>87</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>General Practitioners</td>
<td>0</td>
<td>35</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Outpatient Visits</td>
<td>0</td>
<td>1,073,004</td>
<td>144,049</td>
<td></td>
</tr>
<tr>
<td>Inpatient Discharges</td>
<td>3</td>
<td>21,503</td>
<td>2,893</td>
<td></td>
</tr>
<tr>
<td>Surgeries</td>
<td>13</td>
<td>5,520</td>
<td>2,93</td>
<td></td>
</tr>
<tr>
<td>Births</td>
<td>3</td>
<td>3,658</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>Deaths</td>
<td>0</td>
<td>466</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2: Input and Output Averages by hospital size

<table>
<thead>
<tr>
<th>Bed Capacity</th>
<th># of Hospitals</th>
<th>Specialists</th>
<th>General Practitioners</th>
<th>Outpatient Visits</th>
<th>Inpatient Discharges</th>
<th>Surgeries</th>
<th>Births</th>
<th>Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-30</td>
<td>107</td>
<td>2.4</td>
<td>4.7</td>
<td>53,233</td>
<td>448</td>
<td>29</td>
<td>68</td>
<td>1.3</td>
</tr>
<tr>
<td>31-50</td>
<td>136</td>
<td>6.3</td>
<td>6</td>
<td>93,596</td>
<td>1,283</td>
<td>180</td>
<td>247</td>
<td>5.6</td>
</tr>
<tr>
<td>51-75</td>
<td>28</td>
<td>10.3</td>
<td>7.4</td>
<td>141,388</td>
<td>2,530</td>
<td>462</td>
<td>500</td>
<td>7.8</td>
</tr>
<tr>
<td>76-100</td>
<td>48</td>
<td>13.5</td>
<td>7.8</td>
<td>178,359</td>
<td>3,678</td>
<td>840</td>
<td>589</td>
<td>16.7</td>
</tr>
<tr>
<td>101-150</td>
<td>35</td>
<td>19.8</td>
<td>9.4</td>
<td>220,396</td>
<td>5,394</td>
<td>1,352</td>
<td>846</td>
<td>35.3</td>
</tr>
<tr>
<td>151-200</td>
<td>27</td>
<td>31.6</td>
<td>12.1</td>
<td>318,690</td>
<td>8,122</td>
<td>2,143</td>
<td>1,057</td>
<td>80.1</td>
</tr>
<tr>
<td>200+</td>
<td>21</td>
<td>54.1</td>
<td>18.8</td>
<td>506,868</td>
<td>13,568</td>
<td>3,514</td>
<td>2,034</td>
<td>113.5</td>
</tr>
</tbody>
</table>

### Table 3: Inefficiency estimates.

<table>
<thead>
<tr>
<th>Bed Capacity</th>
<th>CRS</th>
<th>VRS</th>
<th>Scale</th>
<th>Congestion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-30</td>
<td>0.3144***</td>
<td>0.0463***</td>
<td>0.2681***</td>
<td>0.00015***</td>
</tr>
<tr>
<td>31-50</td>
<td>0.2397***</td>
<td>0.1977***</td>
<td>0.0420***</td>
<td>0.00108***</td>
</tr>
<tr>
<td>51-75</td>
<td>0.1858***</td>
<td>0.1733***</td>
<td>0.0125***</td>
<td>0.00202**</td>
</tr>
<tr>
<td>76-100</td>
<td>0.1260***</td>
<td>0.1148***</td>
<td>0.0112***</td>
<td>0.00010**</td>
</tr>
<tr>
<td>101-150</td>
<td>0.1475***</td>
<td>0.1277***</td>
<td>0.0199***</td>
<td>0.00321*</td>
</tr>
<tr>
<td>151-200</td>
<td>0.1707***</td>
<td>0.1190***</td>
<td>0.0516***</td>
<td>0.01254*</td>
</tr>
<tr>
<td>200+</td>
<td>0.1720***</td>
<td>0.0739***</td>
<td>0.0980***</td>
<td>0.00548*</td>
</tr>
<tr>
<td>Overall Average</td>
<td>0.2260***</td>
<td>0.1280***</td>
<td>0.0981***</td>
<td>0.0020***</td>
</tr>
</tbody>
</table>

Overall SD: 0.1766

Significantly different from zero at 1 %(***), 5 %(**) and 10 %(*)

### Table 4: Seemingly Unrelated Regression

<table>
<thead>
<tr>
<th>Equation</th>
<th>Obs.</th>
<th>Parameters</th>
<th>RMSE</th>
<th>R-square</th>
<th>Chi2</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRS</td>
<td>402</td>
<td>5</td>
<td>0.1242</td>
<td>0.175</td>
<td>103.54</td>
<td>0</td>
</tr>
<tr>
<td>Scale</td>
<td>402</td>
<td>3</td>
<td>0.136</td>
<td>0.201</td>
<td>107.09</td>
<td>0</td>
</tr>
<tr>
<td>Congestion</td>
<td>402</td>
<td>3</td>
<td>0.0085</td>
<td>0.589</td>
<td>592.55</td>
<td>0</td>
</tr>
</tbody>
</table>

VRS Coeff Z value P value Corr(VRSμ, Scaleμ) = -0.4341

<table>
<thead>
<tr>
<th>SPARE</th>
<th>0.0011</th>
<th>4.5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TURNOVER</td>
<td>-0.0024</td>
<td>-5.96</td>
<td>0</td>
</tr>
<tr>
<td>RATIO</td>
<td>0.0281</td>
<td>3.31</td>
<td>0.001</td>
</tr>
<tr>
<td>SURGERIES</td>
<td>-0.000074</td>
<td>-6.73</td>
<td>0</td>
</tr>
<tr>
<td>SPECIALIST</td>
<td>0.005</td>
<td>5.62</td>
<td>0</td>
</tr>
<tr>
<td>Constant</td>
<td>0.1357</td>
<td>31.49</td>
<td>0</td>
</tr>
</tbody>
</table>

Scale Coeff Z value P value Corr(Scaleμ, Congestionμ) = -0.1029

| TURNOVER | -0.003 | -8.96 | 0 |
| BED CAPACITY | -0.00063 | -4.29 | 0 |
| SURGERIES | 0.000037 | 4.23 | 0 |
| Constant | 0.2167 | 12.23 | 0 |

Congestion Coeff Z value P value Corr(Congestionμ, VRSμ) = 0.0604

| DEATHS | 0.0003 | 2.9 | 0.004 |
| SPARE | 0.000049 | 1.88 | 0.06 |
| SPECIALIST | -0.0052 | -2.63 | 0.009 |
| Constant | 0.002 | 2.74 | 0.006 |
Using Data Envelopment Analysis to evaluate productivity of public hospitals in Jordan

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ABSTRACT
The use of Data Envelopment Analysis (DEA) exists in several application fields. In this paper, we will present using DEA for evaluating the productivity of the public hospitals in Jordan. We used two widely popular models for measuring efficiency: CCR and BCC. They are used to measure the technical, and cost efficiency from different perspectives of multiple selected inputs and outputs. Each hospital is considered a Decision Making Unit (DMU) and DMUs are clustered based on similarities of certain criteria. The ultimate goal of using DEA for measuring hospitals productivity is to monitor and improve the efficiency of those hospitals that show poor productivity relative to similar ones. Results showed some different indicators between applying BCC and CCR models for some hospitals.  

Keywords: Data Envelopment Analysis (DEA), relative performance, BCC model, CCR model, Health Information Systems (HIS), and data mining.

INTRODUCTION
The first model of DEA was introduced by Charnes, Cooper and Rhodes in 1978 which was actually an enhancement of Farell “border production function” methodology to measure technical efficiency in 1957. Later it, Banker (who introduced the CCR model) and Kemerer use this approach in order to prove the existence of both rising and falling returns to scale [1][2][14]. The BCC model is introduced in 1984 and named after the three researchers (Banker, Charnes and Cooper). DEA is an optimization method of linear programming that uses a nonparametric approach for evaluating relative efficiency and productivity for multiple inputs and multiple outputs of firms or case studies.  

A key aspect of DEA is that it is used to compare efficiency of Decision Making Units (DMUs) for different domains such as bank branches, universities, sales outlets and health care departments from different aspects that does not require the assignment of predetermined weights to the input and output factors (weight can be given to the different inputs and outputs to indicate the importance of those parameters on the final results or decision). DEA employs flexible, nonparametric methods to construct the best-practice and so allows the data to identify themselves in contrast to some other parametric methods such as SFA (Stochastic Frontier Analysis) [3].  

There are numerous DEA models introduced to assess the efficiency of firms. In general, these models differ in their goal orientation (e.g. input-orientation vs. output-orientation), disposability (e.g. strong vs. week), diversification and returns to scale (e.g. CRS vs. VRS). The most well-known models are the BCC developed by Banker, Charnes and Cooper, and CCR developed by Charnes, Cooper and Rhodes.  

The utilization of DEA in health care is widely spread; Banker et al. used DEA to identify the nature of returns to scale to a given patient mix and hospital capacity [15]. They did not determine the returns to scale of individual hospitals; return to scale has important applications in health care organizations, which seeks to attain a specified level of efficiency improvements that determine the effectiveness of resource utilization, such as giving an indicator for adding hospital capacity to improve efficiency in the provided services.  

A DMU is considered as 100% efficient relative to other DMUs if the performance of other DMUs doesn’t show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs.  

RELATED WORK
Barros et al. analyzed the efficiency and productivity growth for a sample number of Portuguese hospitals by observing technological
and efficiency changes [4]. They used DEA and Malmquist productivity index.

A directional distance function is introduced to measure the smallest changes of inputs and outputs in a given direction by defining a reference or goal point to be achieved after performing the frontier approach.

The dataset used is collected from Portuguese Ministry of Health and from the financial accounts of 51 Portuguese hospitals. It takes into account the number of case flows (i.e. the number of patients that leave the hospital), length of stay, consultation and emergency cases as outputs. On the other hand it used three parameters as inputs; number of beds, personnel and total cost.

The results showed that there is a positive technological change for only 8 out of the 51 hospitals analyzed.

Chen et al. evaluated and choose new methods to be used in Taiwan hospitals for two goals: First, to evaluate and choose new medical technologies, and second to improve efficiency, and the monitoring of existing methods [5]. The adopted model is input oriented CCR model integrated with Multi-Criteria Decision Analysis (MCDA) model. The latter is used in order to determine decision making performance indicators. CCR model is used to evaluate the efficiency of existing and new technologies. In the decision making field, seven broad criteria was proposed to be used as alternative strategies when risks are found. Those are: efficiency, effectiveness, safety, revenue, social responsibility, institutional strategy, feasibility and risks. The research preferred to use the DEA approach because it covers multiple inputs-outputs unlike regression or ration analysis which gives a direction of how the efficiency of DMUs could be improved for monitoring and controlling purposes, and computes the relative efficiency for DMUs with no need for defining weights for each DMU in advance. The result showed that MCDA integrated with CCR outperforms other methods in terms of tangible and intangible resources.

Osei et al. measured technical and scale efficiency of 34 hospitals and health centers in Ghana and gives directions that help decision-makers for an effective management in the health sector [6]. The study divided inputs into broad groups and further each one of them is divided into subgroups as the following: personnel, materials, and capital. The output is divided into maternal and child health care visits, deliveries and inpatient discharges. The study used CRS (i.e. Constant Return to Scale) and VRS (i.e. Variable Return to Scale) models to assess the efficiency of the selected hospitals. Measuring efficiency of the DMUs is calculated in three steps; first, the efficiency was estimated through CRS and second, through VRS. Third, scale efficiency was obtained by dividing each hospital's CRS efficiency score by its VRS efficiency score.

Linna et al compared cost efficiency between several Finnish and Norwegian hospitals in order to help decision makers to decide the most suitable payment strategy for health care [7]. They cluster hospitals into several clusters according to some criteria such as size, administrative levels, type of services, etc. They used CRS and VRS for measuring cost efficiency.

The result showed a considerable difference between the Finnish and Norwegian hospitals in term of cost efficiency, input price and length of stay.

Butler et al tried to work on studying the impact of variables’ changes on inefficient DMUs in Michigan hospitals [8]. The variables used in the study include: Number of beds, total services, and number of technical employees are as inputs, and total number of inpatients, number of surgeries, and number of handled operations in the emergency room as outputs.

GOALS AND APPROACHES

DEA involves the use of linear programming methods to construct a non-parametric frontier over the data to calculate relative efficiencies for DMUs under study.

Input-orientation method concerns with decreasing input data proportionally to have more efficient DMUs, while output-orientation method concerns of increasing quality of services provided without affecting the quality of the inputs used. A VRS model composed of three types; first, CRS, which doesn't have to improve inputs or outputs, second, increasing return to scale (RS) that needs to leverage the quality of provided services. Third, decreasing RS will alleviate the input volume.

In this research, standard CRS and VRS (BCC) models in both input and output orientation will be used to evaluate and redirect the management of technical and scale efficiencies.

Multistage DEA method is used to identify the efficient projected frontiers which have mixes of input-output parameters which are invariants and similar as much as possible to those of the inefficient DMUs.

The dataset used in this research to evaluate hospitals efficiency is collected from the Jordanian ministry of health website (i.e. www.moh.gov.jo) and from the ministry information system department.
The indicators of the selected hospitals are divided into two categories:

**Inputs:** Those are composed of equipments and personnel. The personnel are divided into the number of physicians, number of pharmacist, number of nurses and other hospital employees.

**Outputs:** Those are composed of personnel and services. The personnel are divided into: number of inpatients and number of patients treated or outpatients. The services are also divided into outpatients’ visits, number of surgical operations, emergency room visits and length of stay for inpatients. In addition, total revenue is considered as an output indicator [12].

The DEAP software is used as the DEA software in this research. It is a console program written by Coelli in FORTRAN for IBM compatible PCs to calculate technical and cost efficiencies. It is a DOS program but it may run over Windows by using a file manager.

As it was mentioned previously, BCC and CCR standard models were used to evaluate DMUs’ efficiencies. Table1 presents the efficiency results according to CRS (CCR) input and output orientation model.

From Table1, it is observed that 16 hospitals from the 28 are 100% efficient for both input and output orientations of the CCR model. Those hospitals are: Al-basheer, Al-Nadeem, Al-Zarqa, Prince Faisal, National center for mental health, Al-Husain, Princess Basma, Princess Rahma, Princess Badea, Abu-obaida, Mo’ath bin Jabal, Jerash, Al-Eman, women and child care, Al-Mafraq and Al-Rwaished. The least efficient hospital in this method is Queen Rania Al-abdallah hospital.

**Table 1: CCR relative efficiency**

<table>
<thead>
<tr>
<th>Hospital Name</th>
<th>Input TE</th>
<th>Output TE</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL-Basheer</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Prince Hamza</td>
<td>79%</td>
<td>79%</td>
</tr>
<tr>
<td>Dr. Jameel Al-Totanji</td>
<td>97%</td>
<td>97%</td>
</tr>
<tr>
<td>AL-Nadeem</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Salma</td>
<td>74%</td>
<td>74%</td>
</tr>
<tr>
<td>AL-Zarqa</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Prince Faisal Bin AL-Hussein</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>National Center of Psychiatry</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>AL-Hussein / Salt</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>AL-Shuneh (South)</td>
<td>88%</td>
<td>88%</td>
</tr>
<tr>
<td>Princess Eiman’ maadi</td>
<td>93%</td>
<td>93%</td>
</tr>
<tr>
<td>Princess Basma</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Rahma</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Badea</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Raya</td>
<td>80%</td>
<td>80%</td>
</tr>
<tr>
<td>AL-Ramtha</td>
<td>84%</td>
<td>84%</td>
</tr>
<tr>
<td>AL-Yarmouk</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>Abu - Obaidah</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Muaath Bin Jabal</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Jerash</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Al-Iman</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Al-Karak</td>
<td>79%</td>
<td>79%</td>
</tr>
</tbody>
</table>

BCC is another model that is used to compare relative efficiency with input-output orientation. Table 2 presents the efficiency percentage.

From Table2, it is noticed that 20 hospitals are fully efficient when applying input orientation efficiency while 21 hospitals are shown to be efficient when using output orientation. The difference between the two methods occurred in Al-Karak hospital. It is inefficient in input orientation but became efficient when the output orientation is used.

**Table 2: BCC relative efficiency**

<table>
<thead>
<tr>
<th>Hospital Name</th>
<th>Input TE</th>
<th>Output TE</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL-Basheer</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Prince Hamza</td>
<td>92%</td>
<td>95%</td>
</tr>
<tr>
<td>Dr. Jameel Al-Totanji</td>
<td>99%</td>
<td>99%</td>
</tr>
<tr>
<td>AL-Nadeem</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Salma</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>AL-Zarqa</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Prince Faisal Bin AL-Hussein</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>National Center of Psychiatry</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>AL-Hussein / Salt</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>AL-Shuneh (South)</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Eiman’ maadi</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Basma</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Rahma</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Badea</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Princess Raya</td>
<td>90%</td>
<td>83%</td>
</tr>
<tr>
<td>AL-Ramtha</td>
<td>90%</td>
<td>87%</td>
</tr>
<tr>
<td>AL-Yarmouk</td>
<td>94%</td>
<td>92%</td>
</tr>
<tr>
<td>Abu - Obaidah</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Muaath Bin Jabal</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Jerash</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Al-Iman</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Al-Karak</td>
<td>79%</td>
<td>100%</td>
</tr>
<tr>
<td>Ghor Al-Safi</td>
<td>90%</td>
<td>87%</td>
</tr>
<tr>
<td>Al-Mafraq / Gynecology &amp; pediatrics</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Al-Mafraq</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Al-Rueshid</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Maan</td>
<td>91%</td>
<td>67%</td>
</tr>
<tr>
<td>Queen Rania AL_Abedalla</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The efficiency percentages were also increased when we applying BCC model in comparison with the CCR model.

It is noticed that BCC efficiency model is better than CCR for assessing productivity in terms of input and output orientation.

**CONCLUSION AND FUTURE WORK**

In this paper, DEA methods are used to evaluate the productivities of public hospitals in
Jordan in the year 2008. The two methods that were applied are the BCC and CCR DEA models using different indicators of inputs and outputs.

Results indicated that 16 from the 28 hospitals are efficient according to the CCR model in both input and output orientation, while there were 20 hospitals efficient when applying the BCC model input orientation and 21 hospitals achieved efficiency when applying the BCC model output orientation.

Inefficient hospitals can be improved by increasing the quality of services provided, number of physicians, acquire or provide new technologies or services to lessen medical mistakes or their impacts and increase number of patients treated, and improve other care quality indicators.

One limitation in this study and those similar is in taking into consideration those intangible input or output parameters such as quality of service, following safety and emergency standards, etc.

In future, domain experts will be consulted to indicate those relevant input or output indicator for possible weighting for the different indicators and see such impact on the productivity. Clustering will also be used in future to classify different DMUs according to similarities in size, types of services, etc.

REFERENCES


Duncan Mortimer, Stuart Peacock. Hospital Efficiency Measurement: Simple Ratios vs Frontier Methods. 2002. Health Economics Unit, Centre for Health Program Evaluation, School of Medicine, Health Policy & Practice, University of East Anglia.


Financial Applications Stream
Measuring bank branch performance using DEA

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**ABSTRACT**

The aim of this study is to develop a performance model for measuring the relative efficiency and potential improvement capabilities of bank branches by identifying their strengths and weaknesses. Another purpose is to investigate the production and profitability aspects of branches. Under both production and profitability approaches efficiency characteristics of branches, which are grouped according to different size and region, have similar tendencies. In both analyses, it is apparent that branch size and scale efficiency are related to each other. As branch size increases scale efficiency increases, after the most productive scale size, however, as size increases efficiency decreases. Too small and too large branches need special attention. Putting production and profit efficiency scores on two scales reveals the performing characteristics of branches. Each region needs different handling. Branches with low production-low profit efficiency should be evolved towards high production-high profit efficiency region.

**Keywords:** Data Envelopment analysis, Bank Branch

**INTRODUCTION**

Data Envelopment Analysis (DEA) is a non-parametric performance measurement tool that can be used for analysis and decision making in branch banking. It benchmarks the branches, gives potential improvement capabilities, indicates source of inefficiency, and takes management preferences into account when measuring performance.

By making use of DEA, different aspects of branch efficiency can be analyzed. Production, profitability, operation efficiencies can be studied to reveal their performing characteristics. And they can be combined into a single model having multiple dimensions.

In this article, after literature review, implemented methods will be introduced and then empirical results of production and profitability approach will be discussed.

**LITERATURE REVIEW**

After the article of Sherman and Gold (1985), numerous studies have been conducted on bank branch efficiency. Because of publicly available data, the number of bank performance measurement studies is greater than branch efficiency ones. Fethi and Pasiouras (2009) specify 136 studies that use DEA-like techniques to estimate bank efficiency, while only 28 studies about branch efficiency are listed. Of the 28 studies, 17 adopt production and 12 adopt intermediation approach. Yang and Paradi (2004) also, list 42 studies, of which 23 use CCR, 13 use BCC and the remaining 6 use both.

Although early studies measured extent and sources of inefficiencies, recent studies have developed new perspectives to raise awareness about working style of branches (e.g. Portela and Thanassoulis, 2007; Giokas, 2008). These provide valuable information to management to implement required strategies accordingly. And also, hybrid applications of DEA and other methods have been put into practice.

To constitute our models a comprehensive literature review is conducted and 44 studies in 36 articles, published after year 2000, were analyzed. In 31 studies production/operation approach, in 4 studies profitability approach is used. In 24 studies CCR and in 25 studies BCC model is employed. In 32 studies input oriented and in 11 studies output oriented approaches are considered. Average number of inputs is 3.6 and, depending on the accepted approach, the most used inputs are related to employee, other operating expenses, rental area or expenses and other equipments. Outputs have wider range of diversity with an average number of 4.5. The most used outputs are value/number of deposits, loans, non-interest income & commissions and account/transaction numbers.

**METHOD AND DATA ANALYSIS**

In this study 128 bank branches located in Istanbul City and Thrace region of Turkey are handled. 2007 year-end data of inputs and outputs for production and profitability
approaches are listed in Table 1. Management is assumed to have control on all parameters. Since we could not obtain any regular data about service quality and customer satisfaction, they are not covered by the scope of this study.

First, by considering production approach and using output oriented CCR and BCC models, technical, pure technical and scale efficiency of branches are calculated.

Together with the analysis of branches, new strategies are proposed for relatively inefficient branches.

Because of page constraints, results of each model are condensed and presented in Table 2.

### EMPIRICAL RESULTS

#### Production Approach

##### Technical Efficiency

Efficiency analysis of production approach with CCR model shows that 52 (41\%) branches are technically efficient with an average of 0.896 (1st column of Table 2).

In spite of high average efficiency, 76 (59\%) of 128 branches are not technically efficient. This implies that they should produce more outputs at given input level (or use less inputs to produce same outputs), compared to their peers. And also, they should produce at right scale to become efficient. Technical efficiency measures the branches’ overall success at utilizing their inputs by feasible input to output conversion and operating at right returns to scale.

Large branches seem to be technically more efficient than small branches. Since large branches are mostly located in Istanbul, the average efficiency of Istanbul branches becomes higher. Large branches have deposits and credits in greater amounts, which result in more income together with other products. Higher values are transacted with comparatively less personnel and operating costs.

##### Pure Technical Efficiency

In our study, Kolmogorov–Smirnov test showed that CCR and BCC results come from different distributions. Therefore scale efficiency should also be considered.

---

**Table 1: Inputs and outputs of Production and Profitability Approaches and descriptive statistics**

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Average</th>
<th>SD</th>
<th>Coeff. of Var.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inputs of both</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>approaches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Personnel Expenses</td>
<td>1,785</td>
<td>168</td>
<td>710</td>
<td>346</td>
<td>0.487</td>
</tr>
<tr>
<td>Operating Expenses</td>
<td>1,209</td>
<td>69</td>
<td>311</td>
<td>205</td>
<td>0.659</td>
</tr>
<tr>
<td>Loan Losses</td>
<td>2,608</td>
<td>0</td>
<td>93</td>
<td>267</td>
<td>2.871</td>
</tr>
<tr>
<td><strong>Outputs of</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand Deposits</td>
<td>165,959</td>
<td>572</td>
<td>8,345</td>
<td>17,608</td>
<td>2.110</td>
</tr>
<tr>
<td>Time Deposits</td>
<td>455,663</td>
<td>2,538</td>
<td>79,142</td>
<td>73,610</td>
<td>0.930</td>
</tr>
<tr>
<td>Demand FX Deposits</td>
<td>9,050</td>
<td>12</td>
<td>2,017</td>
<td>1,835</td>
<td>0.910</td>
</tr>
<tr>
<td>Time FX Deposits</td>
<td>45,728</td>
<td>112</td>
<td>10,096</td>
<td>8,399</td>
<td>0.832</td>
</tr>
<tr>
<td>Commercial Loans</td>
<td>39,093</td>
<td>90</td>
<td>4,321</td>
<td>5,445</td>
<td>1.260</td>
</tr>
<tr>
<td>Consumer Loans</td>
<td>39,710</td>
<td>769</td>
<td>9,494</td>
<td>6,555</td>
<td>0.659</td>
</tr>
<tr>
<td>No of Total Transaction</td>
<td>857,195</td>
<td>32,356</td>
<td>226,633</td>
<td>124,185</td>
<td>0.548</td>
</tr>
<tr>
<td><strong>Outputs of</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Profitability App.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non Interest Income</td>
<td>4,430</td>
<td>108</td>
<td>1,060</td>
<td>827</td>
<td>0.780</td>
</tr>
<tr>
<td>Net Interest Income</td>
<td>29,520</td>
<td>174</td>
<td>2,485</td>
<td>3,355</td>
<td>1.350</td>
</tr>
<tr>
<td></td>
<td>4,430</td>
<td>108</td>
<td>1,060</td>
<td>827</td>
<td>0.780</td>
</tr>
</tbody>
</table>
BCC model measures the pure technical part of efficiency. By using BCC model, together with CCR, scale efficiency can also be calculated. Pure technical efficiency shows the success of management at input to output “conversion” while scale efficiency reflects the success at working at right scale. Pure technical efficiency values are higher than technical efficiency, as expected. 78 branches become pure technically efficient and the average increases to 0.939 (2nd column of Table 2).

### Table 2: Results of Production and Profitability Approaches with different DEA models.

<table>
<thead>
<tr>
<th>PRODUCTION APPROACH</th>
<th>PROFITABILITY APPROACH</th>
</tr>
</thead>
<tbody>
<tr>
<td>(# of eff. br.)</td>
<td>(# of ineff. br.)</td>
</tr>
<tr>
<td>TE</td>
<td>PTE</td>
</tr>
<tr>
<td>Average</td>
<td>8086</td>
</tr>
<tr>
<td>SD</td>
<td>0.725</td>
</tr>
<tr>
<td>Maximum</td>
<td>1000</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.558</td>
</tr>
<tr>
<td>Large</td>
<td>0.937</td>
</tr>
<tr>
<td>Medium</td>
<td>0.921</td>
</tr>
<tr>
<td>Small</td>
<td>0.851</td>
</tr>
<tr>
<td>Istanbul Cit</td>
<td>0.903</td>
</tr>
<tr>
<td>Thrace Reg.</td>
<td>0.865</td>
</tr>
<tr>
<td>Average</td>
<td>0.929</td>
</tr>
<tr>
<td>TE</td>
<td>104</td>
</tr>
<tr>
<td>PTE</td>
<td>0.688</td>
</tr>
<tr>
<td>Scale</td>
<td>0.616</td>
</tr>
<tr>
<td>Nonrad.TE</td>
<td>0.680</td>
</tr>
<tr>
<td>Mix E</td>
<td>0.625</td>
</tr>
<tr>
<td>Weight.TE</td>
<td>0.616</td>
</tr>
</tbody>
</table>

Pure technical efficiency shows how much a branch can radially increase its outputs when projected to VRS frontier while still remaining at the same input level.

Since input and output combination does not refer to efficient frontier, this also implies some managerial deficiencies. To become purely efficient, a branch should evolve its input-output combination in such a way that it falls on to VRS frontier.

### Scale Efficiency

Scale efficiency measures how much a branch can improve its efficiency by being projected from VRS to CRS, i.e. the ability of further increasing its outputs radially. Scale efficiency reflects the success of the branch whether it operates at the right returns to scale or not.

For a branch to become scale efficient it should increase its outputs further to reach the most productive scale size. Mostly small branches appear to have scale inefficiency because of increasing returns to scale (IRS) characteristics. To be efficient, they should increase their transaction volume. Merging small branches close to each other may also be considered. Scale inefficiency in large branches may result from decreasing returns to scale (DRS). To increase scale efficiency, opening new branches in close hinterland of large branches may also be considered.

Large branches have 0.970, mediums have 0.982 and small ones have 0.922 average scale efficiencies (3rd column of Table 2). Although not listed here, 13 of 15 most inefficient branches are small and the other 2 are big.

These imply that some kind of relationship may exist between branch size and efficiency, and between branch size and RTS.

### Branch Size and RTS Characteristics

On VRS efficient frontier, 18 branches show IRS, 52 show CRS and 8 show DRS. IRS branches mostly include small and DRS branches large ones, as expected. After projecting 6 branches to CRS, 58 will operate at most productive scale size (Table 3). Even after projection to VRS, still 70 branches do not fall on CRS frontier.

### Table 3: RTS characteristics of branches.

<table>
<thead>
<tr>
<th>RTS</th>
<th>Size</th>
<th>Eff. br.</th>
<th>Projected br.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRS</td>
<td>Large</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>2</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Small</td>
<td>15</td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>CRS</td>
<td>Large</td>
<td>17</td>
<td>-</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>19</td>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Small</td>
<td>16</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>DRS</td>
<td>Large</td>
<td>7</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Small</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>78</td>
<td>50</td>
<td>128</td>
</tr>
</tbody>
</table>

![Figure 1: RTS region of branches according to input and output orientation](image-url)
increasing inputs, if it is on DRS region, decreasing inputs may be preferable. Figure 1 shows the RTS region of branches. RTS characteristics of 125 branches are same in both input and output orientation.

Relation between Branch Size and Efficiency

To analyze the relationship between branch size and efficiency, since %100 efficient branches cannot be sorted, only scale inefficient 75 branches are considered. Branches are sorted by size from smaller to larger and by efficiency from inefficient to efficient (Figure 2).

![Figure 2: Branch size order and scale efficiency](image)

When we consider all 75 branches, Spearman’s rank correlation is calculated as 0.305. This indicates not a strong but slightly positive correlation. However when we look at Figure 2, as the size increases the scale efficiency increases and after a certain point, as the size continue to increase efficiency decreases. That’s why it seems more feasible to divide branches into two groups. Spearman’s rank correlation is calculated as 0.510 for the first 50 branches and -0.354 for the next 25. Both represent a stronger relation. In the first part as branch size increases efficiency increases, in the second part as branch size increases efficiency decreases. The small branches at one end and big branches at the other end should be separately handled and analyzed.

Same analysis is conducted with profitability approach and correlations are calculated as 0.404 (for 117 branches), 0.631 (for 80 branches) and -0.579 (for 37 branches), respectively.

Non-radial and Mix Efficiency

CCR and BCC models measures radial efficiency and they radially project non-efficient branches to efficient frontier. In addition to radial, SBM model calculates non-radial efficiency. If radial portion is separated, the residual inefficiency shows mix inefficiency. Mix inefficiency indicates other sources of inefficiencies like slacks etc. beyond technical component.

Since here SBM uses CRS frontier as CCR, the number of efficient and inefficient branches are the same in the both models. But in SBM, average non-radial efficiency decreases to 0.683 (Fourth column of Table 2). Even after some branches projected to efficient CRS frontier, still there exists an improvement potential. Small branches seem to have more sources of inefficiency compared to large ones. But efficiency improvement of large branches, even in small percentages, may be important because small efficiency increase may correspond to large improvement in value.

Mix efficiency of 0.735 implies a further inefficiency additional to technical part.

Efficiency with Weighted Parameters

DEA determines the weight of input and output parameters freely to maximize the efficiency of each unit. This property is one of the advantages of DEA but for some situations relaxing may be required. Sometimes weights assigned by model do not seem to be reasonable and realistic. Management, also, may prefer weight of some parameters to be greater than the others’. When weight restrictions and preferences are imposed on some parameters, efficiency level decreases.

The specified weight range should not be too narrow not to eliminate “benefit of doubt” weighting of DEA. To decide the weights, analyzing the results of free run and some trial and errors may be helpful.

In this study we have some preferences about inputs and outputs and therefore we want to assign certain importance. Looking to the parameters of unrestricted model and taking priorities into consideration, by trial and error a range for input and output weights is specified in ARG Model (Table 4).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assigned Weights (%)</th>
<th>weighted Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personnel Exp.</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>Operating Exp.</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>Loan Losses</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>Demand Dep.</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>Time Deposits</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Demand FX Dep.</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Time FX Dep.</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Com. Loans</td>
<td>5</td>
<td>100</td>
</tr>
</tbody>
</table>

The results showed that it should be preferable to assign a wide range of restrictions and not to impose tight preferences on some parameters.
Thanks to relatively high results of unweighted CCR model, we have a flexibility range to assign weights and improve the discrimination. To limit the effect of non interest income in production approach, 0-25% range is specified. Since the profit margin of fx time deposit is low, max 10% weight is assigned. Other weights are assumed to reflect management preferences. A broad and reasonable restriction range is considered not to eliminate the advantages of DEA.

Comparison of weighted and unweighted results indicates that efficiency changes when some restrictions are imposed. Average technical efficiency decreased from 0.896 to 0.690 as expected. 36 of 52 efficient branches in unrestricted model become inefficient in weighted model.

Profitability Approach

In profitability approach, average efficiency declines to 0.526. Pure technical and scale efficiencies are also comparatively low, 0.641 and 0.842 respectively (Column 7, 8, 9 in Table 2).

According to profitability approach, 117 branches are measured as technically efficient and 104 branches as scale inefficient. Under both production and profitability approaches technical and scale efficiency characteristics of branches, grouped according to different sizes and regions, show similar tendencies.

In both analyses, it is apparent that branch size and scale efficiency are related to each other. As size gets larger, scale efficiency increases and after the most productive scale, as size increases efficiency decreases.

Production and Profitability Comparison

Putting scores on production and profit efficiency scales shows the working characteristics of branches (Figure 3). Since the discrimination is higher, weighted efficiency scores are used for production approach.

The correlation between production and profit efficiency is calculated as 0.696. Generally, a positive interaction between productivity and profitability is reasonable. A productive branch is expected to be profitable also. Branches with low production-low profit efficiency should carefully be investigated and they should be evolved towards high production-high profit efficiency region or should be considered for moving to another location. Depending of the region separation point, 58 branches’ production and profit efficiencies are below 0.70 and 0.50, respectively. They can be classified as non productive and non profitable. Br071 at bottom-left may represent this group. It is the smallest branch and when we deeply analyze it’s data, we realize that outputs of this branch are really very low. It should increase its income outputs 536% more compared to its peers and still there exists slack values in inputs.

Figure 3: Production and profit efficiencies

Branches with low production-high profit efficiency have the potential for more profit and in addition to production side they should be analyzed for higher profits also. Profitability mostly results from comparatively advantageous location, not necessarily from high management quality. 11 branches’ production efficiency is below 0.70, but profit efficiency is above 0.50. Br123 at the top-left is an example of this region. When analyzed, we see that it is a small branch with comparatively low outputs. But because of location, it has a dozen of tax office accounts and export related payments leading to high income with relatively small business volume.

Branches with high production and high profit efficiency are at the target zone that others should achieve. These branches convert inputs into outputs satisfactorily and make higher profits in comparison to their peers. 43 branches’ production and profit efficiency is above 0.70 and 0.50, respectively. Especially 8 of them (Br021, Br028, Br044, Br048, Br055, Br060, Br099 and Br109) are 100% efficient both in terms of production and profitability. When closely analyzed, 5 of them are really big located in the heart of city, with huge amount of transaction and big customers. Bu interestingly, of 100% efficient 8, 2 small and one medium are distinguished with development potential.

The number of branches with high production-low profit efficiency is also relatively low. Because of low profit margin, either due to
high competition or overburdened non-profitable activities, their income is not satisfactory. Nonprofitability may also result from loan loss provisions. These branches should channelize their intensive ‘junk’ transactions into alternative distribution channels and concentrate more on “valuable” products. High production and low profit efficiency may also result from intensive competition. One of the biggest branches BR016 typically represents this group. These branches should be specially handled to achieve profitability.

CONCLUSION

DEA has been used more extensively for measuring the performance of bank branches. It is an important method that should also be utilized as an analysis and decision making tool in branch banking. Its flexibility provides valuable information depending on the analyst’s point of view.

By using DEA the management can realize the properties of a branch and constitutes the strategies accordingly. Different dimensions of branch characteristics can be analyzed too. Under both production and profitability approaches efficiency characteristics of branches, which are grouped according to different size and region, have similar tendencies. In both analyses, it is apparent that branch size and scale efficiency are related to each other. As size gets larger, scale efficiency increases and after the most productive scale size, as size increases efficiency decreases. Branches at small and large ends need special attention. Putting production and profit efficiency scores on two scales reveals the performing characteristics of branches. Each region needs different handling.

Effects of branch segmentation on performance change, the efficiency of different sections within a branch or efficiency of segmented branches by Network DEA and benchmarking the branches of different banks may be the interest of further studies.

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Mexican banking deregulation and productivity change

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ABSTRACT

Commercial Mexican banks are examined during the deregulation of financial markets by utilizing a DEA-type Malmquist index, in order to obtain total factor productivity growth, efficiency change and technical progress during the period 1982-2006.

Keywords: Data Envelopment Analysis, Commercial Mexican Banks

INTRODUCTION

The objective of this paper is to analyze the performance in productivity and efficiency of the commercial Mexican banks during the financial liberalization process. For such purpose, the Malmquist productivity index is used. The period under study is comprised from 1982 to 2006. All the commercial banks were selected, which at the beginning of the period were 20 and at the end thereof, 28. Those banks are: Banamex, Bancomer, Somex, Santander Mexicano, Internacional, Bital, Serfin, Banco del Centro, Mercantil del Norte, Mercantil Mexicano, Proabursa, Bilbao Vizcaya, Bancrecer, Promex, Comerciex, Inverlat, Scotia Bank, Atlantico, Banoro, Confia, Banpais, Orientie, Cremi, Banco Obrero, Banco Union, Citibank, Interacciones, Industrial, Promotor del Norte, Mifel, Invex, Banregio, Sureste, Ixe, Del Bajio, Quadrum, Afirme, Banco de Tokyo, Fuji Bank, Chase Manhattan, J P Morgan, Bank Of America, Abn Amro Bank, Republic of New York, B.N.P., Bank Of Boston, Bansi, Dresdner Bank, Ing Bank, Societe Generale, First Chicago Bank, Alianza, American Express, Nations Bank, Tokyo Mitsubishi, Bank One, Banorte, Comercial Bank, Ge Capital Bank, HSBC, Inbursa, Deutsche Bank, Cred Suisse, and Banco Azteca.8

In order to select the variables, financial intermediation was focused upon; thus, wages, capital and profitable funds, were added as an input variables. On the other hand, credit and profitable assets were added as product variable.

Up to where the author is aware, this is the first paper that applies the methodology of Malmquist’s DEA index for the case of Mexican banks. The paper is organized as follows: 1) References of work that have analyzed banking productivity using Malmquist’s index are presented; 2) the historical background of the banking sector in Mexico is expounded; 3) the model utilized is described; 4) a work hypothesis is developed; 5) the results are presented and 6) conclusions are summarized.

LITERATURE REVIEW


Santander in May 2000 and on September 4, 2002, merged into one bank. Mercantil del Norte remained in the hands of Mexican capital and is owned by the Banorte financial group.

8 During the period being analyzed, some of these banks changed their names and owners and others have disappeared. For example, Banamex is owned by the Citigroup financial group as of August 2002; Bancomer is owned by Bilbao Vizcaya Argentaria, as of June 2002; Bital belongs to HSBC as of November 22, 2002; Somex, to Banco Santander, as of April 21, 1997. Serfin was also purchased by Santander in May 2000 and on September 4, 2002, merged into one bank. Mercantil del Norte remained in the hands of Mexican capital and is owned by the Banorte financial group.
HISTORICAL BACKGROUND

Several inflection points mark the stages of evolution of the Mexican banking sector: bank nationalization occurred in 1982; financial liberalization, began in 1988; privatization of banks took place during 1991-1992; NAFTA, came into effect as of January 1994; the financial crisis, began in December 1994 with the peso devaluation and the rescue of the banks, which began as of 1995 and whose first stage was completed in 1998. As of 1999, to 2006, new rules for the safety of deposits were initiated and the system entered a stage of relative stability.

The global financial crisis as of 2007 could be considered another stage with impacts which will be evaluated in another paper.

THE ALGORITHM

Fare, Grosskopf, Norris and Zhang (1994) defines the geometric mean of the output productivity index with reference to period t and t+1 technology as (1):

\[ M_0(x_{t+1}, y_{t+1}, x_t, y_t) = \left[ \frac{D_0'(x_{t+1}, y_{t+1})}{D_0'(x_t, y_t)} \right]^{1/2} \]

(2)

\[ = \left[ \frac{D_0'(x_{t+1}, y_{t+1})}{D_0'(x_t, y_t)} \right]^{1/2} \]

where (3)

\[ \left( D_0'(x_{t+1}, y_{t+1}) \right) \left( D_0'(x_t, y_t) \right) \]

is the change in relative technical efficiency between periods t and t+1.

And

\[ \left[ \left( D_0'(x_{t+1}, y_{t+1}) \right) \left( D_0'(x_t, y_t) \right) \right]^{1/2} \]

is the shift in technology between the two time periods t, and t+1.

For each production units, define five Malmquist indices for period t+1 relative to period t.

Total factor productivity change index

(5) (TFPCH):

\[ M(t, t+1) = \frac{D_{VRS}'(x_{t+1}, y_{t+1})}{D_{VRS}'(x_t, y_t)} \]

\[ = \left[ \frac{D_{VRS}'(x_{t+1}, y_{t+1})}{D_{VRS}'(x_t, y_t)} \right]^{1/2} \]

\[ = \left( \text{Technical efficiency change index} \right) \left( \text{Technological change index} \right) \]

Technological Change Index

(6) \( \text{(TECHCH)} = \)

\[ \left( \left( D_0'(CRS)(x_{t+1}, y_{t+1}) \right) \left( D_0'(CRS)(x_t, y_t) \right) \right) \]

(7) \( \text{(EFFCH)} = \)

HYPOTHESIS

The null hypothesis is that there is no difference in productivity and efficiency levels before and after deregulation.

The alternative hypothesis is that the financial liberalization process pressured banks to elevate their productivity and efficiency levels, by liberating interest rates and opening competition to foreign banks.

RESULTS

Below is an analysis of the change in the total productivity of factors, as well as an explanation regarding the sources of this change in both efficiency and technical change.

<table>
<thead>
<tr>
<th>YEARS</th>
<th>TFPCH</th>
<th>EFFCH</th>
<th>TECHCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983/1982</td>
<td>1.030</td>
<td>1.015</td>
<td>1.015</td>
</tr>
<tr>
<td>1984/1983</td>
<td>1.067</td>
<td>1.070</td>
<td>0.992</td>
</tr>
<tr>
<td>1985/1984</td>
<td>1.265</td>
<td>1.325</td>
<td>1.272</td>
</tr>
<tr>
<td>1986/1985</td>
<td>0.976</td>
<td>0.975</td>
<td>1.001</td>
</tr>
<tr>
<td>1987/1986</td>
<td>1.040</td>
<td>1.056</td>
<td>0.985</td>
</tr>
<tr>
<td>1988/1987</td>
<td>1.002</td>
<td>0.982</td>
<td>1.020</td>
</tr>
</tbody>
</table>

SUBPERIOD G.MEAN 1.104 1.064 1.037

PRE-DEREGULATION PERIOD 1982/1988

<table>
<thead>
<tr>
<th>YEARS</th>
<th>TFPCH</th>
<th>EFFCH</th>
<th>TECHCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001/2000</td>
<td>0.922</td>
<td>0.136</td>
<td>6.778</td>
</tr>
<tr>
<td>2002/2001</td>
<td>1.136</td>
<td>1.102</td>
<td>1.031</td>
</tr>
<tr>
<td>2003/2002</td>
<td>1.002</td>
<td>5.078</td>
<td>0.197</td>
</tr>
<tr>
<td>2004/2003</td>
<td>1.086</td>
<td>1.414</td>
<td>0.788</td>
</tr>
<tr>
<td>2005/2004</td>
<td>1.073</td>
<td>0.249</td>
<td>4.314</td>
</tr>
<tr>
<td>2006/2005</td>
<td>0.943</td>
<td>4.186</td>
<td>0.226</td>
</tr>
</tbody>
</table>

SUBPERIOD G.MEAN 1.024 1.019 1.005

POST-DEREGULATION PERIOD 2000/2006
It is noted in table 1 that in both periods the total factor productivity change was driven primarily by efficiency change. The big jump in technical change, observed in 2001, should be taken with caution because it is the period when there were changes in ownership of large banks. Rather, this could be interpreted as an effect of bank merger. Tests on the null hypothesis to identify any significant differences in both pre and post deregulation periods were conducted. This examined total factor productivity change and their determinants, and we applied ANOVA statistical test to do this. The results are shown in table 2.

The null hypothesis of no significant difference in total factor productivity change and its determinants, in both periods, is rejected.

We can infer from the probability density functions that the post regulatory period is riskier than the previous with a leptokurtic distribution for the pre deregulation period (see Graph1) and platykurtic for the post deregulation period (see Graph2).

CONCLUSIONS
In this paper we have compared the levels of bank productivity during periods of pre- and post-bank deregulation in Mexico to determine if there is a significant difference in the results achieved. We use the intermediation approach to select variables both of profitable assets and cost from commercial banks during the periods 1982-1988 and 2000-2006 respectively. We found that there is a significant difference in total factor productivity of the Mexican banking sector in the post deregulatory period compared with the previous one. However, these results should be taken with caution because during that period there were significant changes in ownership of banks, as well as mergers and acquisitions. Separating these effects is required to analyze the productivity results. It also noted that the post deregulatory period is less risky. Therefore indicators used should be adjusted for risk to measure productivity, and check for the robustness of our findings obtained so far.

REFERENCES


Optimization of selected portfolios using Data Envelopment Analysis

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ABSTRACT

The selection of assets for the construction of portfolios of investments in the stock market is very important for investors because it can set your profit or loss. This research investigated if the application of the Markowitz model of portfolio optimization in assets selected by Data Envelopment Analysis - DEA generates higher returns to portfolios obtained only with DEA. To meet this goal monthly portfolios (36) were designed. The work started in January 2006 and ended in December 2008. Fundamentalist indicators were used as inputs and outputs of the DEA model. Assurance Region – AR was the choice for the DEA model. When comparing the portfolios with the DEA IBOVESPA the results showed that the portfolio obtained using DEA and Markowitz optimization model delivers better performance than the original DEA portfolios.

Keywords: Portfolio selection; DEA; Stock market; Markowitz Optimization Model

INTRODUCTION

The selection of assets in order to form portfolios in the market of variable revenue is of extreme importance for those who opt for high-risk investments. The difference between the success and failure of the investments invariably lies in the choice of assets that can ensure greater profitability or lower loss during periods of financial market crisis.

In 1952, Markowitz proposed the use of a model of risk optimization and return in the choice of portfolios. Markowitz’s work had its relevance recognized by the academia and served as a basis for the appearance of other studies applied to the forming of portfolios. According to Cohen and Pogue (1967), the majority of these new studies had the objective of simplifying the assessment process of Markowitz’s variance and covariance, having the objective of making the calculations easier and faster. Recently, Lins et al (2007) proposed the use of cluster analysis techniques for the planning of hypothetic portfolios, while Matsumoto et al (2007) worked with the investor’s utility for the same purpose.

A closer study of the application of quantitative methods in the selection of portfolios occurred in Powers & McMullen (2000) and Lopes et al (2006 and 2008a and 2008b), which used a mathematical programming model - Data Envelopment Analysis – DEA – obtaining positive results. This methodology was developed by Charnes, Cooper and Rhodes in 1978 and consists of a model of linear programming, which evaluates the relative efficiency of decision-making units (DMU’s), which use multiple inputs in the production of multiple products. The applications of DEA in finances may be found in the studies conducted by Gregouriou (2007), Choi and Murthi (2001), Basso and Funari (2001), Machado-Santos and Rocha Armada (1997) and Ceretta and Costa Junior (2001). The results achieved by the studies conducted by Lopes et al (2006, 2008a and 2008b) motivated this study, which proposes to advance the use of the DEA methodology for the projection of investment portfolios, including the allocation of weights to the assets by means of Markowitz’s optimization model.
In order to do so, the following research problem is established: Does the use of Markowitz’s optimization model for portfolios projected with DEA methodology generate more efficient investment portfolios?

**DATA ENVELOPMENT ANALYSIS - DEA**

Originally proposed by Charnes, Cooper & Rhodes (1978), Data Envelopment Analysis-DEA is a technique for performance measurement, which has the main objective of determining the relative efficiency of production units considering its proximity to a frontier of efficiency. Through the application of a model of linear programming, this methodology allows for the comparison of the efficiency of several similar decision-making units (DMU’s), which guide a single indicator constructed from various approaches with different performances. (MACEDO, CASA NOVA and ALMEIDA, 2007).

As can be seen in the studies conducted by Zhu (2000), Santos and Casa Nova (2005), Macedo et al. (2006), the DEA methodology is a non-parametric method for the delimitation of the efficiency frontier, because by focusing the efficiency of the relation between inputs and outputs there is no explicit specification required in the functional form of this relationship. The measurement of efficiency calculated through DEA is a generalization of the measure of usual productivity, which is the quotient between the results obtained and the resources used, for each unit being analyzed.

There are various formulations of the DEA models found in the literature according to Coelli et al. (1998). However, two DEA basic models remain as the most used. The first model, which is called CCR (CHARNES, COOPER and RHODES, 1978), also known as CRS (Constant Returns to Scale), evaluates the total efficiency, identifies the efficient and inefficient DMUs and determines at what distance from the efficiency frontier the inefficient units are considering a frontier of returns with a constant scale.

The formulation of the Linear Programming Problem for the DEA-CCR Model (1) considers decision-making units (DMU’s), which produce quantities of inputs. The solution involves the obtainment of the values for the weights of each input and product. The restrictions impose that the application of these weights to all other DMU’s result in efficiency measures lower or equal to one. Thus, the relative efficiency of the units, defined as the weighted sum of the products, divided by the weighted sum of inputs will always result in values between 0 and 1.

This model was later extended by Banker, Charnes and Cooper (1984, p. 1078 to 1092) to include variable returns to scale and was then called BCC or VRS - Variable Returns to Scale (2).

**MARKOWITZ’S OPTIMIZATION MODEL**

The portfolio theory introduced by Markowitz in 1952 (MARKOWITZ, 1952) is a model of quadratic programming for the forming of portfolios. It seeks to maximize the usefulness of an investor who must choose a group of assets to compose a portfolio.

Markowitz affirms that the expected return of a portfolio of assets E(R) is a weighted average of the expected returns from the assets that compose it and that the sum of the participation of the assets in the portfolio must be equal to one (PIZZATTO et al, 2005). The portfolio risk is measured by means of the variance of the returns of the assets and the covariance between them.

Markowitz’s optimization model seeks the percentage participation of each asset (x_i and x_j), which minimizes the portfolio risk defined by f(x). The restrictions impose that the result must offer a portfolio that reaches the expected return (E') while the sum of the asset participation of the portfolio do not exceed one.

The graphic representation can be defined as in Figure 1 where the shaded area represents the possible combinations for a portfolio composed of multiple assets. All the possible combinations are contained within this limited region, in such a way that no individual asset or combination of assets is located outside the shaded area (PIZZATTO et al, 2005).

![Figure 1: Markowitz - Risk x Return](Source: Pizzatto et al, 2005)

- Expected Return from the Portfolio
- Standard Deviation of the Return from the Portfolio

The efficient group of portfolios is situated on the area and can be represented in Image 1 by
means of segment MV-R-X. Portfolios below this line, \( w \) for example, are inefficient because they provide the investor with an expected return lower than \( R \) with the same risk.

**METHOD USED**

The present study is originated from the study of Lopes *et al* (2008), who used DEA to select the annual investment portfolios in the Brazilian variable revenue market.

In this study, two portfolios were constructed from the results of the DEA model and always based on the date of the last day of the previous month to the one being projected. The analysis period went from January 2006 to December 2008.

The initial sample consisted of a group of 732 stocks from companies that participate of the main Brazilian stock exchange - BOVESPA, obtained through the Economática® database. In this study, only stock from open capital companies were used, which had not null participation at the IBOVESPA at the time of the study and that had all the necessary information.

Stage 1 of the study consisted of the survey of the possible important indexes for the analysis. A decision was made for those proposed by Powers and McMullen (2000) and used by Lopes *et al* (2008). Next is the table with the variables that were applied in this study.

**Table 1: List of the Used Indexes**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Description</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price/Profit</td>
<td>Quote of the stock/profit per stock</td>
<td>Input</td>
</tr>
<tr>
<td>Beta</td>
<td>Relationship between the return of the asset and the return of the market portfolio</td>
<td>Input</td>
</tr>
<tr>
<td>Volatility</td>
<td>Standard Deviation of the returns over 36 months</td>
<td>Input</td>
</tr>
<tr>
<td>LPA (PPS)</td>
<td>Profit per Stock</td>
<td>Product</td>
</tr>
<tr>
<td>%12</td>
<td>Return over the last 12 months prior to the period of the portfolio</td>
<td>Product</td>
</tr>
<tr>
<td>%24</td>
<td>Return over the last 24 months prior to the period of the portfolio</td>
<td>Product</td>
</tr>
<tr>
<td>%36</td>
<td>Return over the last 36 months prior to the period of the portfolio</td>
<td>Product</td>
</tr>
</tbody>
</table>

Source: Drafted by the authors.

After the data was collected (Stage 2), in order for the study not to be contaminated by the lack of information and to enable the depiction of reality as faithfully as possible, the exclusion of certain assets that composed the IBOVESPA (Stage 3) was performed. It was established that only the assets that represented non-null data for all the indexes as well as only one type of asset per company (ON or PN) would be part of the sample. The asset that presented greater movement in the stock exchange was maintained.

In Stage 4, collected data was treated by means of mathematical transformations contained in the study by Powers & McMullen (2000).

In stage 5 of this study, the Assurance Region (AR) model with variable returns to the scale was defined as the model to be used. An option was made for the orientation towards the reduction of input seeking the construction of portfolios that prioritized the assets with lower risk (bets and low volatilities). The restrictions to the weights used here established as inferior and superior limits the values of \( 1/5 \) and 5 to the weights of the relationship between two products and every two inputs, according to what is expounded in the study by Powers and McMullen (2000).

The monthly portfolios were obtained by means of the DEA-SOLVER (COOPER, SEIFORD and TONE, 2000) software.

The assets that reached an efficiency index equal to 1 in the DEA result were considered as participants of the projected portfolio (Stage 7).

Stage 8 consisted of the optimization of the DEA portfolios by using the Markowitz (1952) model. The objective was of comparing the performance of a portfolio projected by DEA, and constructed with investments realized in an egalitarian manner (DEA(1/N)), with the one where the assets would have their weights recommended by the Markowitz model (DEA(M)).

The performance of the 72 monthly portfolios was evaluated by means of the comparison with the oscillation of the IBOVESPA (Stage 9), Brazilian index that measures the profitability of a set of assets negotiated with the Brazilian stock exchange. In order to do so, a temporary analysis was used as well as descriptive statistics.

DEA(1/N) and DEA(M). The profitability of these portfolios are compared against IBOVESPA.

**RESULTS**

Table 2 demonstrates the monthly profitability of the portfolios obtained from the application of
By means of Table 2 it can be observed that in the first month of analysis (January, 2006) the portfolios constructed by using only those assets considered to be efficient in DEA, composing it through a egalitarian participation (DEA(1/N)) and with optimized weights (DEA(M)), reached a profitability lower to the São Paulo Stock Exchange Index - IBOVESPA. During this month, DEA (1/N) obtained a better performance than DEA (M), surpassing it by 5.45%. During the year of 2006, the DEA (1/N) portfolio surpassed Ibovespa in 7 months (58.3%) while DEA (M) surpassed it in 5 (41.67%). The year of 2007 was the worst for DEA (1/N) and DEA (M) portfolios, because they reduced the number of times they surpassed IBOVESPA (33.3% and 41.67%, respectively). When the two portfolios are compared, it can be observed that DEA (1/N) surpasses DEA (M) in 5 months during the year of 2006 and in 7 months during the year of 2007.

By analyzing 2008, the result is that the DEA (1/N) and DEA (M) portfolios show good results. DEA (1/N) surpasses IBOVESPA by 6 months, while DEA (M) surpasses it by 7 months. It is important to emphasize the positive and superior results obtained by the portfolios in the months of October and November of 2008, during moments of crisis in the financial market in Brazil and in the world. In the month of October, the DEA (1/N) portfolio presented profitability 9.79% greater than IBOVESPA while the DEA (M) portfolio presented profitability 16.47% greater. The same occurs in November of 2008 with a greater advantage for the DEA (M) portfolio – 15.39%.

Table 3 demonstrates the profitability of the annual portfolios formed by using the same DEA (1/N) and DEA (M) criteria. Here we have a study of the behavior of an investment made at the beginning of the year, in the portfolios selected by DEA and maintained without operations of purchase and sale of assets during that entire year. Therefore, we have a portfolio, which was purchased on
January 2 and maintained until December 31 of the same year.

We can observe that DEA (1/N) surpasses DEA(M) during the years of 2006 and 2007 while during periods of crisis, the portfolio with optimized weights surpasses in by 13.36%. We can also observe the weak performance of the DEA portfolios compared to IBOVESPA during the years of 2006 and 2007, a situation which is reversed in 2008, when both DEA models have a greater profitability, because they are less affected by the 2008 crisis.

<table>
<thead>
<tr>
<th></th>
<th>DEA (1/N)</th>
<th>DEA(M)</th>
<th>IBOVESPA</th>
<th>DEA(1/N)-</th>
<th>DEA(M)-</th>
<th>IBOVESPA-</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>25.5</td>
<td>20.19</td>
<td>32.9</td>
<td>5.31</td>
<td>-7.4</td>
<td>-12.71</td>
</tr>
<tr>
<td>2007</td>
<td>10.95</td>
<td>4.39</td>
<td>38.44</td>
<td>6.56</td>
<td>-27.49</td>
<td>-34.05</td>
</tr>
<tr>
<td>Sum</td>
<td>8.89</td>
<td>10.4</td>
<td>30.46</td>
<td>-1.51</td>
<td>-21.57</td>
<td>-20.06</td>
</tr>
<tr>
<td>Average</td>
<td>2.96</td>
<td>3.47</td>
<td>10.15</td>
<td>-0.51</td>
<td>-7.19</td>
<td>-6.68</td>
</tr>
</tbody>
</table>

**FINAL CONSIDERATIONS**

In this study, we have evaluated the possibility of using DEA in the construction of portfolios for investments in the Brazilian stock exchange – BOVESPA. For this purpose, two models were applied to the data of assets belonging to IBOVESPA: DEA Assurance Region with equal weights for the assets considered to be efficient (DEA (1/N)) and DEA Assurance Region with weights optimized by the Markowitz model (DEA (M)). We studied the hypothesis that the portfolios optimized by the model presented a better performance that the one formed by egalitarian participation. With this study, we expected to find an efficient way of choosing the assets that offered profitability superior to IBOVESPA.

The results indicate that the DEA (M) model presents a better monthly profitability than DEA (1/N), as well as surpassing IBOVESPA by 1.86% in the difference between the sum of monthly profits.

Regarding the profitability of annual portfolios formed by the DEA (1/N) and DEA (M) criteria, the result is that both achieve good performance only in 2008; because despite presenting drops in their profits, the drop is even lower than IBOVESPA’s.

If the DEA methodology was already considered as an integral part of a set of techniques truly capable of selecting assets for the composition of portfolios, the inclusion of Markowitz’s optimization model in this analysis constitutes another step in the search of better results in the variable revenue market. The portfolio obtained with the inclusion of Markowitz’s optimization model, produces a portfolio with greater profitability, and more importantly, reduces the losses during periods of crisis.

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The effect of 2001 financial crisis on Turkish banking sector efficiency

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ABSTRACT

Turkish banking system faced significant changes in consequence of Financial Liberalisation Program, since 1980. In 1980’s, new financial products were permitted, interest and foreign exchange rates were freed and foreign banks were welcomed to the market. In 1990’s traditional banking functions tended to disappear as a result of profitable arbitrage activities and profitable government papers. As a result of macroeconomic instability and weak banking system, November 2000 and February 2001 crises which were caused deepest effects in real sector were experienced. This paper examines banking efficiency between 2001 and 2007 for determining the effects of 2000-2001 Financial Crises. Data Envelopment Analysis which is a non-parametric linear programming-based technique for measuring deposit banks efficiency excluding banks under the deposit insurance fund was employed. DEA method calculated for both production and intermediation approaches as constant and variable rates to scale assumptions by using financial ratios. The basic founding suggests that state-owned banks were not affected by the financial crises and they are the most efficient banks in selected years. Another finding of the study is that Turkish banking system is more efficient in intermediation approach according to production approach.

Keywords: Banking, Efficiency, Liberalization, Data Envelopment Analysis

INTRODUCTION

Before 1980, the main development strategy was import substitution; the economy was relatively closed and heavily regulated by the government in Turkey. The characteristics of the financial system such as deposit and loan rates, negative real interest rate were centrally determined. But 1980 is the turning point of the financial structure of Turkey. A new structural adjustment program was adopted, interest rates were freed, new financial products and institutions were permitted and liberalization started to deepen financial markets. Major steps taken in the structural reforms included: liberalization of deposit and loan rates; establishments of governmental securities (1985) and Interbank markets (1986); introduction of open market operations with government securities (1987); re-opening of the Istanbul Stock Exchange (1985); and its becoming operational (1986); and the liberalization of the foreign exchange regime; liberalization of capital movements and acceptance of the convertibility of the Turkish Lira in 1989 and 1990 (Mercan et. al, 2003:188) As a reflection of internationalization policies, foreign banks, joint ventures and partnerships between domestic and foreign banks were also welcomed to the system (Isik and Hassan, 2002). During the 1990s, the banking system rapidly assumed the role of an intermediary between foreign financial investors and the government and the primary function of the banking system became to lend the government the short term credit obtained from the foreign markets (Yilmazkuday and Akay, 2008). As a result of profitable arbitrages traditional banking activities tended to disappear. Not only financial institutions were not deepen enough but also financial openness decreased the effects of global risks, Turkey experienced several economic crisis such as; 1991 Gulf Crisis, 1994 Crisis, 1997 Far East Crisis, 1998 Russian Crisis, 1999 The earthquakes of August, 2000 and 2001 liquidity crisis.

Among these crises, 2000-2001 crises were the most destructive ones for Turkish economy. The basic causes of the crises were fragile banking sector, high and inertial inflation, weakness of the banking system unsustainable nature of dept burden. The crisis started from banking sector and spread to real sector. The banking system has been used to earn easy money borrowing from individuals and foreigners and then lending government with the exorbitant interest rates. When interest rates fell down, they had to find new costumers and they gave cheap and risky credits to individuals and firms. But, meanwhile, since the global interest rates rose they are caught in naked position and in urgent need for liquidity. The banks in question,
desperate for liquidity, began to fire sales of government securities, causing interest rates to shoot up and international investors to exit the market (Ozutku, 2003).

There are exponential surveys which study the efficiency of banking industry in Turkey from different points of view. Some focused on the effects of banks scale (Kaya and Dogan; 2005, Cihangir; 2005 Denizer and Tarmiclar; 2000, Mercan et al; 2003, Chambers and Çifter; 2005), some studies productivity (Onuç and Aktaş; 2007) and some searched the effect of ownership (Atan and Çatalbaş; 2005, Yayla et al; 2005) on banking efficiency.

TURKISH BANKING SECTOR

As a result of financial liberalization after 1980, competition in banking sector was promoted and entry into the sector simplified and sector experienced significant development in the number of banks and employment.

Number of Banks, Branches and Employees

The number of banks in Turkey was 61 as of the end of 2001. At the end of 2008 it was 45. In the year 2001 the number of private banks decreased 50 percent in comparison with 2008.

<table>
<thead>
<tr>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposit</td>
<td>State</td>
<td>Private</td>
<td>Foreign</td>
<td>Deposit</td>
<td>State</td>
<td>Private</td>
<td>Foreign</td>
</tr>
<tr>
<td>46</td>
<td>3</td>
<td>22</td>
<td>15</td>
<td>40</td>
<td>3</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>34</td>
<td>3</td>
<td>17</td>
<td>13</td>
<td>33</td>
<td>3</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

Source: www.tbb.org

The number of branches and employees continued to rise in accordance with the development in the number of branches.

<table>
<thead>
<tr>
<th>2002</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposit</td>
<td>State</td>
<td>SDIF*</td>
</tr>
<tr>
<td>118329</td>
<td>138570</td>
<td>153212</td>
</tr>
<tr>
<td>40159</td>
<td>39223</td>
<td>41056</td>
</tr>
<tr>
<td>5886</td>
<td>333</td>
<td>325</td>
</tr>
<tr>
<td>5416</td>
<td>25794</td>
<td>36707</td>
</tr>
<tr>
<td>4942</td>
<td>4573</td>
<td>5322</td>
</tr>
<tr>
<td>Total</td>
<td>123271</td>
<td>143143</td>
</tr>
</tbody>
</table>

* Savings Deposit Insurance Fund.

Source: www.tbb.org

Banks’ employees in total rose by 1833 in state owned banks, 10913 in foreign banks and 1904 in private banks. Deposit banks employed 97 percent of banks’ employees and the rest was employed by development and investment banks.

The number of branches and employees dropped in 2000-2003 period as a result of the transfer to the Saving Deposits Insurance Fund and/or closure of banks with weak financial strength within the framework of the restructuring program, implementation of restructuring program in state-owned banks, bank mergers, as well as banks’ efforts to limit the operational costs; however it started to increase from the second quarter of 2003 (The Banks Association of Turkey, 2008).

Balance Sheet

Total assets of the banking sector rose 16 percent to 561 billion TL which is equal to 484 billion USD at the end of 2007 as compared to the end of 2006. Total deposits grew by 14 percent in 2007. The rate of increase was 21 percent in TL deposits, and 4 percent in foreign exchange deposits (The Banks Association of Turkey, 2008).

Graph 1: Size of Balance Sheet

DEA METHODOLOGY

DEA methodology is a linear programming application and the basics of DEA built upon the work of Farrell (1957) and developed by Charnes et al (1978). DEA compares efficiency of inputs and outputs of DMUs. Each DMU gets the same outputs by using the same inputs. In this paper, each bank represents a DMU. Although commercial banks are homogenous with respect to their
organizational structure, goals and objectives, they vary significantly in size and production level (Denizer et al, 2000:17). That makes the scale of the banks important in counting efficiency or inefficiency. So that two DEA models are employed in this study. The CCR model (Charnes et al., 1978), constant rates to scale, includes both technical and scale efficiency. The BCC model (Banker et al., 1984), variable rates to scale, assumes that the source of the inefficiency is only technical inefficiency. The ratio of the efficiency scores such as $S_{im}/d_{im}$ measures relative scale efficiency of bank m. If $S_{im}$ equal to 1; bank m is operating as efficient scale size and if less than 1; bank m is scale inefficient bank (Denizer et al, 2000).

The DEA input-oriented and output oriented models were chosen. Assume that there are $I$ inputs converted to $J$ outputs by $N$ number of decision making units (DMUs). Particularly, let’s think that $m$th DMU converts $x_{im}$ inputs to $y_{jm}$ outputs.

**DEA CRS (Constant Rates to Scale) Input Oriented Model**

s.t. $\min \Theta$

$$\sum_{j=1}^{J} y_{jm} \geq \theta_{0}$$

$$\sum_{i=1}^{I} x_{im} \leq \gamma_{0}$$

$$\gamma_{jm} \leq \gamma_{0}$$

$$\sum_{j=1}^{J} v_{jm} = 1$$

$$m \in \{2, \ldots, N\}$$

**DEA VRS Output Oriented Model**

$$\max \sum_{j=1}^{J} y_{jm}$$

Subject to:

$$\sum_{i=1}^{i} x_{im} \geq \theta_{0}$$

$$\sum_{j=1}^{J} \gamma_{jm} = 1$$

$$\sum_{j=1}^{J} v_{jm} = 1$$

$$m \in \{2, \ldots, N\}$$

The parameter $c_1$ is unconstrained in sign.

$c_1 > 0$: increasing returns to scale,

$c_1 = 0$: constant return to scale,

$c_1 < 0$: decreasing returns to scale.

**DEFINITION OF VARIABLES AND DATA**

A financial institution brings the economic agents who want to lend and borrow together. Banks are the most important intermediaries in financial institutions. There are basically two approaches in banking literature to explain the banking activity and calculate the efficiency; the production approach and intermediation approach. In production approach, banking activities can be describe as; using traditional production factors such as labour, capital and land to turn into loans and deposits. In intermediation approach the deposits which are collected and funds borrowed from financial markets are transformed into loans and investments.

Selection of variables to measure financial efficiency affects the results significantly. In light of this simple explanation, this study utilizes the financial ratios below:

<table>
<thead>
<tr>
<th>Table 4: Variable List</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inputs</strong></td>
</tr>
<tr>
<td><strong>Production</strong></td>
</tr>
<tr>
<td><strong>Personnel per Branch</strong></td>
</tr>
<tr>
<td><strong>Other Operating Expenses/TA</strong></td>
</tr>
<tr>
<td><strong>Intermediation</strong></td>
</tr>
</tbody>
</table>
Financial ratios of commercial banks are achieved by the Bank Association of Turkey to compute the efficiency between 2001-2007.

**FINDINGS AND DISCUSSION**

This study examines the impact of 2000 and 2001 financial crises on Turkish bank’s efficiency under production and intermediation approaches in terms of bank groups such as; state-owned, privately-owned, foreign banks and deposit banks as a whole. Development and investment banks and SDIF banks are not counted.

The efficiency scores for the deposit banks can be seen on Table 5. The efficiency score of the deposit banks which is calculated for production approach is 0.632 for constant rates to scale and 0.724 for variable rates to scale in 2001. After 2001, the efficiency scores started to decrease. For comparison, intermediation function of the deposit banks is more efficient than production function.

The results of production approach for CRS are presented on Table 6 and VRS are presented on Table 7. The private banks are more efficient than state owned and foreign banks in 2001. CRS efficiency score of the private banks 0.800 and VRS efficiency score 0.816. After 2001 efficiency scores are decreasing for state-owned and private banks. Foreign banks are not efficient in production function in all years.

In intermediation approach (Intermed), state owned banks are more efficient than private and foreign banks, private and foreign banks were expected to have more efficient scores. One of the reasons for this is public confidence in state owned banks for deposits safety. Because state supports the public banks and also public banks give loans more easily than private banks.

**CONCLUSION**

Employing DEA input and output oriented methods, we estimate the production and intermediation efficiencies of Turkish deposit banks over 2001-2007. The efficiency measures that we find, which is unexpected is; state owned banks are more efficient than private and foreign banks. State-owned banks were not affected by the financial crises as expected. And another finding is deposit banks are more efficient in intermediation approach.

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WTO membership and bank efficiency: evidence from China

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ABSTRACT
We use a non-parametric approach to examine the impact of recent banking reforms and WTO accession on the bank cost efficiency in China. We compare the efficiency of different bank ownership groups prior to and after joining the WTO between 1998 and 2006. We find that, on average, domestic banks outperform their foreign counterparts over the sample period in terms of overall and allocative efficiency, but they fall behind in terms of overall technical efficiency. Further, the efficiency of domestic banks has declined post WTO membership, while foreign banks have enjoyed an improvement in their cost efficiency post WTO accession in 2001.

Keywords: Data Envelopment Analysis, WTO membership, bank efficiency, china

INTRODUCTION
The adoption of an “open-door” policy in 1978 followed by accession to the World Trade Organization (WTO) on December 11, 2001, and subsequent external trade liberalization allowed the Chinese economy to grow consistently in the past two decades. At the end of 2006, the size of China’s economy measured by nominal Gross Domestic Product (GDP) was the third largest in the world after the United States and Japan. Factoring in the price differentials and measuring the purchasing power parity adjusted GDP, many economists believe that China’s GDP is actually the second largest economy in the world after the United States. It is expected that this growth momentum will continue at least for the next twenty years (Holz 2006). As of this writing, all signs of macroeconomic variables are pointing to further growth of the economy.

Without any doubt, the Chinese financial system has served well the economy in allocating economic resources to more productive uses. Although there are equity and bond markets in China, the major financiers for non-financial firms are banks and the current financial system is mostly dominated by the banking sector.

Existing research indicates that there is a positive relationship between the sustainability of economic growth and the development of the financial sector in emerging economies. In the past twenty years, China has introduced a series of banking reforms to increase the efficiency and viability of its banking sector, which dominates the financial system. This study investigates whether the efficiency and productivity of the banking sector in China has improved in recent years, especially after accession to the WTO in December 2001. We also examine the potential determinants of Chinese banking sector inefficiency. These issues are of major importance since China has agreed to open its financial system to full foreign competition as of December 11, 2006.

While previous studies have investigated the efficiency of a small number of banks in China (for instance see Berger et al. (2006) who analyze efficiency and bank ownership of 38 banks), we present in this paper a more accurate analysis of the Chinese banking efficiency status by employing a larger sample of 62 banks and by including an updated dataset that reaches 2006.

METHODOLOGY AND DATA
In order to assess the efficiency of Chinese banks between 1998 and 2006, we estimate several efficiency indices by employing a non-parametric methodology proposed by Farrell (1957) and expanded by Färe et al. (1985).

The main efficiency index that belongs to this set is overall efficiency (OE) which is measured as the ratio of the minimum potential total production cost to the total observed production cost. The OE index is then decomposed into two efficiency indices: allocative efficiency (AE) and overall technical
efficiency (OTE). OTE determines over-utilization inputs relative to a best practice frontier that exhibits constant returns to scale (CRS) property. AE evaluates the extent to which a bank misallocates resources i.e. utilizes input mix inconsistent with the cost minimization principle.

The OTE is further decomposed into two more indices: pure technical efficiency (PTE) and scale efficiency (SE). PTE is determined by comparing the actual inputs employed by a bank relative to a best practice frontier that exhibits constant returns to scale (CRS), increasing returns to scale, and decreasing returns to scale properties. SE is calculated as the ratio of OTE to PTE and it measures the extent to which a bank deviates from operating at the optimal size of operation.

Let bank k be an observation in a sample of K Chinese banks, the efficiency indices described above can be written as:

\[
\begin{align*}
OE_k &= OTE_k \times AE_k \\
PTE_k &= PTE_k \times \psi \\
SE_k &= \frac{OTE_k}{PTE_k}
\end{align*}
\]  

(1)

To compute the OE empirically for bank k in year t, we first solve the linear programming model 1 below to obtain the potential minimum total cost of production:

\[
\begin{align*}
& \text{min } p \times x \\
& \text{subject to } y_k \leq z Y, x_k \geq z X, z \in R^K \\
& k = 1, \ldots, K
\end{align*}
\]  

(Model 1)

Where \(C^*_k\) is potential minimum total cost, \(p\) is a vector of input prices, \(y_k\) is a vector (1 x n) of outputs produced by bank k, \(x_k\) is a vector of inputs (1 x m) utilized by bank k, \(Y\) is a matrix of observed outputs (K x m) produced by all banks in the sample, and \(X\) is a matrix of observed inputs (K x n) employed by all banks in the sample, and \(z\) is a (1xK) intensity vector.

The OE for bank k is calculated as

\[
OE_k = \frac{C^*_k}{C^*_k}
\]

where \(C^*_k\) is a defined earlier and \(C_k\) is the actual total cost incurred by bank k.

In order to estimate OTE for the kth bank, we solve the following LP model:

\[
\begin{align*}
& \text{min } \varsigma \\
& \text{subject to } y_k \leq z Y, x_k \geq z X, z \in R^K \\
& k = 1, \ldots, K
\end{align*}
\]  

(Model 2)

Where 0 ≤ \(\varsigma\) ≤ 1 is the OTE index for bank k relative to a best practice frontier that exhibits CRS.

In model 2, vector \(z\) consists of a set of weights given to each bank included in the pooled sample by the model in construction of common efficient frontier relative to which OTE of bank k is computed.

To obtain PTE, \(\psi\), for bank k, we resolve Model 2 with \(\sum_{k=1}^{K} z_k = 1\) as an additional constraint and replacing \(\delta\) by \(\psi\). The SE for bank k is then:

\[
SE_k = \frac{OTE_k}{PTE_k} = \frac{\delta}{\psi}
\]  

(2)

If \(SE_k = 1\), (\(\delta = \psi\)), bank k is scale efficient, i.e. it lies on the frontier that exhibit CRS. If 0 ≤ \(SE_k < 1\), then k is scale inefficient and an additional linear program should be solved to identify the source of scale inefficiency of this bank. More specifically, we resolve Model 2 after replacing \(\sum_{k=1}^{K} z_k = 1\) by \(\sum_{k=1}^{K} z_k \leq 1\) and \(\delta\) by \(\omega\). The efficiency score \(\omega\) measures the efficiency of bank k relative to a best practice frontier that displays non-increasing returns to scale property. Färe, Grosskopf and Lovell (1985) show that, if \(SE_k \neq 1\) and \(\omega = \psi\), the source of scale inefficiency of bank k is decreasing returns to scale (DRS), or if \(SE_k \neq 1\) and \(\omega = \psi\), then the scale inefficiency of this bank is due to increasing returns to scale (IRS).

The major source of the data for this study is Thompson’s BankScope and it covers a period of 9 years from 1998 to 2006. We categorize Chinese banks based on their ownership organizational structure into domestic (and its sub-groups; Big-Four state owned, joint-stock, and city commercial) and foreign banks. Every attempt has been made to include as many banks in our sample as possible. Table 1 provides the means of outputs, inputs, input prices and total assets for all banks included in the sample, as well as for different ownership categories.

**EMPIRICAL RESULTS**

Table 2 reports summary statistics of efficiency indices for both groups. This table, in addition, contains the summary statistics of efficiency indices for the three ownership sub-
categories that constitute domestic banks, including Big Four banks, Joint-Stock Banks, and City Commercial banks.

First, we note that the average of overall efficiency for all banks is 87.02 percent, indicating an average potential total production cost saving of 12.98 percent over a nine-year period, if all banks had been fully overall efficient. Furthermore, the 12.98 percent cost inefficiency of these banks over the sample period is mainly due to overall technical inefficiency (88.60 percent) while allocative inefficiency plays a little part in the overall inefficiency.

Second, the comparison of efficiency indices between foreign and domestic banks reveals that while domestic banks are slightly more overall and significantly more allocatively efficient, foreign banks are found to be more overall technically, pure technically, and scale efficient. For instance, the mean value of OE of domestic banks is 0.56 percent higher than that of foreign banks, suggesting that domestic banks could produce the same output level with 0.56 percent lower total production cost than foreign banks. Furthermore, the higher mean value of allocative efficiency for domestic banks indicates that these banks chose more optimal input mix consistent with cost minimization compared to foreign banks. However, domestic banks suffer significantly from over-utilization of input since their OTE index is 4.84% less that of foreign banks. The breakdown of OTE shows that foreign banks operate closer to the best practice frontier that exhibits CRS, IRS, and DRS, and also operate at a more optimal scale of operation during the period studied.

This analysis is based on what was explained in the methodology section, namely that overall technical efficiency can be decomposed into pure technical efficiency and scale efficiency. As can be seen from the indices reported in Table 2, the overall technical inefficiency for foreign banks is to some extent due to purely technical inefficiency since these banks operate more at an optimal scale (0.9710 versus 0.9531) compared to foreign banks. This, however, is not the case for domestic banks whose PTE and SE are at the same level (SE=0.9371 versus PTE=0.9374).

Inspection of standard deviations of efficiency indices for all banks shows that the volatility of OE has been highest and that of AE has been lowest over the period under study for all banks. We also note that the efficiency indices of foreign banks are generally more dispersed relative to domestic banks, except for SE where the domestic banks group demonstrates notably a higher level of variability.

Third, the analysis of the efficiency indices given in Table 2 for three ownership sub-categories that constitute domestic banks suggests that the Big Four banks have been the most efficient among all the domestic bank sub-categories in terms of all efficiency indices expect for allocative efficiency. More specifically, in the case of OE, OTE, PTE, and SE the Big Four are the most efficient, whereas in the case of AE, City Commercial banks are more efficient than other domestic banks. The index of overall efficiency has an average of 95.25 percent for Big four group (most overall efficient) and 85.15 percent for City Commercial (least overall efficient). However, further examination of the results reveals that the Big Four banks experienced an input mix sub-optimization as indicated by their AE level (AE=0.9871) compared to other domestic banks (0.9908 and 0.9929 for Joint-Stock and City Commercial banks respectively). Nonetheless, the Big Four represent the most technically efficient banks (OTE=0.9650 as compared to 0.8874 and 0.8574 for Joint-Stock and City Commercial banks respectively), albeit their overall technical inefficiency is caused mainly by scale inefficiency (SE=0.9730) rather than pure technical inefficiency (PTE=0.9917). On another note, the figures show that Joint-Stock banks are ranked second among domestic banks in terms of all efficiency measures except for AE where they rank first. We also note that the efficiency rankings are almost clear-cut among the three domestic banks sub-categories other than for the AE in which the Big Four have the lowest score.

Fourth, the examination of standard deviations of efficiency indices shows that, for all banks, the volatility of OE has been highest and that of AE has been lowest over the period studied. The figures of Table 2, furthermore, illustrate that the efficiency indices of foreign banks show considerably more variability compared to those of domestic banks except in the case of SE where Domestic banks have a higher standard deviation. We note that among Domestic banks, the Big Four banks have the lowest standard deviations in all efficiency indices except for AE, while the results between Joint-stock and City commercial are mixed.

In order to provide more evidence to verify the impact of membership in the WTO on the efficiency of Chinese banks, we divide the sample period (1998 to 2006) into two sub-periods: the pre-WTO membership sub-period (1998-2001) and the post-WTO membership sub-period (2002-2006). The mean values of the efficiency indices for both sub-periods are
displayed in Table 3. It is apparent that, for all Chinese banks, all efficiency indices (except for PTE) are higher in the pre-WTO membership sub-period compared to the post-WTO membership sub-period. In fact, the PTE component of OTE is consistently improved for all banks operating in China post WTO accession. However, these results do not hold alike for foreign and domestic banks. In the case of foreign banks, it is interesting to note that every efficiency index is higher post-WTO membership compared to pre-WTO membership while the reverse is true for Domestic banks. It seems that foreign banks are reacting more positively than Domestic banks to the WTO accession as figures related to the domestic banks sub-categories displayed in Panels C1, C2 and C3 indicate. As can be seen from these panels, the efficiency indices are in general higher during the pre-WTO membership sub-period relative to the post-WTO membership sub-period, with the exception of PTE for the Big Four and City commercial banks group, and of OE, AE, and PTE for the Joint-Stock banks group.

To sum, the analysis shows that the efficiency of domestic Chinese banks has weakened after China’s accession to the WTO in 2001, while foreign banks have become more cost efficient in regards to all the measured efficiency indices.

SUMMARY AND CONCLUSIONS

The findings of the study provide evidence that over the period under study, domestic banks in China were generally more overall efficient than foreign banks, also exhibiting less variability in their efficiency scores. Among domestic banks, the state-owned Big Four have been the most efficient followed by Joint-Stock and City Commercial banks. Foreign banks, in contrast, were found to be more overall technically, pure technically, and scale efficient than domestic banks over the period under study. Furthermore, the Big Four banks experienced a clear rising trend in their cost efficiency prior to WTO accession, but the trend is reverted after the WTO membership date, while Foreign banks have generally enjoyed a rise in their cost efficiency post WTO accession in 2001.

REFERENCES


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## Table 2: Means of efficiency measures of Chinese banks ownership groups relative to common frontier, for the sample period 1998-2006

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### Notes
- Y1 = Net Loans, Y2 = Total Deposits, Y3 = Total Other Earning Assets, X1 = Loanable Funds
- X2 = Other Inputs, P1 = Unit price of Loanable Funds, P2 = Unit price of Other Inputs
- TA = Total Assets
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OE = Overall Efficiency; AE = Allocative Efficiency; OTE = Overall Technical Efficiency; PTE = Pure Technical Efficiency; SE = Scale Efficiency; N = Number of Observations
Designing dynamic fuzzy Data Envelopment Analysis model for measuring efficiency of the investment corporations in Tehran stock exchange

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ABSTRACT
The aim of this article is to design a Dynamic Data Envelopment Analysis (DDEA) Model for measuring the efficiency of the investment corporations in Tehran Stock Exchange during 2004-2008. Dynamic model is a model by which one can measure the efficiency of an organization by itself and by using data in different periods in order to resolve the limitations of the same organizations. To do so, apart from comparison and ranking of twenty-seven active companies in stock exchange, the efficiency of a firm was measured twenty-four times each three months and it was revealed that within which period the firm had the best performance. Furthermore, not only certain data, but also uncertain data are used which can guarantee the better result. This research has been done on twenty-seven investment corporations and also one corporate in twenty-four time periods. The result indicates that the efficiency of the three corporations from twenty-seven corporations and four time periods from twenty-four periods are on the frontier of efficiency  
Keywords: Efficiency, Fuzzy, Dynamic Data Envelopment Analysis, Investment Companies

INTRODUCTION
Most of the researchers believe that measuring the efficiency of a firm or an organization is of the highest importance. To measure the efficiency of units, different tools can be used. However, there are two major methods, in general: parametric methods, and non-parametric methods. In parametric methods certain production function will be estimated using different statistical and econometric methods. Then, using the function the efficiency will be measured. In non-parametric method it is not necessary to estimate the production function. DEA is a non-parametric method which compares the relative efficiency of units with each other. To know the shape of production function is not important in this method and there is no restriction for the number of inputs and outputs [1].The problem with using DEA method is the lack of such tools for measuring the units individually which was eliminated by including different time periods as deciding units.  
Investment corporations are active sectors in financial market. Financial intermediary has grown significantly in the world since the establishment of the first investment firm in United States in 1924. The corporations’ major duty is to invest the stock purchasers’ capital effectively. Although investors are all familiar with the importance of such corporations, it does not mean that they rely on them totally [2]. Investment corporations are financial intermediaries which sell stocks to all groups of people and invest sales revenue in various stock exchange baskets. Each sold stock is the relative proportion representative of stock exchange basket which is managed on behalf of stock-owners by the investment firm. Providing the stock basket, the corporations try to reduce risks for the investors[3].The major role of financial intermediaries is to change less desirable financial assets to other financial assets (negative assets). In doing so, one of the four major economic approaches should be used: reducing risk by providing various investments, reducing the cost of making contract and data processing, providing payment mechanisms such as cheque, debit card, and electronic transaction [4].One of the major elements for the economic development and progress of a country is investment. Besides, investment corporations play an important role in stock exchange markets. The efficiency of such markets impact investment markets and because the improvement of economy in a country is based on efficiency of
such corporations, therefore the performance and activity of such corporations should be determined using suitable tools.

THEORETICAL BASES AND REVIEW OF RELATED LITERATURE

To measure the efficiency because of its impact on assessment of corporations and organizations performance is a major concern for researchers. In the same way, managers, engineers, and economists attempted to design quantitative methods to measure it and among which DEA is one of the most important methods. It should be said that the characteristics of other methods and measuring tools like index and ratio methods, function production method, utility approach, and unit cost method can be found in DEA[5]. DEA is a method used for measuring the efficiency and productivity of the units. These units are used for resources to gain desirable outputs. What is significant in this method is that predetermined weights (i.e. index method) and explicit specification between inputs and outputs (i.e. regression method) are not needed [6]. Also, using organization and production units data as deciding units, DEA is a linear planning method that produces the frontier of efficiency. Such frontier will be determined relying on input and output data and constant linear planning results. In fact, the degree of inefficiency of any deciding units is the interval it takes from the frontier of efficiency [7].

Since DEA models are special mathematical planning and particularly linear planning method, so that all that about fuzzy linear planning in general is also true about DEA models. However, the major problem is that DEA model have primary definitions which simply cannot be changed into linear planning by using fuzzy parameters [8].

There have been some researches about the same method. For example, Sengupta was the one who formulated fuzzy planning model in which restrictions and target function cannot be satisfied completely. Sengupta’s DEA method takes into account several inputs and outputs and provides two fuzzy planning versions for DEA. First, he uses linear membership function, and in the second version he uses non-linear membership function [9]. As he suggests the breach limitation and target function have certain values, but such assumption is not applicable in most cases [10]. Triantis and Seaver proposed a clustering approach as a tool to determine efficiency behaviour limit or exorbitant efficiency behaviour [11]. Tanaka and Guo suggested an approach based on alpha cutting measure which changes a fuzzy DEA model into two level linear planning models. Such approach based on certain restricting situation results in productivity, and therefore it is not a general method. Besides, using such method needs solving several linear planning problems in order to estimate membership function to assess the performance of a decision making unit [12]. Cooper, Park, and Yu developed the imprecise DEA. In their method, a combination of precise and imprecise data with explicit limit and precise data are used [13]. Leon, Liern, Ruiz, and Sirvent using some ranking method based on alpha cutting measure. They changed BCC into a fuzzy DEA model. Fuzzy BCC with $m+s+1$ limit will be converted into a primary linear planning model with $4m+4s+1$. Also, using their methods, several linear planning problem should be resolved to estimate the membership function and then measure efficiency of a decision making unit [14].

Jahanshaloo, Soleimani, and Naserabdi suggested a comparison of fuzzy numbers and developed a SBM model to a fuzzy DEA model. In this method, it is necessary to use non-linear multi-target planning to assess each decision making unit. Nevertheless, it can be changed into a linear planning model using the fixed value of a variant [15].

Lertworasirikul, Fang, Joines, and Nuttle recommended a feasible approach to deal with fuzzy DEA model which is based on solving CCR model. Again, this method is not applicable in BCC model and hence their model is not a general method [16].

Kao and Liu suggested a method using alpha cutting measure which changes fuzzy DEA model into primary firm model. Using such method, it is needed to solve several linear planning problems to estimate membership function and then assess performance of a decision making unit [17].

Tanaka, Ichihashi, and Asai used fuzzy triangular numbers. Applying alpha cutting measure, they used a pair of linear planning to assess efficiency of the unit under consideration [18]. Memarian, Saati, and Jahanshaloo in his article finds a common set of weights for inputs and outputs of all units. To find a common set of weights in fuzzy DEA model, they assigned high cut of a membership function to assess the performance of a decision making unit [19]. Soleimani-Damaneh, Jahanshaloo, and Abbasbandi try to eliminate such drawbacks, theoretical, and calculating
problems to measure the current performance, and provide a new approach using fuzzy DEA which is the basic theoretical principle of the present research [20].

METHODOLOGY

Since this research paper is concerned about designing and providing a conceptual and a suitable model for assessment of dynamic efficiency of organizations, the analytical-descriptive research method is chosen. Therefore, some library researches about the related literature of basic DEA model and fuzzy logic were conducted. Then, the suggested mathematical model was chosen by using DEA model and logic. Besides, the data collection of items includes tangible inputs and outputs based on studying relevant documents.

STATISTICAL COMMUNITY

The statistic community in this research includes those affirmed corporations Tehran Stock Exchange Market. Among 34 active corporations 27 of them were chosen. The requested data were chosen by studying the documents, financial statements, and reports of the corporations on TSEM and the corporations own websites.

CONCEPTUAL MODEL

To measure the efficiency of organization as a comprehensive conceptual model is suggested by using DEA model. First, fuzzy DEA is used to calculate the efficiency of 27 corporations and then they were ranked according to their efficiency levels. Lastly, using fuzzy dynamic DEA model and data of a firm each 3 months for 24 times, the efficiency was calculated and it was ranked. The above mentioned model is called dynamic efficiency measurement model since it includes time entity as a variant. The theoretical principle of the model is based on the performance of a decision-making unit in different time periods. As a result, each organization as a decision-making unit can be measured by itself and its performance in each will be considered as a decision-making unit which can be used for creating the efficiency frontier in DEA model. Therefore, measuring the performance of an organization in different periods of time and comparing them using AP, BCC models, the comparison between non-similar units will be feasible.

Fuzzy model

If we assume a set of decision-making units (DMU) including fuzzy input $\tilde{x}_i$ and output $\tilde{y}_j$ which belong to a set of fuzzy positive numbers, then fuzzy input axis a BCC model will be represented as:

\[
\min \theta \\
\text{s.t.} \\
\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \leq \theta \tilde{x}_{io} \quad i = 1, \ldots, m \\
\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{ro} \quad r = 1, \ldots, s \\
\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 
\]

If fuzzy LR numbers will be assigned to input and output values then:

\[
\tilde{x}_{ij} = (x_{ijL}, x_{ijM}, x_{ijU}, \delta_{ij}) \\
\tilde{y}_{rj} = (y_{rjL}, y_{rjM}, y_{rjU}, p_{rj}, q_{rj}) \\
L_{i1} = \ldots = L_{in} := L_i \\
\hat{L}_{r1} = \ldots = \hat{L}_{rn} := \hat{L}_r \\
R_{i1} = \ldots = R_{in} := R_i \\
\hat{R}_{r1} = \ldots = \hat{R}_{rn} := \hat{R}_r 
\]

Model (1) will be converted into:

\[
\min \theta \\
\text{s.t.} \\
\theta(\tilde{x}_{io} + \tilde{x}_{io}) - \sum_{j=1}^{n} \lambda_j (x_{ijL} + \tilde{x}_{ij}) + (\sum_{j=1}^{n} \lambda_j \beta_{ij} - \theta \beta_{io}) \int_0^1 \Gamma^{-1}(\alpha) d\alpha \\
+ (\theta \delta_{io} - \sum_{j=1}^{n} \lambda_j \delta_{ij}) \int_0^1 R^{-1}(\alpha) d\alpha \geq 0 \\
\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n 
\]

Now we have to place the following:

\[
t_i = \int_0^1 L^{-1}(\alpha) d\alpha, \quad t_r = \int_0^1 \hat{L}^{-1}(\alpha) d\alpha, \\
k_i = \int_0^1 R^{-1}(\alpha) d\alpha, \quad k_r = \int_0^1 \hat{R}^{-1}(\alpha) d\alpha 
\]

Which the above mentioned model will be represented as (3):

\[
\min \theta \\
\text{s.t.} \\
\sum_{j=1}^{n} \lambda_j (x_{ijL} + \tilde{x}_{ij} + k_i \delta_{ij} - \theta \beta_{ij}) \leq \theta (x_{ja} + \tilde{x} + k_i \delta_{io} - \theta \beta_{io}), \\
\sum_{j=1}^{n} \lambda_j (y_{rjL} + \tilde{y}_{rj} + k_r q_{rj}) \geq \sum_{j=1}^{n} \lambda_j y_{ro} + k_r q_{ro}, \\
\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n 
\]

It is obvious that such model is always feasible and it allocates (0,1] to optimum interval.

DMU0 is an efficient unit if $\theta^* = 1$ and it is an inefficient if $\theta^* < 1$ and this is like DEA model theory suggested by Cooper and others. Also, there is one efficient unit in comparison to other units at least. There are one or several
and it bears testimony to the first DEA theory.

Furthermore, it is obvious that such approach to convert all DEA models to a fuzzy model without any condition precedent and if we consider the change of the variant:

\[
\hat{x}_{ij} = x_{ij} + k_i y_{ij} - t_i \beta_{ij} ; \\
\hat{y}_{rj} = y_{rj} + y_{rj} - r_r p_{rj} + k_r q_{rj}
\]

Model (4) will be represented as

\[
\text{Min } \theta \quad (4)
\]

s.t.

\[
\sum_{j=1}^{n} \lambda_j \hat{x}_{ij} \leq \hat{x}_{io} , \quad i = 1, \ldots, m \\
\sum_{j=1}^{n} \lambda_j \hat{y}_{rj} \geq \hat{y}_{ro} , \quad r = 1, \ldots, s \\
\sum_{j=1}^{n} \lambda_j = 1 , \quad \lambda_j \geq 0 ; \quad j = 1, \ldots, n
\]

The above model is a standard DEA model so that the efficiency of decision-making units can be determined. In fact, to assess the efficiency of decision-making units, according to our approach, needs solving a linear planning model (LP) while in other approaches one has to solve several LPs [20].

**Inputs and Outputs**

One of the major steps in designing a suitable operational assessment system for corporations is the definition and recognition of suitable indexes and variants. From economic standpoint, the assessment indexes for an investment firm are mostly the same assessment indexes of other corporations. Indexes such as price-earnings ratio (P/E), Debt-to-equity ratio, Earnings-per share (EPS), Dividends-per share, yield-per share, the final value of share in a specific day, the number of stock traders, costs and debts of a firm, surplus value per share, and etc[21]. As a result, firstly, some input and output variants were chosen by studying some documents and then among the primary variants three input variants and two output variants as the most important and reliable indexes for performance assessment and ranking of corporations based on our discussion with some experienced scholars and experts in capital market. The input variants are:

- The current costs of a firm
- The current debts of a firm
- Portfolio value of a firm

And output variants are:

- The performance of each share which can be calculated using equation:
  
  \[
  \text{Share performance} = \frac{\text{DPS} + (P_t - P_0)}{P_0}
  \]

- Earnings-per share (EPS)

**ANALYSIS AND MODEL IMPLEMENTATION**

Firstly, the data were collected and summarized studying the related documents, financial sheets, firm reports on Tehran Stock Exchange Market websites to analyze and implement the model. Then, using Matlab and EMS software, Dynamic Fuzzy DEA was applied and the corporations were ranked according to their efficiency and afterwards the efficiency of a sample firm was calculated and ranked using Dynamic Fuzzy DEA in each three months for 24 times.

**Analysis and Fuzzy model implementation**

Implementing the fuzzy DEA in table (1) shows that among 27 investment corporations only 3 of them (DMU1, 3, 11) are on efficiency frontier, and are the most efficient ones. These units are considered as paragon or reference of inefficient units, that is, if inefficient units want to reach to efficiency frontier they have to reach to set of reference data.

**Table (1): Implementing the fuzzy DEA**

<table>
<thead>
<tr>
<th>Row</th>
<th>DMU</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DMU1</td>
<td>737.47%</td>
</tr>
<tr>
<td>2</td>
<td>DMU2</td>
<td>699.21%</td>
</tr>
<tr>
<td>3</td>
<td>DMU3</td>
<td>111.07%</td>
</tr>
<tr>
<td>4</td>
<td>DMU4</td>
<td>84.28%</td>
</tr>
<tr>
<td>5</td>
<td>DMU5</td>
<td>76.79%</td>
</tr>
<tr>
<td>6</td>
<td>DMU6</td>
<td>74.29%</td>
</tr>
<tr>
<td>7</td>
<td>DMU7</td>
<td>63.29%</td>
</tr>
<tr>
<td>8</td>
<td>DMU8</td>
<td>38.81%</td>
</tr>
<tr>
<td>9</td>
<td>DMU9</td>
<td>34.82%</td>
</tr>
<tr>
<td>10</td>
<td>DMU10</td>
<td>32.41%</td>
</tr>
<tr>
<td>11</td>
<td>DMU11</td>
<td>30.32%</td>
</tr>
<tr>
<td>12</td>
<td>DMU12</td>
<td>29.21%</td>
</tr>
<tr>
<td>13</td>
<td>DMU13</td>
<td>23.09%</td>
</tr>
<tr>
<td>14</td>
<td>DMU14</td>
<td>20.80%</td>
</tr>
<tr>
<td>15</td>
<td>DMU15</td>
<td>18.70%</td>
</tr>
<tr>
<td>16</td>
<td>DMU16</td>
<td>18.15%</td>
</tr>
<tr>
<td>17</td>
<td>DMU17</td>
<td>15.46%</td>
</tr>
<tr>
<td>18</td>
<td>DMU18</td>
<td>7.03%</td>
</tr>
<tr>
<td>19</td>
<td>DMU19</td>
<td>6.93%</td>
</tr>
<tr>
<td>20</td>
<td>DMU20</td>
<td>5.92%</td>
</tr>
<tr>
<td>21</td>
<td>DMU21</td>
<td>5.43%</td>
</tr>
<tr>
<td>22</td>
<td>DMU22</td>
<td>5.19%</td>
</tr>
<tr>
<td>23</td>
<td>DMU23</td>
<td>2.84%</td>
</tr>
<tr>
<td>24</td>
<td>DMU24</td>
<td>1.64%</td>
</tr>
<tr>
<td>25</td>
<td>DMU25</td>
<td>1.3%</td>
</tr>
<tr>
<td>26</td>
<td>DMU26</td>
<td>0.50%</td>
</tr>
<tr>
<td>27</td>
<td>DMU27</td>
<td>0.14%</td>
</tr>
</tbody>
</table>

**Analysis and Implementation of Dynamic Fuzzy DEA**

To analyse Dynamic Data and implementing Dynamic Fuzzy DEA the data of a sample corporation in three months in a 24-time period is used.

Implementing Dynamic Fuzzy DEA in table (2) shows that the second, third, fourth, and twenty-fourth time periods (T2,T3,T4,T24) are on efficiency frontier and so that are chosen as
paragon or reference units for other time periods. As it can be seen in Table (2), the first column shows the row, the second the DMU, and the third the efficiency.

Table (2): Implementing Dynamic Fuzzy DEA

<table>
<thead>
<tr>
<th>Row</th>
<th>DMU (Times periods)</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T3</td>
<td>58.47%</td>
</tr>
<tr>
<td>2</td>
<td>T24</td>
<td>252.71%</td>
</tr>
<tr>
<td>3</td>
<td>T4</td>
<td>148.53%</td>
</tr>
<tr>
<td>4</td>
<td>T24</td>
<td>111.18%</td>
</tr>
<tr>
<td>5</td>
<td>T14</td>
<td>84.29%</td>
</tr>
<tr>
<td>6</td>
<td>T14</td>
<td>75.03%</td>
</tr>
<tr>
<td>7</td>
<td>T15</td>
<td>70.66%</td>
</tr>
<tr>
<td>8</td>
<td>T5</td>
<td>69.98%</td>
</tr>
<tr>
<td>9</td>
<td>T18</td>
<td>64.55%</td>
</tr>
<tr>
<td>10</td>
<td>T6</td>
<td>53.65%</td>
</tr>
<tr>
<td>11</td>
<td>T9</td>
<td>50.38%</td>
</tr>
<tr>
<td>12</td>
<td>T22</td>
<td>47.94%</td>
</tr>
<tr>
<td>13</td>
<td>T20</td>
<td>42.73%</td>
</tr>
<tr>
<td>14</td>
<td>T8</td>
<td>40.82%</td>
</tr>
<tr>
<td>15</td>
<td>T19</td>
<td>33.27%</td>
</tr>
<tr>
<td>16</td>
<td>T21</td>
<td>32.26%</td>
</tr>
<tr>
<td>17</td>
<td>T13</td>
<td>21.78%</td>
</tr>
<tr>
<td>18</td>
<td>T17</td>
<td>12.47%</td>
</tr>
<tr>
<td>19</td>
<td>T12</td>
<td>8.63%</td>
</tr>
<tr>
<td>20</td>
<td>T10</td>
<td>6.47%</td>
</tr>
<tr>
<td>21</td>
<td>T7</td>
<td>5.83%</td>
</tr>
<tr>
<td>22</td>
<td>T16</td>
<td>4.29%</td>
</tr>
<tr>
<td>23</td>
<td>T11</td>
<td>2.16%</td>
</tr>
<tr>
<td>24</td>
<td>T23</td>
<td>0.09%</td>
</tr>
</tbody>
</table>

CONCLUSION

To measure the efficiency and to rank the investment corporations of Tehran Stock Exchange Market Dynamic Fuzzy DEA model was used in the first place. However, one of the major drawbacks of DEA is that the similar units cannot be compared with each other and it is also about some other models by which one can measure the efficiency and productivity. Therefore, to eliminate such drawback, we used time variant, and that is why the model is called Dynamic model. The theoretical principle of the research is based on performance of a decision-making unit indifferent time periods, so that each organisation as a decision-making unit which can be used for creating the efficiency frontier in Dynamic Fuzzy DEA model.

In doing so, on the one hand, a sample firm each three months in a 24-time period was analysed using Dynamic Fuzzy DEA and was ranked based on efficiency 24-time period. On the other hand, the values of inputs and outputs of different agencies may differ under different condition and as a result have imprecise values. Therefore, to find their real condition, real problem, and realistic results, fuzzy approach was used.

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Human Development & Quality of Life Applications Stream
The human development cost efficiency

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ABSTRACT
This paper investigates the Human development efficiency using data envelopment analysis (DEA). The analysis concerns a sample of 149 countries for whom the data is available. We consider health expenditure, education expenditure and natural resources depreciation through CO2 emissions as inputs, and we consider human development components as outputs. We use a VRS input oriented multiplier model. Our results show that efficient countries belong to different levels of development and that natural resource depreciation affects the human development cost efficiency more than the other components.

Keywords: DEA, HDI, efficiency, CO2.

INTRODUCTION
The year 2010 marks the 20th anniversary of the human development report (HDR). This report gives the state of development in the world through several indicators. The Human Development Index (HDI) is the oldest and the most referenced index. The HDI has been constructed every year since 1990 to measure average achievements in basic human development in one simple composite index and to produce a ranking of countries [5]. It is based on three indicators: longevity, as a measure of life expectancy at birth; educational attainment, as measured by a combination of adult literacy (2/3 weight) and combined primary; secondary and tertiary enrolment ratios (1/3 weight); and standard of living, as measured by real GDP per capita (PPPP).

Since health, education and standard of living are not free, we consider in this paper the human development cost efficiency, where the human development components are considered as outputs and public spending on health, public spending on education and natural resources depreciation; and more precisely CO2 emissions are considered as inputs. The term cost efficiency is used because the inputs used are cost. In other words we are not interested by a cost DEA model.

Our main contribution resides in the subject itself since to our knowledge this is the first time human development is treated under this angle and in the way results are interpreted as it is explained in Section 3.

The rest of this paper is organized as follow, In Section 2, we present the inputs/outputs and the model. In Section 3, a detailed description of the data is given and In Section 4, the efficiency results are analysed. Conclusion follows in the last section.

PRESENTATION OF THE INPUTS/OUTPUTS AND THE MODEL

Choice of inputs and outputs
The world has been facing since World War II and the industrial revolution many problems such as climate change, disparities between people within the same country or between countries, etc. The HDI does not reflect these problems, that are why many other indexes such as genuine index, the poverty index has been considered. In this paper instead of considering other indexes, we consider some input components that contribute to human development. These components are: State commitment to health; which contributes to high life expectancy, state commitment to education since it contributes to the education index, and as a third input, we consider a very important contributor to industrial development and comfortable life; natural resources depreciation (Air, Water, Forest, etc.). We choose to represent natural resources depreciation by CO2 emissions since it is the most used at an international level and data is available for it. It is worth to mention that even though the three input components contribute directly to human development, we will distinguish through our interpretations between the appreciated contributors (expenditure on health and on education) and the non
appreciated contributor (natural resources depreciation which have a negative effect on future generations).

In summary, we consider as inputs:
- Per capita public expenditure on health (PEH)
- Per capita public expenditure on education (PED)
- Per capita CO2 emissions (CO2)

The outputs that represent the human development components are:
- Per capita Gross Domestic Product (GDP)
- Life expectancy index (LEI)
- Education index (EdI)

We consider GDP per capita rather than GDP index because the GDP index is based on the logarithm of the GDP which reduces the real difference between countries.

The DEA Model

Data Envelopment Analysis (DEA), is a non-parametric method based on linear programming. This method aims to measure the relative efficiency of Decision Making Units (DMU). It was first introduced by Charnes, Cooper and Rhodes (CCR) in 1978 [2]. The CCR model assumes constant return to scale (CRS) where the DMU’s are operating under optimal conditions and an increase in input is accompanied by a proportional increase in output. The Variable Return to Scale (VRS) version of the model was introduced by Banker, Charnes and Cooper in 1984 [1]. In the basic model, efficiency is a weighted sum of inputs divided by a weighted sum of outputs. In this model, the decision variables are the respective weights of inputs and outputs.

The objective of this paper is to measure efficiency and to compare the sensitivity of the model to the different input components. Since we are dealing with countries with different levels of development, we are not interested by benchmarks or by producing a virtual objective DMU for inefficient countries. Fore a detailed description of DEA models, see the book of cooper et al. [3] and the DEA homepage [7].

In our analysis we are interested by the scores and the virtual inputs, thus we use a multiplier input oriented VRS model (See model 2.2 of Chapter 2 of the Handbook of Data Envelopment Analysis [4]). The choice of the VRS model is justified by the fact that countries with low human development could improve their inputs with a proportion higher than their inputs, while for countries with a high human development it is more difficult for them to improve their outputs proportionally.

DATA DESCRIPTION

Our analysis concerns the year 2004 given that our main source is the 2007/2008 human development report [6] and that the 2009 report was not as comprehensive as the earlier report. The number of DMU considered in this analysis is a sample of 149 countries for whom the data is available. This sample is made of countries with different levels of development.

In the following Table 1 gives a statistical description of the inputs and the outputs for the countries under consideration.

Table 1 – statistical description of the inputs and the outputs for 149 countries

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CO2</td>
</tr>
<tr>
<td>Average</td>
<td>5.40</td>
</tr>
<tr>
<td>Min</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>79.30</td>
</tr>
<tr>
<td>Median</td>
<td>2.50</td>
</tr>
<tr>
<td>St.Dev</td>
<td>7.60</td>
</tr>
<tr>
<td>VC</td>
<td>1.61</td>
</tr>
<tr>
<td>1 Quart</td>
<td>0.80</td>
</tr>
<tr>
<td>3 Quart</td>
<td>7.60</td>
</tr>
<tr>
<td>LB</td>
<td>0</td>
</tr>
<tr>
<td>UB</td>
<td>17.80</td>
</tr>
</tbody>
</table>

LB: Lower bound = 1 Quartile – 1.5IQR
UB: Upper bound = 3 Quartile+1.5IQR
IQR = 3rd Quart - 1st Quart

From the data description we see that the variability coefficient (VC) is greater than 15% for all inputs and outputs which means that the data is not homogeneous and the variability among countries is high. The variability among countries is higher for inputs than for outputs. The highest variability concerns the input “CO2 emissions” and the output “GDP per capita”.

The data description shows that for all input components and for the GDP output component the maximal observed value (Max) is higher than the upper bound (UB). In fact, eight countries are emitting more than 17.8 tons of CO2/Capita, 25 countries are spending more than 1343 $PPP/Capita on health, 10 countries are spending more than 1800 $PPP/Capita on education, and finally only four countries have a GDP/Capita higher than 36030 $PPP. Also, several countries have a relatively low education index and low health index since eight countries have a health index lower than the lower bound (LB = 0.29) and four countries have an education index lower than 0.37.

In our analysis, we are considering public spending on health and public spending on education as inputs, which means that countries with high commitment to health and education (the appreciated costs) risk not being efficient. Further more, countries with very
low commitment to health and education could appear efficient given that these countries have “no negligible” outputs.

ANALYSIS OF EFFICIENCY RESULTS

As we mentioned in the first section, our main contribution resides in the way our results are interpreted. We are not adopting an envelopment form since we are not interested by generating benchmarks for non efficient countries. In fact, we believe that in our context to be or not to be efficient is not the question for many countries, but the real question is whether the causes of inefficiency are the appreciated or the non-appreciated costs. Our approach is based on the following: If average efficiency decreases due to the removal of an input component and furthermore the score of a given country decreases by a proportion greater than the average decrease, this means that the input under consideration is relatively low for most of the countries. On the contrary, if efficiency is unchanged, this means that the input under consideration is relatively high and occupying a considerable part among the inputs of the country under consideration.

Presentation of global scores

In our analysis, we consider a sample of 149 countries. First we determine the human development efficiency using the three input components (All). In the second step, we consider only PEH and PED as inputs (No CO2). We consider in a third step CO2 emission and PED (No PEH) and finally we consider CO2 emission and PEH (No PED). Table 3 compares the average efficiencies and the number of efficient countries with the different sets of inputs. It presents also the number of countries operating under decreasing return to scale (DRS) or operating under increasing return to scale (IRS).

The efficient countries under CRS model are Chad, Equatorial Guinea, Guinea, Madagascar, Myanmar, Nepal and Tanzania. With exception of Equatorial Guinea, they are all countries with low human development they have very low inputs and low outputs. The efficiency of Equatorial Guinea which has a medium human development is due to its output GDP/capita. The efficient countries under VRS model have different levels of human development. In fact Norway and Iceland which are always among the top five in terms of human development are among those countries.

Only 10 countries are operating under increasing return to scale when all input components are considered. All of them are African countries with low human development. They are Benin, Burkina-Fasso, Congo, Eritrea, Ethiopia, Guinea-Bissau, Mozambique, Niger, Sierra-Leon and Zamb. The number of countries operating under IRS conditions increases when CO2 emission is not considered as input.

Under both CRS and VRS, the average efficiencies as well as the efficiency of all countries decrease when CO2 emission is not considered as input. This could appear surprising since CO2 emission is adding too much to the cost of human development, but this could be explained by the fact that there is a big variability in CO2 emission among countries and further more, with an average of 5.4 and a Median of 2.5 (see table 1), most countries have relatively a low CO2 emission and they appear efficient in the first step thanks to that. Average VRS efficiencies as well as the number of efficient countries are less affected when the input components PEH and PED are removed. When the component Health spending is removed, average efficiency under CRS decreases with only 1%. Thus, average scale efficiency increases. Table 4 summarizes the average changes in efficiencies as well as the number of countries with no efficiency changes under different conditions.

Table 4 – Average changes in efficiencies

Table 3 – Comparison of average efficiencies for 149 countries

<table>
<thead>
<tr>
<th>CRS</th>
<th>All</th>
<th>No CO2</th>
<th>No PEH</th>
<th>No PED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nbr</td>
<td>7</td>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>VRS</td>
<td>Avg</td>
<td>82%</td>
<td>69%</td>
<td>76%</td>
</tr>
<tr>
<td>Nbr</td>
<td>56</td>
<td>28</td>
<td>48</td>
<td>49</td>
</tr>
<tr>
<td>Scale</td>
<td>Avg</td>
<td>54%</td>
<td>48%</td>
<td>57%</td>
</tr>
<tr>
<td>Nbr</td>
<td>7</td>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>DRS</td>
<td>132</td>
<td>132</td>
<td>135</td>
<td>133</td>
</tr>
<tr>
<td>IRS</td>
<td>10</td>
<td>15</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

Interpretation of global results

In our approach, we are more interested by identifying the countries that are giving priorities to what we called a non appreciated input (CO2 emissions) and the countries that are not giving enough interest to the appreciated inputs (PEH and PED).

From table 4, we see that globally, 13 countries are not relatively competitive because of their CO2 emissions (they are...
emitting too much CO2 compared to their appreciated spending). This number could be extended to 37 countries under VRS (see appendix for a list of countries). Under CRS, the average decrease in efficiencies when the component PED is removed is the same as the average decrease in efficiencies when the component CO2 is removed; which means that globally, the world is giving equal attention to CO2 emissions and education spending. But this is an average in a very diversified world and could not be taken as a general truth.

The positive point concerns spending on health (PEH). In fact, we can see in the column “No PEH” of table 4 that its removal does not have a considerable effect on efficiencies (an average decrease of 3% and 8% with respectively 110 countries and 77 countries not affected). This means that it is not the component PEH that was making countries efficient in the first place (When All input components were used).

To go forward in our interpretation, it is worth to mention that given that the major drawbacks of the DEA approach are “errors in data” we need to assume that the data collected by the UNDP and published in their report does not contain many errors and that the human development components adopted by the UNDP represent the true human development. On this basis, we suggest that:

- Countries operating under IRS should invest much more in education, health and emit more CO2 to insure some industrial development since they are very fare from the degree of harmful emissions. In fact, among these countries, only Congo seems to relatively emit CO2 rather than spending on education and/or on health.
- The thirteen or thirty-seven countries identified in table 3 should try to better equilibrate their spending. It is worth to mention that not all of them are countries with high human development. The thirteen or thirty-seven countries identified in table 3 should try to better equilibrate their spending. It is worth to mention that not all of them are countries with high human development.
- The world is globally spending on health. But disparities between nations are high. From our results, we could say that half of the nations are giving enough attention to their populations’ health. What about the other half? The above analysis is very “global”. In the next section we try to take a closer look to some selected countries.

**Comparative analysis between countries**

In this section we propose to conduct a comparative analysis of five groups of countries with similar levels of development and/or similar geopolitical positions.

In the following, each table compares two countries and is composed of three parts. The first part is devoted to the original data (inputs and outputs of the two countries). Part two presents the virtual inputs and outputs generated by the model. It is worth to mention that the virtual inputs and outputs represent the generated optimal solution and that the model could have multiple solutions. The last part presents the decrease in efficiencies after removing each of the input components.

**The first two countries** are Norway and Iceland. They both belong to the Scandinavian region and they have always been among the top in terms of human development. Both of them are not efficient under CRS but are efficient under VRS what ever the input components used. Globally, Norway has higher inputs than Iceland and a higher GDP per capita.

<table>
<thead>
<tr>
<th>Country</th>
<th>PEH</th>
<th>PED</th>
<th>LEI</th>
<th>EdI</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway</td>
<td>0</td>
<td>0.91</td>
<td>0.99</td>
<td>38454</td>
<td></td>
</tr>
<tr>
<td>Iceland</td>
<td>0</td>
<td>0.93</td>
<td>0.98</td>
<td>33051</td>
<td></td>
</tr>
</tbody>
</table>

**Decrease in scores of Norway and Iceland (%)**

<table>
<thead>
<tr>
<th>Component</th>
<th>CRS</th>
<th>VRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEH</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PED</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LEI</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>EdI</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Under CRS, the virtual outputs are made of only one component which is the education index and the virtual inputs are made of the components CO2 emission and education spending. Under VRS, their virtual outputs are more diversified while the virtual inputs are composed only of the CO2 emission component for Iceland.

The scores under CRS (which represent also the scale efficiencies) are not affected when the component PEH is removed but are affected when any of the other two components is removed. This means that for both countries their relative spending on health is higher then their relative spending on education or their level of CO2 emissions.

**The second group** is made of Chad and Guinea. Both of them are African countries with a low level of human development. Both are efficient under CRS and under VRS when all input components are used. Health
spending for Chad is higher then health spending of Guinea but the outputs of Guinea are higher then the outputs of Chad.

### Table 5 (b)

<table>
<thead>
<tr>
<th>CO2</th>
<th>PEH</th>
<th>PED</th>
<th>GDP</th>
<th>LEI</th>
<th>EdI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>31.35</td>
<td>43.89</td>
<td>2090</td>
<td>0.31</td>
<td>0.29</td>
</tr>
<tr>
<td>0.1</td>
<td>15.26</td>
<td>46.00</td>
<td>2180</td>
<td>0.48</td>
<td>0.34</td>
</tr>
</tbody>
</table>

**Virtual Inputs and outputs of Chad and Guinea**

<table>
<thead>
<tr>
<th>CO2</th>
<th>PEH</th>
<th>PED</th>
<th>GDP</th>
<th>LEI</th>
<th>EdI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.653</td>
<td>.347</td>
<td>0</td>
<td>.214</td>
<td>.786</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>.168</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.570</td>
<td>.430</td>
<td>0</td>
<td>.026</td>
<td>.329</td>
<td>0</td>
</tr>
</tbody>
</table>

**Decrease in scores of Chad and Guinea**

<table>
<thead>
<tr>
<th>Initial score</th>
<th>No CO2</th>
<th>No PEH</th>
<th>No PED</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>Chad</td>
<td>100</td>
<td>53</td>
</tr>
<tr>
<td>Guin</td>
<td>100</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>VRS</td>
<td>Chad</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Guine</td>
<td>100</td>
<td>17</td>
<td>0</td>
</tr>
</tbody>
</table>

Under CRS, the virtual inputs and outputs of Chad are made respectively of only one component, spending on education for the inputs and GDP for the outputs. The virtual inputs of Guinea are made of CO2 emissions and education spending and its virtual outputs are composed of GDP and LEI. Under VRS, the compositions of the inputs and outputs are unchanged for Guinea but the virtual inputs of Chad are made of the PEH component only. The scores under both VRS and CRS are not affected when the PED component is removed. But they are mostly affected when the component CO2 is removed. This means that both countries are relatively spending on education without excessive natural resources depreciations.

For Guinea, CRS efficiency is affected when the component PEH is removed, which means that this country is relatively not spending on health.

The third group is composed of the USA and the Russian federation. They belong to different continents but they both influence world policy. They both have a high level of human development (higher for the USA). Both are not efficient what ever the model and the components used. The inputs and outputs of the USA are higher than the inputs and outputs of Russia. Under CRS, their virtual inputs are made of the components CO2 emissions and education spending and their virtual outputs are made of GDP only. Under VRS, Russia inputs are much more diversified while for the USA the virtual inputs are made of Health spending only. Under VRS, USA outputs are more diversified while Russia virtual outputs are made of two components; GDP and education index.

Under CRS, their scores are mostly affected when the component PED is removed and they are not affected when the component PEH is removed which means that these countries are relatively spending on health more than education. On the contrary, under VRS the scores are mostly affected by the removal of the PEH input component.

The fourth group is made of two Gulf States which are Qatar and the UAE. Both joined the group of countries with a high level of human development lately. They are among the top in terms of CO2 emissions (1st and 3rd). Globally, their inputs and outputs are not very different.

### Table 5 (c)

<table>
<thead>
<tr>
<th>CO2</th>
<th>PEH</th>
<th>PED</th>
<th>GDP</th>
<th>LEI</th>
<th>EdI</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.6</td>
<td>2738</td>
<td>2341</td>
<td>39676</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>10.6</td>
<td>367</td>
<td>356</td>
<td>9902</td>
<td>0.67</td>
<td>0.95</td>
</tr>
</tbody>
</table>

**Virtual Inputs and outputs of USA and Russia**

<table>
<thead>
<tr>
<th>CO2</th>
<th>PEH</th>
<th>PED</th>
<th>GDP</th>
<th>LEI</th>
<th>EdI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1.524</td>
<td>1</td>
<td>3.180</td>
</tr>
<tr>
<td>.074</td>
<td>.140</td>
<td>0</td>
<td>1.155</td>
<td>0</td>
<td>8.105</td>
</tr>
</tbody>
</table>

**Decrease in scores of USA and Russia (%)**

<table>
<thead>
<tr>
<th>CRS</th>
<th>Initial Score</th>
<th>No CO2</th>
<th>No PEH</th>
<th>No PED</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>26</td>
<td>48</td>
<td>0</td>
<td>62</td>
</tr>
<tr>
<td>Russ</td>
<td>29</td>
<td>18</td>
<td>0</td>
<td>57</td>
</tr>
<tr>
<td>VRS</td>
<td>78</td>
<td>0.1</td>
<td>13</td>
<td>0.1</td>
</tr>
<tr>
<td>Russ</td>
<td>74</td>
<td>6.5</td>
<td>4.7</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Under CRS, their scores are mostly affected when the component PED is removed and they are not affected when the component PEH is removed which means that these countries are relatively spending on health more than education. On the contrary, under VRS the scores are mostly affected by the removal of the PEH input component.
input CO2 emissions. They are mostly affected by the non consideration of spending on education. What we could say is that both countries are relatively depreciating natural resources more than they are spending on education.

Finally, we compare two countries with a medium level of human development, Tunisia and Jordan. As is the case for USA and Russia, both are not efficient what ever the model and the inputs used. Tunisia is spending more on education and has a smaller education index while Jordan is emitting more CO2 and has a smaller GDP.

Table 5 (e)

<table>
<thead>
<tr>
<th>Inputs and outputs of Tunisia and Jordan</th>
<th>CO2</th>
<th>PEH</th>
<th>PED</th>
<th>GDP</th>
<th>LEI</th>
<th>EdI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>2.3</td>
<td>218</td>
<td>567</td>
<td>7768</td>
<td>0.81</td>
<td>0.75</td>
</tr>
<tr>
<td>Virtual Input</td>
<td>2.9</td>
<td>230</td>
<td>230</td>
<td>4688</td>
<td>0.78</td>
<td>0.86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Virtual Inputs and outputs of Tunisia and Jordan</th>
<th>CO2</th>
<th>PEH</th>
<th>PED</th>
<th>GDP</th>
<th>LEI</th>
<th>EdI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>1.64</td>
<td>0.051</td>
<td>0.784</td>
<td>0.253</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Virtual Input</td>
<td>3.42</td>
<td>0.658</td>
<td>0.282</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Decrease in scores of Tunisia and Jordan (%)</td>
<td>0.445</td>
<td>0.555</td>
<td>0</td>
<td>0.810</td>
<td>2.93</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.71</td>
<td>0.065</td>
<td>0.466</td>
<td>0.314</td>
<td>0</td>
<td>1.017</td>
</tr>
</tbody>
</table>

Under CRS, the virtual inputs for Tunisia are composed of the three input components while they are made of CO2 emission and education expenditure for Jordan. The virtual outputs of Tunisia are made of GDP and LEI, while the virtual outputs for Jordan are made of GDP and EdI.

The difference in efficiencies for the two countries is much higher under VRS than under CRS. The results are that scale efficiency is higher for Jordan. Tunisia is more affected by the removal of the CO2 component under VRS than under CRS. It is much more affected by the removal of the component PEH under VRS also. Jordan is not affected by the removal of the PEH component under both CRS and VRS.

CONCLUSION

We showed in this paper that CO2 emission, “today main question” of sustainable development is affecting human development efficiency more than other cost component, such as spending on health and spending on education. We identified the countries that even if they appear not to be efficient, should spend more on health and education and should not worry about CO2 emissions. On the other hand, about thirty countries should worry about their excessive emissions. In fact this study confirms the position of most African countries which were in conflict with several developed countries during the world environmental summit in Copenhagen in 2010. We believe thus this analysis could be extended in many ways (Model, Inputs and interpretation, etc.), and that it is not enough to draw conclusions.

References


Analysis of the changes in quality of life in Kazakhstan using DEA/Malmquist index

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ABSTRACT

This work analyzes the changes of quality of life (QOL) in Kazakhstan at the national and regional levels during the years 1998-2008 (sustained growth and emerging economy periods). The results are in agreement with other previous works on economic development and poverty, but in addition the use of DEA/Malmquist index methodology allowed for direct comparison across the regions and years by using a multi-dimensional synthetic index that included socio-economic indicators from different QOL domains. During the 11 years period, 3 regions (Atyrau oblast and Almaty and Astana cities) were permanently placed in the QOL frontier while the rest of regions showed different frontier shifts across the time. In general, Kazakhstan has experienced a steady and positive improvement in QOL, reaching its peak (80% better than the starting year) in 2008. Besides the identification of “best QOL” and “not-best QOL” regions, it was also found that gross regional product (GRP) and air pollution had strong influence in ranking the regions. Regional economic development policies are especially important for regions with poor Malmquist index (Akmola, Almaty, East-Kazakhstan, Kostanay, West-Kazakhstan and North Kazakhstan oblasts), whereas environmental policies (e.g. targeting reduction of air pollutants) could cause significant positive impact on enhancing QOL in oblast like Karaganda, Pavlodar and East-Kazakhstan.

Keywords: Kazakhstan, quality of life, DEA, Malmquist index

INTRODUCTION

The overall assessment of human experience has been expressed by the term quality of life (QOL) across multiple disciplines, including environmental science, economics, sociology, psychology, and medicine. Understanding and evaluating the quality of life is a complex task, as the concept of quality of life is multi-dimensional (involving environmental, social, and economic concerns) and multi-scale (involving both objective and subjective interrelating elements) (Costanza et al., 2007; Kazana and Kazaklis, 2008). Nowadays, improving QOL is a common aim of international development. However, identifying robust QOL indicators, or providing a coherent and robust definition of the concept, remains problematic (Bloom et al., 2001).

One of the major problems of constructing quality of life indicators is determining an appropriate aggregation method for incorporating multi-dimensional quality of life variables into an overall index (Somarriba, 2009). When multiple social indicators are employed, the indicator's weighted sum is generally taken as an integrated measure of all indicators. Unfortunately, it is difficult both to define such a weighting a priori, and to interpret it when derived through a particular multivariate technique. Thus a Data Envelopment Analysis (DEA) to evaluation of quality of life can be applied, as a well-known and respected methodology that is able to avoid dealing with fixed weights (Hashimoto 2009).

In regard to the applications of DEA framework in QOL research, important contributions to obtain synthetic indicators of well-being and quality of life have been made by Hashimoto and Ishikawa (1993), Hashimoto and Kodama (1997), Zhu (2001), Murias et al (2006), Hashimoto et al (2009), Somarriba (2009), among others.

The objective of the present research is to investigate the dynamics of changes in QOL in Kazakhstan and its regions during the transition to a market oriented economy by an objective assessment of QOL using the DEA/Malmquist index methodology.

DATA

In this study, five data panels were created, where each one represented the 16 administrative regions of Kazakhstan in 11 years (1998-2008). Each of the 16
administrative regions (14 oblasts and two main cities) was considered as a separate decision-making unit (DMU) in a DEA CCR-input oriented model, employing three inputs and two outputs. The used outputs: Gross Regional Product (GRP per capita by oblast, reported in thousands of Kazakhstani tenge (KZT)) and number of doctors (per 10,000 population); along with the inputs: unemployment rate (%), crime level (per 10,000 population) and emission of the most widespread pollutants produced by stationary sources of air pollution per capita (thousand tons) (Table 1), pretend to cover the main domains in QOL (e.g. economy, health, safety and environment). All the data was collected from the Statistical Agency of Kazakhstan (http://www.stat.kz). It is important to note that inputs represent negative social indicators, where the smaller the value, the better; and outputs represent positive social indicators, where the greater value, the better. In the current study, due to the limitations of the model for the number of observations, it was possible to use only five social indicators, which cover main domains of the quality of life. The panel data was normalized to mean of 5 and variance of 100 in order to perform subsequent DEA computations.

**METHODOLOGY**

In this study of quality of life in Kazakhstan we followed the DEA/Malmquist index approach. This index has experienced several developments during the last decade (Hashimoto and Ishikawa1993; Hashimoto and Kodama 1997; Hashimoto et al., 2009). Mathematically, the CCR model, in its weak efficiency and ratio form, generates an efficiency score for decision making unit(s) (DMU) of interest $j_0$, $g_{j0}$ ($0 \leq g_{j0} \leq 1$). It is formulated as the following fractional program:

$$\text{Maximize } \frac{g_{j0}}{y_{j0}} = \frac{\sum_{j=1}^{n} y_{j0}}{\sum_{j=1}^{n} y_{j0}}$$

subject to

$$u_i, v_i \leq 1, \quad j = 1, \ldots, n,$$

where: $n =$ number of DMUs, $y_j = [y_{j1}, \ldots, y_{jm}] =$ output vector for DMU $j$, $y_{j0} =$ output $r$ from DMU $j$, $u = [u_1, \ldots, u_r] =$ vector of output weights, $u_i =$ weight assigned to output $r$, $t =$ number of outputs, $x_j = [x_{j1}, \ldots, x_{jm}] =$ input vector for DMU $j$, $x_{j0} =$ input $t$ to DMU $j$, $v = [v_1, \ldots, v_m]$ = vector of input weights, $v_i =$ weight assigned to input $i$, and $m =$ number of inputs.

Eq. (1) can be converted to the following linear program (LP), which is essentially the CCR model in its weak efficiency, input-oriented and envelopment form:

$$\text{Minimize } g_{j0} = \Theta$$

Subject to

$$\sum_{j=1}^{n} \lambda_j v_j = y_{j0},$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} - \Theta x_j \leq 0, \quad j = 1, \ldots, n,$$

($\Theta$ Unconstrained),

(2)

where: $\lambda_j = 1, \ldots, n =$ model's decision variables. We can generate DEA scores $g_{j0}$ ($= \Theta$) for all DMUs by solving Eq. (2) $n$ times, setting each DMU as the target DMU $j_0$ in turn. Here, DMUs $j_0$ with the optimum $g_{j0} = 1$ are judged DEA efficient, while those with $g_{j0} < 1$ are defined as DEA inefficient. In other words, DMUs $j_0$ with $g_{j0} = 1$ might thus be judged to have a “best QOL,” while those with $g_{j0} < 1$ will simply possess a “not-best QOL” (Hashimoto et al., 2009).

**DEA/MI analysis**

DEA/MI analysis measures the Malmquist (productivity) index within a DEA framework. In a single input and output DEA case (Fig.1),

![Figure 1: DEA efficiency changes with frontier and rear shifting over time.](image)

where DMU $j_0$ is at point $A$ in period $\alpha$, and line OCD represents the CCR DEA frontier, the input-oriented efficiency of DMU $j_0$ is then measured by $PC/PA$ ($<1$, DEA inefficient). When point $A$ is on the frontier, its score is 1 (DEA efficient). Suppose that, in period $\beta$ ($\beta > \alpha$), DMU $j_0$ has moved to point $B$ and the frontier itself has also shifted to line OEF. The efficiency change in DMU $j_0$ can be measured by the ratio of its DEA score in period $\beta$ to that in period $\alpha$; however, the frontier has shifted, so that we must compute the geometric mean of ratios for the two frontiers in those same periods. This is the DEA (CCR input-oriented)/Malmquist index for DMU $j_0$ between periods $\alpha$ and $\beta$, given in Eq. (3):
\[
MI_{j_{0}}[\alpha, \beta] = \left( \frac{Q_{j0}^{+}Q_{PA}^{+}Q_{PE}^{+}}{PC_{j0}^{+}} \right)^{\frac{1}{2}} \quad (3)
\]

Here, MI \(> 1\) implies a gain in DEA efficiency by DMU \(j_{0}\) from period \(\alpha\) to \(\beta\), while \(MI = 1\) and \(MI < 1\) imply the status quo and loss, respectively.

Transforming Eq (3), the Malmquist index can be decomposed into two components as follows:

\[
MI_{j_{0}}[\alpha, \beta] = \left( \frac{PF_{j0}}{PC_{j0}} \right)^{\alpha} \times \left( \frac{Q_{j0}^{+}Q_{PA}^{+}Q_{PE}^{+}}{PC_{j0}^{+}} \right)^{\beta} \quad (4)
\]

\[
= CU_{j_{0}}[1, 1] \times FS_{j_{0}}[1, 1] \quad (5)
\]

\[
= \frac{Q_{j0}^{+}Q_{PA}^{+}Q_{PE}^{+}}{PC_{j0}^{+}} \times \left( \frac{Q_{j0}^{+}Q_{PA}^{+}Q_{PE}^{+}}{PC_{j0}^{+}} \right)^{\beta} \quad (6)
\]

where the first term on the right hand side (RHS) of Eq. (4), expresses the Catch-Up (CU, also known as the technical efficiency change) index. When \(CU > 1\) suggests that DMU \(j_{0}\) has moved closer to the period \(\beta\) frontier than to that for period \(\alpha\). \(CU = 1\) and \(CU < 1\) then apply when the same distance, or more, have been covered, respectively. The second term on the RHS of Eq. (4) is defined as the Frontier Shift (FS, also known as the technological change) index. When FS \(> 1\) this means a gain in the DEA frontier shift from period \(\alpha\) to \(\beta\) as measured from DMU \(j_{0}\). That is to say, the frontier has moved outward, generating more output but with less input (Fig.1). As in previous cases, FS \(= 1\) and FS \(< 1\) imply no change and loss (shift backward), respectively.

Since \(PC_{E}PIA\) in Fig.1 is, for example, the DEA score \(\theta = g(j_{0})\) of the period \(\alpha\) DMU \(j_{0}\) measured by means of the period \(\beta\) frontier, we denote it as \(\theta[D^{\beta}, F^{\beta}]\). Then, from Eq. (6), we get:

\[
MI_{j_{0}}[\alpha, \beta] = \left( \frac{Q_{j0}^{+}Q_{PA}^{+}Q_{PE}^{+}}{PC_{j0}^{+}} \right)^{\beta} \quad (7)
\]

In Eq. (2), letting \(x_{j}^{\alpha}, y_{j}^{\alpha} = x_{j}, y_{j}\), respectively, in period \(\alpha\), \(\theta[D^{\beta}, F^{\beta}]\) can be obtained as the optimum of the following LP, which is the classical DEA model:

Minimize \(\theta\)

subject to \(\sum_{j=1}^{n} \hat{\lambda}_{j} y_{j}^{\alpha} \geq y_{j_{0}}^{\alpha}\),

\(\sum_{j=1}^{n} \hat{\lambda}_{j} x_{j}^{\alpha} - \theta x_{j_{0}}^{\alpha} \leq 0\),

\(\hat{\lambda}_{j} \geq 0, \quad j = 1, ..., n\),

(\(\theta\) Unconstrained), \(\theta[D^{\beta}, F^{\beta}]\) can also be obtained using the LP in Eq. (8) by replacing \(\alpha\) with \(\beta\).

While \(\theta[D^{\beta}, F^{\beta}]\) is obtained as the optimum of Minimize \(\theta\)

\[
subject to \sum_{j=1}^{n} \hat{\lambda}_{j} y_{j}^{\beta} \geq y_{j_{0}}^{\beta},
\sum_{j=1}^{n} \hat{\lambda}_{j} x_{j}^{\beta} - \theta x_{j_{0}}^{\beta} \leq 0,
\hat{\lambda}_{j} \geq 0, \quad j = 1, ..., n,
(\(\theta\) Unconstrained),
\]

this forms the DEA exclusion model. Finally, it is possible to obtain \(\theta[D^{\beta}, F^{\alpha}]\) by again using the DEA exclusion model of Eq. (9) with \(\alpha\) and \(\beta\) switched.

It is important to highlight that the catch-up index compares closeness of region \(j_{0}\) to the QOL frontier in each period. The frontier shift index expresses movement of the QOL frontier between two periods, while the Malmquist index measures the change in QOL for region \(j_{0}\) taking both the frontier shift and catch-up into consideration. Note that while \(CU_{j_{0}}\) and \(MI_{j_{0}}\) represent the movement of region \(j_{0}\) FS_{j_{0}} represents the shift of the QOL frontier, which is composed of those regions having a DEA best QOL. This led to Hashimoto et al. (2009) to propose the average frontier shift index of all regions, i.e., the QOL frontier shift measured from the average oblast, as an appropriate indicator of QOL change at the national level of Kazakhstan.

Normally, MI, CU and FS indices, for year \(\beta\), are compared to those in the preceding year, i.e., \(\alpha = \beta - 1\). However, such annually successive indices do not seem appropriate when looking across the full multi-year sample period. Then, for a proper computation of the cumulative change the cumulative indices (in our case \(CU_{j_{0}}[1998, \beta], FS_{j_{0}}[1998, \beta]\) and \(MI_{j_{0}}[1998, \beta], \beta = 1998, ..., 2008\), were calculated according to Hashimoto et al. (2009). Here, the cumulative index values when \(\beta = 1998\) are all 1.

**DEA ANALYSIS OF QUALITY OF LIFE CHANGES**

Large differences within the 5 indicators used in our QOL study were found across the oblasts and years (Table1). This was quite evident especially in the cases of air pollution and GRP (86 and 11-fold between the 11 years oblasts average minimum and maximum values, respectively), suggesting sharp differences in the well-being of the population at the different regions. In the current study, we used panel social-indicators data from 14 oblasts and two main cities (Astana and Almaty) of Kazakhstan and obtained the catch-up \(CU_{j_{0}}[\alpha, \beta]\), frontier shift \(FS_{j_{0}}[\alpha, \beta]\), and Malmquist \(MI_{j_{0}}[\alpha, \beta]\) indices from period \(\alpha\) to \(\beta\) (1998 to 2008).

The cross-section DEA analysis by year (Table 2) shows that Atyrau oblast along with Almaty...
and Astana cities were in the frontier for all the 11 years period (or most of the years in the case of Astana). This is, these regions located on the “efficiency frontier” by producing more outputs with fewer inputs. In terms of quality of life, DMU’s with score of 1 are judged to have a “best QOL”, while those with scores < 1 will simple posses a “not-best QOL”. Other 4 oblast also reached the frontier but only in two or three years (East-Kazakhstan, West-Kazakhstan, Mangistau and South-Kazakhstan oblast). Poor DEA scores were constant in Akmola, Almaty, Kostanay and North-Kazakhstan oblasts. The cumulative MI, CU and FS gave origin to a set of QOL shift graphs that can be visualized in Fig. 2. In the case of Almaty city and Atyrau oblast, the cumulative catch-up indices were all 1. For Almaty city, the MI reached its peak (40% better than the start year 1998) in 2002. Atyrau had more ups and downs with a peak of 90% better than the first year in 2008, but with 4% worse than the first year in 2004. Astana has shown a steady improvement in QOL during the first years of the study (1998-2001), reaching its peak (100% better than the first year) in 2001 and being in the frontier since 2000. Some oblasts like Aktube, Mangistau and Karaganda have experienced onward FS during the time with CU indices ≥ 1 that resulted in placing them near the QOL frontier. By contrast, oblasts like Akmola, Almaty, East-Kazakhstan, Kostanay, West-Kazakhstan and North Kazakhstan, in spite their own improvements in QOL compared to the first year (between 30 and 400% better), their CU indices were frequently ≤ 1, and consequently they were placed far from the QOL frontier. The case of South Kazakhstan oblast is of interest because it firstly was placed in the frontier or near the frontier until 2005, then its CU index moved off the QOL frontier showing a progressive decline in QOL during 2006-2008. Looking at the social indicators data from the two groups “best QOL” (Atyrau oblast and Almaty and Astana cities) vs. “not-best QOL” (Akmola, Almaty, East-Kazakhstan, Kostanay, West-Kazakhstan and North Kazakhstan oblasts) (data not shown), we found that the “best QOL” group have advantages over the “not-best QOL” group in GRP, number of doctors and air pollution (less emission of pollutants except for Atyrau). Conversely, the number of crimes and unemployment rate were comparable or slightly higher in the “best QOL” group than in the “not-best QOL” group. This suggests important influence of GRP and air pollution in ranking the regions under the existence of a similar unemployment rate and number of crimes scenarios. The relationship of GRP with higher QOL found in the present study is in agreement with Agrawal (2008), who concluded that there are evidences about relationship between economic growth and poverty reduction where higher growth rates of GDP per capita are strongly associated with larger declines in poverty across the oblasts. The computation of the geometric average cumulative FS indices (this is because the FS index’s multiplicative nature) of all oblasts $FS_d(1998, β), β = 1998, …, 2008$, described the movement made Kazakhstan’s QOL frontier from the point of view of average oblasts and main cities (Astana and Almaty) performance. Hence, it is possible to note from the last graph in Fig. 2 that Kazakhstan’s QOL frontier has moved onwards (e.g. Kazakhstan’s QOL improved through the study period). Kazakhstan’s QOL reached a peak in 2008 of 80% better than the start year 1998 (with an annual change rate of 5.3%). Kazakhstan had a fast improvement of QOL during the years 1998-2000 which corresponds to the end of the period of negative economic growth (1990-1997) and the start of the phase of strong and sustained growth (Agrawal, 2008). This phase of strong and sustained growth is characterized by a stable improvement of QOL from 2001 to 2006 (with a small decline in 2004). The final onward move of QOL observed from 2006 to 2008 supports the conclusions of many observers that Kazakhstan has essentially completed its transitional phase, suggesting that since 2006 Kazakhstan can be considered an emerging economy with a market system in many sectors including banking and finance (Agrawal, 2008; Kalyuzhnova and Kambhampati, 2008) which is reflected in an overall improvement of well-being of the population.

**IMPLICATIONS FOR THE FUTURE**

The present work analyzed the changes of QOL in Kazakhstan during the years 1998-2008, both at the national and regional levels. Our work contemplated two important economic periods known as: sustained growth and emerging economy. The results are in close agreement with other previous works on economic development and poverty, but in addition the use of DEA/Malmquist index methodology allowed for direct comparison across the regions and years by using a multi-dimensional synthetic index that included socio-economic indicators from different QOL domains. Besides the identification of “best QOL” and “not-best QOL” regions, the analysis also permits the description of which indicators contribute more to QOL. In our
case, we found that GRP and air pollution have heavy weights in ranking the regions. Although regional economic development policies should continue as the main engine for improving overall well-being, especially in oblast with poor MI scores (Akmola, Almaty, East-Kazakhstan, Kostanay, West-Kazakhstan and North Kazakhstan oblasts), policies targeting important improvements in environmental issues (e.g. significant reduction of air pollution) could cause significant positive impact on enhancing QOL in oblast like Karaganda, Pavlodar and East-Kazakhstan.

REFERENCES

ACKNOWLEDGEMENTS
The authors want to thank Dr. Canan Yildirim at Kadir Has University (Istanbul, Turkey) for the introduction of the DEA methodology.
### APPENDICES

#### Table 1: Summary statistics of each indicator for Kazakhstan from the period 1998-2008

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Average</th>
<th>Minimum (Year)</th>
<th>Maximum (Year)</th>
<th>Std Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRP (thousands tenge)</td>
<td>486.4</td>
<td>42.3 (1998)</td>
<td>3626 (2008)</td>
<td>572.744</td>
</tr>
<tr>
<td>Number of doctors (per 10,000 population)</td>
<td>36.7</td>
<td>18.5 (1998)</td>
<td>87.4 (2003)</td>
<td>14.827</td>
</tr>
<tr>
<td>Crime level (cases per 10,000 population)</td>
<td>88.8</td>
<td>39.7 (2007)</td>
<td>171.8 (2001)</td>
<td>29.170</td>
</tr>
</tbody>
</table>

#### Table 2: Cross-section DEA by year

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Akmola Oblast</td>
<td>0.363</td>
<td>0.540</td>
<td>0.488</td>
<td>0.469</td>
<td>0.387</td>
<td>0.471</td>
<td>0.456</td>
<td>0.401</td>
<td>0.411</td>
<td>0.326</td>
<td>0.286</td>
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<tr>
<td>Aktobe Oblast</td>
<td>0.744</td>
<td>0.744</td>
<td>0.926</td>
<td>0.743</td>
<td>0.606</td>
<td>0.624</td>
<td>0.632</td>
<td>0.632</td>
<td>0.624</td>
<td>0.680</td>
<td>0.735</td>
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<td>Almaty Oblast</td>
<td>0.591</td>
<td>0.519</td>
<td>0.484</td>
<td>0.446</td>
<td>0.381</td>
<td>0.499</td>
<td>0.439</td>
<td>0.393</td>
<td>0.455</td>
<td>0.439</td>
<td>0.263</td>
</tr>
<tr>
<td>Anyrau Oblast</td>
<td>1</td>
<td>0.971</td>
<td>0.991</td>
<td>1</td>
<td>0.767</td>
<td>0.654</td>
<td>0.745</td>
<td>0.730</td>
<td>0.803</td>
<td>0.833</td>
<td>0.595</td>
</tr>
<tr>
<td>East-Kazakhstan Oblast</td>
<td>0.668</td>
<td>0.605</td>
<td>0.745</td>
<td>0.734</td>
<td>0.585</td>
<td>0.594</td>
<td>0.673</td>
<td>0.632</td>
<td>0.618</td>
<td>0.632</td>
<td>0.416</td>
</tr>
<tr>
<td>Zhambyl Oblast</td>
<td>0.797</td>
<td>0.774</td>
<td>0.884</td>
<td>0.856</td>
<td>0.687</td>
<td>0.872</td>
<td>0.878</td>
<td>0.821</td>
<td>0.701</td>
<td>0.630</td>
<td>0.646</td>
</tr>
<tr>
<td>Karaganda Oblast</td>
<td>0.760</td>
<td>0.665</td>
<td>0.618</td>
<td>0.751</td>
<td>0.653</td>
<td>0.725</td>
<td>0.829</td>
<td>0.919</td>
<td>0.967</td>
<td>0.923</td>
<td>0.796</td>
</tr>
<tr>
<td>Kostanay Oblast</td>
<td>0.489</td>
<td>0.438</td>
<td>0.390</td>
<td>0.387</td>
<td>0.325</td>
<td>0.310</td>
<td>0.322</td>
<td>0.300</td>
<td>0.328</td>
<td>0.323</td>
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<tr>
<td>Kyzylorda Oblast</td>
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<td>0.774</td>
<td>0.884</td>
<td>0.856</td>
<td>0.687</td>
<td>0.872</td>
<td>0.878</td>
<td>0.821</td>
<td>0.701</td>
<td>0.630</td>
<td>0.646</td>
</tr>
<tr>
<td>Mangistau Oblast</td>
<td>0.990</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.864</td>
<td>0.743</td>
<td>0.827</td>
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<td>0.794</td>
<td>0.931</td>
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<tr>
<td>Pavlodar Oblast</td>
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<td>0.498</td>
<td>0.509</td>
<td>0.739</td>
<td>0.568</td>
<td>0.570</td>
<td>0.665</td>
<td>0.609</td>
<td>0.623</td>
<td>0.649</td>
<td>0.606</td>
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<tr>
<td>North-Kazakhstan Oblast</td>
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<td>0.397</td>
<td>0.418</td>
<td>0.373</td>
<td>0.389</td>
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<td>0.388</td>
<td>0.383</td>
<td>0.363</td>
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<tr>
<td>South-Kazakhstan Oblast</td>
<td>0.885</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Astana city</td>
<td>0.772</td>
<td>0.955</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Almaty city</td>
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<td>1</td>
<td>1</td>
<td>1</td>
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</table>
Figure 2: Cumulative catch-up (CU), frontier shift (FS), and Malmquist (MI) indices for 14 oblasts and two main cities; and country average cumulative FS
TFP Change in Tunisian public employment offices

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ABSTRACT
This study uses data envelopment analysis (DEA) and a distance function approach to derive Malmquist Productivity for 10 public Graduate Employment Offices (GEO) and 70 Multiservice Employment Offices (MEO) during 2006-2009. The results of this decomposition have been discussed and interpreted. Further the bootstrap estimate is used to perform statistical inference of the results.

Keywords: Tunisian public employment offices, Total Factor Productivity

INTRODUCTION
The public services represent a significant segment. Thus, any variation in the productivity of this sector leads to a significant variation in productivity across the economy. In other words, the performance of public services influences the rest of the economy. This is particularly evident for education, research, public health, infrastructure, transport, employment etc. However, a high level of productivity is regarded as a good sign of management resource. The public services have a number of special features which are an undeniable interest to address analytically and simplified a number of theoretical and operational issues raised by these services. The core of these issues is related to management of human resources, integration of new technologies, quality, productivity and performance. In this context, the efficiency and productivity of public employment service has drawn increased attention in labor market policy debates in recent years. Measuring and analyzing efficiency and productivity change over time of public employment services can gives authorities’ guidelines to improve results of employment policy. The improvement of public employment services productivity can increase their efficiency level. Consequently, this is a means of lowering the high level of structural or non-cyclical unemployment. The most popular method of using DEA efficiency scores for doing comparisons over time is the Malmquist index.

METHODOLOGY
In the economic theory, productivity can be considered as an alternative measure of firm’s performance. The fundamental definition of productivity is the relation between inputs and the resulting outputs. In the case of multiple outputs and multi inputs production technology Total Factor Productivity (TFP) is used as measure for productivity. This latter, is defined as the ratio of aggregated output to aggregated input at a given point of time. Several measures of productivity have been advanced in the economic literature (Coelli et al., 2005). The main approaches are the index numbers such as the Tornqvist index, the Fisher index, the Luenberger index and the Malmquist TFP index, depending on the aggregation of inputs and outputs (Boussemart and al. 2003). But, the Malmquist index is the most commonly used one, which measures the TFP change between two periods by calculating the ratio of the distances of each data point relative to a common technology. Hence, a priori definition of a distance function is primordial. In this respect, the variability of outputs occurs because of several reasons, i.e., efficiency change (shifting toward the frontier), the scale effects (more factors being accumulated) and/or technological change (shifting of the frontier). Then, we can define the productivity change as the change of TFP over time.

Malmquist TFP Index
As we have noted bellow, the Malmquist TFP index can be used to calculate productivity change between two particular periods (in periods t and s) by calculating the ratio of two distances function concerning each data point relative to a common technology. Analytically, the output oriented Malmquist index is defined as:

\[
\overline{TFP} = m_{0}(y_{t}, x_{t}, y_{s}, x_{s})
\]

Three components can be derived from the Malmquist index by using output distance function; efficiency change, technical change...
and scale change (Coelli et al. 2005). We compute the technical efficiency change between the period’s $t$ and $s$ as follows:

$$TE_{t \rightarrow s} = \frac{D_{0}(y_{t}, x_{t})}{D_{0}(y_{s}, x_{s})}$$

Then, the technological change is defined as:

$$TA = \left[ \frac{D_{0}(y_{t}, x_{t})}{D_{0}(y_{s}, x_{s})} \right]^{1/2}$$

The Malmquist TFP index is thus the product of the two components, i.e.

$$TFP = TE_{t \rightarrow s} \times TA$$

Further if we used variable returns to scale, the Scale efficiency change (SEC) can also be calculated as follows:

$$TFP = TE_{t \rightarrow s} \times TA \times SEC$$

INPUTS AND OUTPUTS

Regarding various studies (Sheldon (2003), Althin and Behrenz (2005) and Vassiliev and al, (2006)), the public employment services output is measured by total unemployed persons placed. Public employment services are supposed to assist in the matching of unemployed to employers and thereby reduce the unemployment rate. In other words, the employment service operates primarily as a placement driven labor exchange. Placements services produced by employment offices are linked to stock of unemployed and job vacancies. The link is provided by the matching function, popularized through the work of Blanchard and Diamond (1989) and Pissarides (1990). The economic activities of employment is represented, as a production function in which the stocks of unemployed and job vacancies serve as inputs to generate flows of hires. In addition, the link is presented by the work of Oi (1992); the output of public services can be an “interactive output” in which recipients of services can participate in producing this output. The intervention of the client makes the service always different, adapted to specific needs and can affect performance. For example, job seekers who express their wishes regarding the characteristics of job (area, region, qualifications required, ...) contributing to the improving the performance of this service. In addition, the individual characteristics of job seeker may be able to increase the performance of services which it employs. For example, a job seeker experienced, skilled and qualified can positively influence the performance of public service employment. Firms that are also a social partner of employment offices may also participate in the production of the placement service. Indeed, firms with specification and definition of their needs of employees facilitate the task of employment offices and increase the number of hires. Indeed, the number of unemployed persons and the number of job vacancies which are two inputs of public employment office model depends on local economic conditions and them not under the office’s control. Moreover, the public services provided by employment offices are produced by labor because it is the dominant cost component in placement service production. The labor factor corresponds to efforts of counselors in matching job seekers to job vacancies. In line with the focus of the studies of Sheldon (2003) and Vassiliev and al, (2006) and in this modeling framework, efficiency coincides with achieving a maximum number of hires with a given the number of counselors, stocks of unemployed and job vacancies.

DATA AND DESCRIPTIVE STATISTIC

Our data pertain to 80 Tunisian employment offices (10 Graduate Employment Offices (GEO) and 70 Multiservices Employment Offices (MEO)) that operated during the period January 2006 to December 2008 (i.e., 36 months). The data were provided by the National Agency for Employment and Independent Work (NAEIW). They consist of monthly number of hires, number of unemployed persons, number of job vacancies and number of counsellors. Table 1 shows descriptive statistics of inputs and output of GEO.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hires</td>
<td>83</td>
<td>95</td>
<td>2</td>
<td>706</td>
</tr>
<tr>
<td>Number of unemployed persons</td>
<td>2158</td>
<td>1701</td>
<td>724</td>
<td>9206</td>
</tr>
<tr>
<td>Number of job vacancies</td>
<td>84</td>
<td>117</td>
<td>1</td>
<td>659</td>
</tr>
<tr>
<td>Number of counsellors</td>
<td>19</td>
<td>1</td>
<td>18</td>
<td>23</td>
</tr>
</tbody>
</table>

Source: Statistic of NAEIW

Also, in the labour market covered by the GEO, average the number of hires per offices is equal to 83 and the average the number of job vacancies per offices is 84. We also note that the mean of number of unemployed is about 2158. Regarding the share of labour market covered by the MEO, statistics presented in Table shows that the mean of number of hires is equal to 143 per offices and the mean of number of job vacancies received is about 160 jobs.
However the analysis by the offices reveals the existence of major differences in terms of placement. There are some employment offices having between 200 and 370 numbers of hires and other employment offices made between 10 and 40 numbers of hires. Indeed, Table 2 gives the mean number of counsellors among the 70 MEO offices is 6 counsellors per office. Besides, the 6 counsellors per MEO office manage 1115 unemployed persons. This is equal to one counsellor per 185 unemployed persons.

Table 2: Descriptive statistic of MEO

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hires</td>
<td>143</td>
<td>118</td>
<td>2</td>
<td>1585</td>
</tr>
<tr>
<td>Number of unemployed persons</td>
<td>1115</td>
<td>579</td>
<td>219</td>
<td>4101</td>
</tr>
<tr>
<td>Number of job vacancies</td>
<td>160</td>
<td>138</td>
<td>2</td>
<td>1787</td>
</tr>
<tr>
<td>Number of counsellors</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>

Source: Statistic of NAEIW

The favorable productivity level of GEO was a result of a positive technological change (+1%) accompanied by an improvement in scale efficiency (+0.1%). This could be due to the integration of new communication technology in the activities of GEO. Since 2003, computers are increasingly used to support placement services of GEO, as much as other administrative purposes, including office procedures and staff. The potential to share and transfer data in great volumes are providing technical infrastructure for the integration of employment service functions and to develop linkages between programs. This evolution will give more quality to the GEO services. The use of information technology in placement services makes information and services more accessible. In addition, the use of new technology will bring more facility for labor mobility, which is very weak in Tunisia. The use of advanced technology increases the penetration rate and performance of the employment services. However, the level of efficiency change of GEO is less than one (-0.7%) because the level of pure efficiency change deteriorated by 0.8%. This means that the majorities of GEO are technically inefficient and they must increase their output to improve efficiency level and consequently improve productivity level. Furthermore, over the period of analysis, the technical change level increased. As shown in Table 3, the year of 2007 is the year that TFP and its components were the highest caused by an increase in the number of hires per GEO compared to 2006 and 2008.

As opposed to GEO, the TFP change of MEO in Table 4 shows an average deterioration per office with about 1.12%. This unfavorable productivity level was a result of a negative efficiency change and its components; pure technical efficiency and scale efficiency respectively by about 2%, 1.4% and 0.7%. This implies that the majorities of MEO were technically inefficient and they were not operating at the right scale. In addition, like the GEO the MEO increased the technical change between 2006-2008 by 0.8%. The year 2006 was characterised by an improvement in technological (2.9%) and in efficiency (0.9%).

Table 3: Malmquist index summary: GEO

<table>
<thead>
<tr>
<th></th>
<th>effch</th>
<th>techch</th>
<th>pech</th>
<th>sech</th>
<th>TFP change</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>0.959</td>
<td>0.939</td>
<td>0.962</td>
<td>0.907</td>
<td>0.956</td>
</tr>
<tr>
<td>2007</td>
<td>1.053</td>
<td>1.007</td>
<td>1.015</td>
<td>1.037</td>
<td>1.133</td>
</tr>
<tr>
<td>2008</td>
<td>0.997</td>
<td>1.008</td>
<td>1.001</td>
<td>0.996</td>
<td>1.025</td>
</tr>
<tr>
<td>2006-2008</td>
<td>0.993</td>
<td>1.010</td>
<td>0.992</td>
<td>1.001</td>
<td>1.004</td>
</tr>
</tbody>
</table>

Table 4: Malmquist index summary: MEO

<table>
<thead>
<tr>
<th></th>
<th>effch</th>
<th>techch</th>
<th>pech</th>
<th>sech</th>
<th>TFP change</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>1.009</td>
<td>1.029</td>
<td>1.006</td>
<td>1.003</td>
<td>1.038</td>
</tr>
<tr>
<td>2007</td>
<td>0.996</td>
<td>0.995</td>
<td>0.997</td>
<td>0.999</td>
<td>0.990</td>
</tr>
<tr>
<td>2008</td>
<td>0.967</td>
<td>0.982</td>
<td>0.971</td>
<td>0.996</td>
<td>0.942</td>
</tr>
<tr>
<td>2006-2008</td>
<td>0.980</td>
<td>1.008</td>
<td>0.986</td>
<td>0.993</td>
<td>0.988</td>
</tr>
</tbody>
</table>

RESULTS AND INTERPRETATION

In this study, we employed the DEAP 2.1 program described in Coelli (1996), to compute the distance functions using non-parametric Data Envelopment Analysis. Then we calculated Malmquist productivity index (TFP change) for each GEO and MEO, technical change (techch) and efficiency change (effch) which it is further decomposed to pure efficiency change (peffch) and scale change (seffch). If the value of the Malmquist index or any of its components is less than one, then there is deterioration. On the other hand, if the value is bigger than one, then there is improvement in the relevant performance. In fact, the estimated Malmquist indices indicate that between 2006 and 2008 TFP in GEO increased by 0.4% and TFP in MEO decreased by 1.2% (Table 3 and 4).
As presented in Table 4, between 2006, 2007 and 2008 TFP change decreased because of the negative effects of efficiency change and technical change. In fact, there are only 5 GEO (50% of GEO) and 25 MEO (36% of MEO) that had a TFP change higher than 1. The majority of these offices are located in regions having more opportunity of employment. As a result, they increased their efficiency change compared to the other offices. Accordingly, high efficiency level is the main factor that can explain differences in TFP change between GEO and MEO.

Furthermore, the monthly change in TFP of GEO increased slightly (0.4% on average) over the entire period. Technical change plays an important role in productivity growth. There are fluctuations regarding TFP change (efficiency and technical change) over the period of analysis. This may be explained by business cycle effects experienced by the Tunisian economy between 2006 and 2008. The highest increase in TFP occurred in 2007, as there was a decrease in productivity in 2008. As can be seen from the highest increase in TFP is in March 2007, Mai 2007 and September 2007 respectively by about 75.6%, 49.4% and 58%.

As opposed to GEO, The TFP in MEO decreased slightly (1.2% on average) over the entire period. TFP levels in MEO varied between 0.466 and 2.137. The highest increase in TFP occurred in 2006, as there was a decrease in productivity in 2007 and 2008. The highest increase in TFP was in May 2006, July 2006 and December 2006 respectively by about 48.6%, 44.9% and 51.9%. In addition, it seems that the highest increase in TFP over the period of analysis was in October 2007 by about 2.137.

The low level of TFP change for 5 GEO and 45 MEO is due to the low level of efficiency change. In fact, Tunisian public employment offices are operating in a wrong size, low number of human resources and budget. The most GEO and MEO offices offer only registration services and perform poorly in their traditional placement functions.

DETERMINANTS OF TFP

Bootstrapping Malmquist productivity index

The second step of our study attempts to explain the variations in TFP indices across Tunisian employment offices. We try to identify determinants of total factor productivity of GEO. The usual approach to analyzing effects of the operating environment on productivity is to regress TFP indices on vector of exogenous variables:

$$ TFP_i = X_i \beta + \varepsilon_i \quad (1) $$

where

- $X_i$: the vector of exogenous variables associated with employment office $i$
- $\varepsilon_i$: a normally distributed error term with zero mean and variance $\sigma^2$.

According to Simar and Wilson (2007) the estimations will be inconsistent and biased because first each TFP indices estimate depends on all observed inputs and outputs: $\varepsilon_i$ is serially correlated. Second, the environmental variables are correlated with both inputs and outputs: $\varepsilon_i$ is not independent from $X_i$.

The approach of Simar and Wilson (2007) is used to estimate model (1). The methodology used for efficiency scores can be easily adapted to this case. The process can be summarized as follows steps (for more details see Simar and Wilson (2007)):

1. We estimate $\hat{\beta}$ of $\beta$ and $\hat{\sigma}^2$ of $\sigma^2$ by the method of maximum of likelihood in the truncated regression of $TFP_i$ on $X_i$.
2. Loop over the next three steps (2.1) to (2.3) $L$ times to obtain a set of bootstrap estimates $A = \{(\hat{\beta}, \hat{\sigma}^2)\}_{b=1}^L$.
3. For each $i = 1, ..., m$, draw $\varepsilon_i$ from the $N(0, \hat{\sigma}^2)$ distribution with left-truncation at $1 - X_i \hat{\beta}$.
4. Again for each $i = 1, ..., m$, compute the model $\hat{TFP}^*_i = X_i \hat{\beta} + \varepsilon_i$.
5. Use the maximum likelihood method to estimate truncated regression $\hat{TFP}^*_i$ on $X_i$.
6. Use the bootstrap values in $A = \{(\hat{\beta}, \hat{\sigma}^2)\}_{b=1}^L$ and the original estimates $\hat{\beta}$ and $\hat{\sigma}^2$ to construct estimated confidence intervals for each element of $\beta$ and $\sigma^2$ as described below.

The explanatory variables that we introduced as explanatory factors in the model to explain TFP change between all GEO and between all MEO are as follows: number of counselor by job seekers (CJS), number of job vacancies per job seekers (JVJS), share of job-seekers in long term unemployment (LTUJS), size of office(SIZE), share of women in job-seekers(WJS), seasonal variable(D1) and trend (T). The model is as follow:

$$ TFP_i = \beta_0 + \beta_1(CJS)_i + \beta_2(JVJS)_i + \beta_3(LTUJS)_i + \beta_4(SIZE)_i + \beta_5(WJS)_i + \beta_6D_1 + \beta_7T_i + \varepsilon_i \quad (2) $$

Results and interpretation

The results for GEO with bootstrapping estimation are presented in Table 3. Estimation shows that variables used in the model explain
only 35% the TFP change between the 10 employment offices. The variables: counsellor per job-seekers and dummy variables (June, July and August) are significant and has a negative effect on productivity of GEO. For the first variable, the number of job seekers per counsellor is higher and so the labour productivity decrease. If this report is large, then the productivity of counsellors decrease and TFP of GEO decrease. The International Labour Organization (ILO) has proposed the following standard: one counselor for 80 jobseekers registered as unemployed. Later this report was revised to a counselor for every 100 job-seekers. In Tunisia and average per GEO this report is equivalent to a counselor for average 114 jobseekers for the GEO. This ratio is higher than the standard set by the ILO.

### Table 5: Determinants of TFP index of GEO

| Variable                           | Coef. | P>|z| |
|------------------------------------|-------|-----|
| Counselor per job seekers          | -2.483| 0.08 |
| Share of women in job-seekers      | -1.231| 0.27 |
| Size of office                     | 0.090 | 0.86 |
| Jobs vacancies per job-seekers     | -0.029| 0.96 |
| Share of job-seekers in long       | 3.664 | 0.17 |
| Trend                              | -0.001| 0.35 |
| Dummy variable                     |       |     |
| June                               | -0.233| 0.04* |
| July                               | -0.290| 0.08*** |
| August                             | -0.363| 0.01* |
| Constant                           | 3.408 | 0.01* |

*/***/***: statistically significant at the level: 1%/5%/10%

The second variables are characterising the situation of labour market in Tunisia. There is a decline in the activities of firms in the summer; consequently it has a negative effect on the activities of placements and productivity of GEO. The specifics exogenous conditions exercise a strong influence over the efficiency and productivity of each employment offices. In other word, TFP depends on labour marker conditional and on the number of counsellors managing situation of job seekers.

### Table 6: Determinants of TFP index of MEO

| Variable                           | Coef. | P>|z| |
|------------------------------------|-------|-----|
| Counselor per job seekers          | 0.488 | 0.08 |
| Share of women in job-seekers      | -0.319| 0.26 |
| Size of office                     | -0.145| 0.29 |
| Jobs vacancies per job-seekers     | 0.238 | 0.54 |
| Share of job-seekers in long       | 0.085 | 0.22 |
| Trend                              | -0.002| 0.32 |
| Dummy variable                     |       |     |
| June                               | 0.155 | 0.07** |
| July                               | 0.118 | 0.00** |
| August                             | 0.224 | 0.00** |
| Constant                           | 1.251 | 0.00** |

*/***/***: statistically significant at the 1%, 5%, 10%

Regarding the results in Table 4, the size has no effect on productivity of MEO. Only counsellors per job seekers and dummy have a significant effect on productivity. The specific exogenous economic conditions and the number of counsellors managing situation of job seekers have a strong influence over the efficiency and productivity of GEO and MEO.

### CONCLUSION

Through the nonparametric Malmquist index approach; this study analyzes TFP change in the Tunisian public employment offices (GEO and MEO) from 2006 to 2008. The results indicate that efficiency change plays a main role in contributing to TFP change in the GEO and MEO. The findings of the study also indicate that the differences of efficiency change within GEO and within MEO appear to be the main reason of the employment offices disparity in productivity. In addition, results show that technical change contributes positively in improving productivity of GEO and MEO. Furthermore, in this study we explained disparities in TFP change within GEO and within MEO. We have estimated two models and we have applied recently developed approach to bootstrapping Malmquist productivity indices. Results indicate that the number of job seekers per counselors, the size and the Tunisian economic conditions in summer (months of June, July and August) explain the difference in TFP change within GEO. Besides, the size has no effect on productivity of MEO because they have almost similar size. However, the external operating environment has a significant influence on the productivity of MEO. In addition, due to the hard economic situation, the global difficulties, the novelty of the services and the constant need for updating capacity building among the staff of public employment offices is needed. For this reason, one can say that specific guidance is required to promote efficiency and productivity in the GEO and MEO. The solutions proposed affect the internal organization and human resources, the reciprocal relationship counselors-job seekers and counselors-firms and employment policy. For internal organization we propose a specialization of employment offices by sector activity rather than the actual specialization by qualification of job seekers (skilled and unskilled). Next, employment offices must establish a relationship of trust with employers in order to increase the rate of job seekers who are employed. Then the collaboration of inter-offices plays a key role in improving the efficiency of offices. This collaboration results in maintaining a strong and effective contact between offices through an established network. Finally, employment policy must be combined with measures related to firms that are considered more effective than others in terms of hires. Thus, we recommend an
intensification of programs that promote direct contact of job seekers with labour market for instance short term training in firms.

REFERENCES


Environment & Agriculture Applications Stream
Using DEA to measure environmental efficiency of dairy farms South Africa

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ABSTRACT
The findings show that there is still room to reduce emissions of nitrogen to the environment. For example farms 3, 18 and 30 need to reduce their N surpluses by 333.3 kg to attain environmental efficiency. It was found that there minimal “over-usage” (over production) of milk thus reducing milk output on its own will not necessarily lead to improved environmental efficiency. Farm size, herd size, and nitrogen fertilizer applied, present the best scope of thus improving environmental efficiency of the dairy farms by reducing nitrogen surplus. Reducing imported feed and relying more home grown feed is another approach. Milk production would have to be reduced by 0.8 litres per hectare; farm size by 73.69 ha; herd size by 33 cows, nitrogen fertilizer application by 74.3 kilograms per hectare; and imported feed by 13.4 kilograms of dry matter per hectare. The positive correlation between technical and environmental efficiencies indicates that improving environmental efficiency could be associated with improvements in technical efficiency. Thus, policies aimed at improving efficiency, such as educating farmers in best practice technology, could have substantial rewards.

Keywords: Data Envelopment Analysis, environmental efficiency, South Africa

INTRODUCTION
In South Africa the extent of the environmental problems caused by nitrogen surplus is unknown thus not well documented. Subsequently, there is not legislation governing the use of nitrogenous fertilizer in farming. However, the absence of any policy regulatory framework with regard to nitrogen surplus does not preclude the existence of such a problem neither does it discount the potential of excess nitrogen becoming a serious problem. Given the paucity and/or lack of reported work in the area of the environmental damage caused by nitrogen surplus, this study is pioneering work in South Africa and provides both a theoretical and methodological contribution the field of efficiency studies.

This paper attempts to measure environmental efficiency of dairy farms in terms of efficiencies in the utilization of nitrogen as indicated by surplus nitrogen production. Nitrogen surplus is the difference between the applied nitrogen plus the nitrogen contained in marketable products and the nitrogen that remains on the farm (excess nitrogen that was not used in the production of the desirable outputs – milk, pasture and meat products).

Admittedly, the idea of measuring environmental efficiency in agriculture is not new and considerable work has been done in this regard (for example, Hoang and Coelli, 2009; Coelli et al, 2007; Roberts et al, 2007; Wossink and Denaux, 2006; Färe et al, 2005; Reinhard et al, 2002; Hadley et al, 1999; Chung and Färe, 1995). However, the methods used are varied and evolve rapidly owing to the difficult of coming up with one method that could be used universally in agriculture. Ball et al (2001) aptly stated that agriculture, unlike most other industry is diverse and this diversity rests in the complexity in measuring environmental efficiency and the need for unique methods for each sector. In this chapter an appropriate methodology for measuring environmental efficiency in the dairy industry will proposed and applied to the dairy industry in the KwaZulu-Natal Midlands employing the DEA approach. The rest of the paper is organized as follows:

A BRIEF REVIEW OF ENVIRONMENTAL EXTENSION OF DEA

Literature review
The complications introduced to efficiency analysis by trying incorporate environmental efficiency have already been discussed in Section 1 and these can be summarized as:

The efficiency level of any decision making unit (DMU), a farm in this particular study, measured via the DEA is defined by its input and output quantities, where more output from less output, other things being constant, gives a higher degree of efficiency (Dyckhoff and Färe, 1995). However, the methods used are varied and evolve rapidly owing to the difficult of coming up with one method that could be used universally in agriculture. Ball et al (2001) aptly stated that agriculture, unlike most other industry is diverse and this diversity rests in the complexity in measuring environmental efficiency and the need for unique methods for each sector. In this chapter an appropriate methodology for measuring environmental efficiency in the dairy industry will be proposed and applied to the dairy industry in the KwaZulu-Natal Midlands employing the DEA approach. The rest of the paper is organized as follows:
when environmental efficiency is considered because environmental efficiency by definition is concerned with the either use or production of environmentally undesirable input(s) or output(s), respectively.

There has been considerable work done in measuring efficiency using the DEA approach and a number of studies have looked at incorporating some environmental efficiency aspect. Allen (1999) carried out a review of 29 studies using the DEA to measure efficiency and found 17 out the 29 that incorporated environmental efficiency, although Allen (1999) referred to this environmental efficiency as ecological efficiency.

THE PROPOSED APPROACH

Initially the data for 37 farms from 2000 to 2007 was used to calculate environmental efficiency using model. However, this approach of using the favoured variables was aborted because the results gave unreasonably environmental efficiencies of close to unitary (signifying full environmental efficiency) for all the farms. The next step was to use cross-sectional data for 34 farms participating in a pasture utilization improvement study group with good nitrogen data. But this also did not work well the data was not sufficiently large. The last resort was to use the more nitrogen related data (quantities of nitrogen applied by each farm – farm specific, quantities of nitrogen containing feed imported into the farm in terms of concentrates and purchased silage, etc). Using these more nitrogen related variables gave better results, albeit at the cost of model misspecification and this was adopted following the approaches of Dyckhoff and Allen (2001) and Allen (1999).

Given that DEA looks at the quantity of output versus input usage, in other words, the DEA measures efficiency as the ability of converting inputs into output, it was necessary to do some manipulation of the data, particularly the nitrogen surplus quantities. In order for DEA to correctly analyze environmental efficiency the nitrogen surplus values were inverted, that is the inverse of the values were used. Using the inverse facilitates identifying farms with high N surplus values being less environmentally benign thus less environmentally efficient and those farms with low N surplus values as being more environmentally efficient (taking into consideration the use of inputs, especially N fertilizer and feed concentrates). This approach of taking the reciprocal of the nitrogen surplus has been used successfully before in other studies (for example, Dyckhoff and Allen, 2001).

PRODUCTION TECHNOLOGY

Before introducing the proposed analytical approach it would suffice the give a brief discussion of a production system that incorporates a polluting or an environmentally-detrimental output.

A production process in which aggregate fertilizer consumption (F), output – milk and other product (Y) and nitrogen surplus (N) are respectively taken as input, desirable output, and undesirable output is considered. The production technology can be described as

\[ T = \{(F, Y, N); F \text{ can produce } (Y, N)\} \] (2)

Note that finite amount of input can only produce finite amounts of outputs as T is often assumed to be a closed and bounded set in production theory (Färe and Primont, 1995). Additionally, F and Y in T are supposed to be strongly or freely disposable, i.e. if \((F, Y, N) \in T\) and \(F' \geq F\) (or \(Y' \leq Y\)) then \((F', Y, N) \in T\) (or \((F, Y', N) \in T\)).

Two assumptions have to be made in order to reasonably model a production process in which both desirable and undesirable outputs are jointly produced (Zhou and Ang, 2007). The two such assumptions were introduced by Färe et al (1989) and these are as follows:

(i) Outputs are weakly disposable, i.e., if \((F, Y, N) \in T\) and \(0 \leq \theta \leq 1\), then \((F, \theta Y, \theta N) \in T\).

(ii) Desirable and undesirable outputs are null-joint, i.e., if \((F, Y, N) \in T\) and \(N=0\), then \(Y=0\).

It is necessary to expand on the two assumptions made above. Under Assumption (i), the implication is that the reduction of nitrogen surplus is not free and a proportional reduction in production (milk in this case) and nitrogen surplus is feasible. Conversely, Assumption (ii) implies that nitrogen surpluses must also be produced when milk is produced, which carries the connotation that the only way to eliminate all nitrogen surpluses is to do away with the production process altogether.

The preceding discussion was centred on defining and conceptually locating the suggested technology for modelling the joint production of desirable (Y) and undesirable (N) outputs. Other studies have attempted model the joint production of desirable and undesirable outputs. For example, Färe et al (2005) referred to this technology a polluting technology. The conceptualization process of model has been sufficiently developed.

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9 In line with other related studies, the weak disposability of N surpluses as a kind of undesirable output is implicitly assumed although N surpluses are still unregulated in South Africa. Additionally, the growing concern on environmentally benignity of production makes the treatment of nitrogen surpluses as weakly disposable a logical assumption (Zaim and Taskin, 2000).
However, what is still needed is further characterization of the polluting technology within a parametric or a nonparametric framework in empirical studies. The nonparametric approach will be adopted for this study. In the nonparametric construction, the polluting technology can be constructed by the piecewise linear combinations of the observed data. Assume that there are \( k = 1, 2, \ldots, K \) entities (e.g. farms) and for entity \( k \) the observed data are \( (F_k, Y_k, N_k) \). Then the piecewise linear polluting technology \( T \) can be formulated as follows:

\[
T = \left\{ (F, Y, N) : \sum_{k=1}^{K} z_k F_k \leq F \right\}
\]

\[
\sum_{k=1}^{K} z_k Y_k \geq Y
\]

\[
\sum_{k=1}^{K} z_k N_k = N
\]

\[
z_k \geq 0, k = 1, 2, \ldots, K\}
\]

\( T \) is the environmental DEA technology exhibiting constant returns to scale (CRS) since it is formulated in the DEA framework (Zhou et al, 2008).

DATA

In this chapter data describing the production activities of 34 specialized dairy farms in the KwaZulu-Natal Midlands for 2008 were used. The 34 farms used for the analysis of environmental efficiency, using nitrogen, were part of a pasture-utilization study group and the dataset included quantities of fertilizer used thus being suitable for the study. The data was obtained from Allan Penderis of Tammac Consulting who is responsible for coordinating the pasture-utilization study.

The balanced cross-section had a total of 1074 observations, and so each farm appears on average 31.65 times. The period 2008 was chosen by default because data were available for only this one year even though a panel with a number of years would have been ideal. With panel data it would have been possible to do Malmquist productivity indices for environmental efficiency. However, in reality, the analyses that can be done are also determined by the available data. The dataset of 34 farms was selected because detailed information describing the nitrogen flows at each farm was available and the dataset used in the previous chapters was deficient of such information. The inputs and the output specified were based upon the production process of dairy farms in the area.

Of particular interest partly because the work reported in this chapter is concerned with environmental efficiency and also because this has not been covered in any of the previous empirical chapters is how the data for the nitrogen containing inputs and outputs were derived. Firstly the input quantity index comprises of nitrogen inputs and outputs. The output quantity index consists of milk, meat, livestock and roughage sold. These all contain nutrients, which are depicted in Figure 4. The nitrogen surplus is represented in Figure 4 as the sum of ‘nutrient exchange with the soil’ and ‘ammonia from land’. The nitrogen surplus, the difference between nitrogen input and nitrogen contained in desirable outputs, is measured in kilograms. The characteristics of the data set are summarized in Table 1.

The variables used for the environmental efficiency analysis were: roughage (pasture) area allocated to the dairy herd measured in hectares; mature herd size (number of cows); all concentrates used (kilograms of dry matter per hectare - kg/DM/ha); nitrogen fertilizer applied (kg/ha); output in milk produced (l/ha/year) and; nitrogen balance/ surplus (kg/ha).

Table 1: Summary of the environmental variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farm size</td>
<td>ha</td>
<td>Max</td>
</tr>
<tr>
<td>Herd size</td>
<td>No.</td>
<td>Max</td>
</tr>
<tr>
<td>Concentrates</td>
<td>kg/DM/ha</td>
<td>Max</td>
</tr>
<tr>
<td>Nitrogen (N)</td>
<td>kg/ha</td>
<td>Max</td>
</tr>
<tr>
<td>Milk produced</td>
<td>l/ha/year</td>
<td>Max</td>
</tr>
<tr>
<td>N Balance</td>
<td>kg/ha</td>
<td>Max</td>
</tr>
</tbody>
</table>

Table 1 shows the summary statistics of the 34 farms studied. It can be observed that the farms came in different sizes with a wide range as indicated by the roughage area allocated to the dairy herd. The average area was 223.6ha with the smallest farm being only 75.8ha and the biggest farm was almost eight times larger than the smallest at 650ha. Although there was a big difference between the smallest and largest farm, the standard deviation of 131.5ha indicates that most of the farms were distributed around the means of 223.6ha. Looking at the productive herd size (mature herd size) the picture is similar to that of the farm size (roughage area allocated to the dairy herd) with the largest farm, in terms of herd size, having 776 mature cows and the smallest having only 96. The average mature herd size was 337 cows with a standard deviation of 151.6. All the 34 dairy farms used concentrate feed to supplement grazing with supplies roughage and energy to the cows. Notwithstanding that that dairy production in South Africa is predominately pasture-based the use of concentrate feed (normally referred to as concentrates in the dairy industry for
simplicity purposes) is quite common. Concentrates are mainly used to provide protein, since milk production can deplete a cow’s protein reserves, vitamins and minerals. The amount of concentrates used varied widely across the farms with an average of 8.8 Kilograms of dry matter per day (kg/DM/day), a maximum of 21 and a minimum of 1.1 kg/dm/day.

Of particular interest to the purposes of this chapter are the nitrogen utilization and milk output because these two indicators have a substantial bearing on the nitrogen balance in the farms. The milk yield/production exhibited a wide range with the least producing farm having 3597 litres per hectare per year (l/ha/year) and the highest having almost five times the amount of the least producing at 17174 l/ha/year. The amount of nitrogen fertilizer used was no different from the variables already discussed in that there were marked inter-farm differences with a standard deviation of 90.8 kg of nitrogen per hectare. The most nitrogen -frugal farm only used 69 kg while the most lavish used 482 kg of nitrogen in 2008. All the differences discussed above ultimately manifested themselves in the differences in the nitrogen balances between farms. The mean nitrogen balance of 578.5 was higher than the mean fertilizer nitrogen applied of 482 which indicates that there is scope to reduce nitrogen application without stifling production. Next results from the DEA are presented and discussed in Section 6.

RESULTS

Standard cost and returns analysis for the dairy farms

Before the DEA application, a standard costs and returns analysis was performed for all 34 farms to see if the average economic results of the nitrogen fertilizer technology yield useful information. Table 2 presents the analysis. Gross value production and total costs of nitrogen fertilizer use for the dairy farms are provided. The analysis of costs of nitrogen fertilization as a proportion of gross value of production (a product of milk output and milk price per litre) gives an indication of the magnitude of the cost of applying nitrogen to the soil. The costs: returns ratio undergirds the importance of using only the required quantities of the nitrogen input and guarding against prophylactic application. Caution should be taken, however, that in the quest to minimize the amounts of nitrogen applied, thereby minimizing nitrogen surplus, one should not skimp on applying the required levels to achieve the desired output levels of pasture yields.

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Value</th>
<th>Std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N fertilizer usage</td>
<td>kg/ha</td>
<td>228.18</td>
<td>90.83</td>
</tr>
<tr>
<td>Cost of N fertilizer use</td>
<td>R/ha</td>
<td>3088.14</td>
<td>1408.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Costs and returns</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross value of production</td>
<td>R/ha</td>
<td>3161698</td>
<td>2200128</td>
</tr>
<tr>
<td>Costs of N fertilization</td>
<td>R/ha</td>
<td>690399.3</td>
<td>483406.1</td>
</tr>
<tr>
<td>Costs : Returns Ratio</td>
<td></td>
<td>0.22</td>
<td></td>
</tr>
</tbody>
</table>

ENVIRONMENTAL EFFICIENCY

Before looking at environmental efficiency\(^\text{10}\), which is the raison d’entre for this chapter, a brief analysis and discussion of technical efficiency\(^\text{11}\) is warranted to put things into perspective. It is important to establish if environmental efficiency and technical efficiency are mutually exclusive for the farmers in the sample, that is, to see if environmental efficiency can be achieved without sacrificing technical efficiency. Table 3 shows both the technical and environmental efficiencies of the dairy farms studied. Technical efficiency is shown in the second column. Eight farms were found to be fully technically efficient, i.e. having technical efficiency of unitary (1) and the average technical efficiency of the sample was moderate at 0.779 (77.9%) implying that there is scope to improve efficiency by more than 22 percent (22.1%). Farms 1, 8, 22, 24, 26, 27, 30, and 31 were the efficient farms thus defined the production frontier. It is interesting to observe the wide range in the efficiency of the farm with the poorest performing farm (farm 18) having an efficiency score of 0.459 when compared to the efficiency farms (1.000). Another interesting point to note is that a lot of the farms, however, were distributed around the mean as indicated by the median value of 0.735 (only 0.044 less than the mean).

Next is a look at the environmentally-adjusted efficiency scores, where nitrogen surplus was used as the polluting output (undesirable output) or environmentally-detrimental by-product of the dairy production system (nitrogen emission to the environment). These environmental efficiency scores are shown in the third column of Table 3. Unlike with technical efficiency where eight farms were

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\(^{10}\) Environmental efficiency refers to the efficiency value obtained using two outputs, namely, milk output as the desirable output, and nitrogen surplus as the undesirable output.

\(^{11}\) Technical efficiency here refers to the simple efficiency value obtained using one output, milk output as the dependent (output) variable.
efficient, only four farms were found to be environmentally efficient. Interestingly, only two farms were both technically and environmentally efficient and these were Farms 1 and 24.

The average environmental efficiency was 0.738 (73.8%) and was lower than the technical efficiency average meaning that the farms were less environmentally efficient than they were technically efficient. This observation is hardly surprising as most farmers are more preoccupied with technical efficiency than being environmentally benign in their production. The apparent lack of focus on environmental efficiency could be a result of the absence of any incentives to be ecologically friendly as there is no current legislation to that effect in South Africa. The other possible reason could be the difficulty of recommending the correct amount of nitrogen fertilizer to be applied due the lack of an accurate soil nitrogen predicting method. Nitrogen is currently applied following general broad guidelines determined by the crop being grown and soil type. This does not take into account the inherent nitrogen content of the soil and residual amount of nitrogen from previous applications, animal excretion, atmospheric and biological fixation of nitrogen.

Table 3: Technical and environmental efficiencies

<table>
<thead>
<tr>
<th>Farm</th>
<th>Technical Efficiency</th>
<th>Environmental Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.503</td>
<td>0.459</td>
</tr>
<tr>
<td>18</td>
<td>0.459</td>
<td>0.43</td>
</tr>
<tr>
<td>Mean</td>
<td>0.779</td>
<td>0.738</td>
</tr>
</tbody>
</table>

The results discussed can also be presented graphically as shown in Figure 2. The graphical representation of the results facilitates a better visualization of distribution of efficiencies across farms. From the results, it can be observed that farms tend to be more efficient technically than they are environmentally.
efficiency and environmental efficiency being dependent is rejected because the results show that they independent of each other although highly correlated.

### Table 4: Spearman and ktau correlation between technical efficiency and environmental efficiency

<table>
<thead>
<tr>
<th>Number of observations</th>
<th>34</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman's rho</td>
<td>0.9944</td>
</tr>
<tr>
<td>Test of Ho: technical efficiency and environmental efficiency are independent</td>
<td>0.0000 (continuity corrected)</td>
</tr>
</tbody>
</table>

The next step is to look at the environmental efficiency results as presented in Table 5. Table 5 shows the results of environmental efficiency suing only the undesirable output, N surplus, instead of the two-output approach that was adopted in the previous section. The results reported in Table 5 show that the average environmental efficiency of the 34 farms was 58.4 percent (0.584) which is quite low and not widely dispersed as shown by the standard deviation of 0.285. There are five farms that define the environmental efficiency frontier, namely: Farm 1, 4, 10, 15, and 27. These farms are therefore considered as benchmarks for the 34 farms analyzed.

The worse performing farm was Farm 30 which recorded an environmental efficiency of 0.109. A closer look at Farm 30 reveals a number of tell-tale observations: 1) Farm 30 was the largest farm with a total roughage area allocated to the dairy herd of 650ha and a mature herd size of 776 cows. 2) Although the quantity of nitrogen fertilizer used per hectare was modest by sample standards, 191.6 kg of nitrogen compared with the mean of 228.18 kg nitrogen. However, when taking into consideration the total quantity of nitrogen fertilizer applied on farm, Farm 30 was by far the largest consumer of nitrogen. This is to be expected as farm size is postulated to have a positive effect on the quantity of nitrogen applied. 3) Farm 30 imported almost 40 percent (38.4%) of the total feed for dairy production which was substantially high considering that the dairy farms in the sample are predominantly pasture-based. From the total sample, there appears to be a strong relationship between the percentage of feed brought into the farm from outside sources (imported) and the quantity of nitrogen surplus.

Interestingly, the best performing farm in terms of environmental efficiency was Farm 18 which was of the same size as Farm 30, the worst performer. Farm 18 also had 650ha of roughage area allocated to the dairy herd (basically this is the extent of pasture). However, the management was different in that Farm 18 had a smaller mature herd size of 491 cows compared with the 776 cows for Farm 30. Having discussed both best and worst performing farms in terms of environmental efficiency, the next step is to look at the distribution of the farms within environmental efficiency categories. Figure 7 is a graphical rendition of the distribution of the 34 farms according to their environmental efficiencies.

The categories were divided into intervals 0.1 exclusively (efficiency scores falling within the defined range, excluding the starting and end points). There were no farms falling with the first category of 0.0 > 0.1, only one farm in the 0.1 > 0.2. The bulk of the farms fell within the 0.2 > 0.3 and 0.3 > 0.4 categories with seven farms in the first and 6 in the second. Thirteen farms fell between category 0.4 > 0.5 to 0.9 > 1.0 and there were five farms that were environmentally efficient. The DEA defines efficiency according the data provided thus any efficiency is relative to the best farm within the sample. Thus care should be taken in understanding the environmentally efficient farms. The efficient farms are only efficient when compared among their peers. Having mentioned peers, it becomes necessary to discuss the subject and also to look at what he results show in relation to peers.

### Peers and peer weights

Table 6 shows the peers for each farm and the weights that these peers account for. For each inefficient farm there are peers which serve as comparators against which the farm is measured. Efficient farms do not have any peers other than themselves as they are on the environmental efficient frontier thus defining the efficiency. It stands to reason that the weight will be unity in the case of efficient farms. Only the efficient farms serve as peers for the inefficient farms and in this instance farms 1, 4, 10, 15, and 27 are the peers. Farm 1, for example, was a peer for 13 farms making it the most used farm as a comparator. Turning to peer weights, the higher the weight the more important that particular farm is as a peer for the inefficient farm in question. This means that the inefficient farm is better off comparing itself to the peer with the highest weight in order to improve its environmental efficiency by emulating its peers. The identification of peers is important in that the peers’ production technology, in this case pollution minimizing.
technology, can be studied and implemented by the inefficient farms. The next section deals with slacks.

Figure 4: Distribution of farms according to environmental efficiency

In the current study both milk production and nitrogen surplus can be viewed as outputs with milk output a desirable output and nitrogen surplus being the undesirable output (sometimes referred to as the ‘bad’ or pollutant in other literature). The objective here would be to reduce the amount of nitrogen surplus to zero, if possible, thus attaining the materials balance equilibrium (Coelli et al, 2007; Tyteca, 1995), thus the aim is minimizing nitrogen inputs without adversely affecting the production of milk and other desirables (meat and pasture). The results of the undesirable output slacks are presented in Table 7.

Table 5: Environmental efficiency of the dairy farms

<table>
<thead>
<tr>
<th>Farm</th>
<th>TE</th>
<th>Environ. efficiency Farm</th>
<th>TE</th>
<th>Environ. efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
<td>18</td>
<td>0.46</td>
</tr>
<tr>
<td>2</td>
<td>0.60</td>
<td>0.46</td>
<td>19</td>
<td>0.76</td>
</tr>
<tr>
<td>3</td>
<td>0.69</td>
<td>0.28</td>
<td>20</td>
<td>0.61</td>
</tr>
<tr>
<td>4</td>
<td>0.62</td>
<td>1.00</td>
<td>21</td>
<td>0.72</td>
</tr>
<tr>
<td>5</td>
<td>0.53</td>
<td>0.39</td>
<td>22</td>
<td>0.83</td>
</tr>
<tr>
<td>6</td>
<td>0.61</td>
<td>0.29</td>
<td>23</td>
<td>0.43</td>
</tr>
<tr>
<td>7</td>
<td>0.96</td>
<td>0.45</td>
<td>24</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>0.80</td>
<td>0.36</td>
<td>25</td>
<td>0.64</td>
</tr>
<tr>
<td>9</td>
<td>0.72</td>
<td>0.27</td>
<td>26</td>
<td>0.83</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>1.00</td>
<td>27</td>
<td>0.64</td>
</tr>
<tr>
<td>11</td>
<td>0.59</td>
<td>0.82</td>
<td>28</td>
<td>0.62</td>
</tr>
<tr>
<td>12</td>
<td>0.78</td>
<td>0.38</td>
<td>29</td>
<td>0.64</td>
</tr>
<tr>
<td>13</td>
<td>0.86</td>
<td>0.79</td>
<td>30</td>
<td>0.99</td>
</tr>
<tr>
<td>14</td>
<td>0.67</td>
<td>0.23</td>
<td>31</td>
<td>1.00</td>
</tr>
<tr>
<td>15</td>
<td>0.77</td>
<td>1.00</td>
<td>32</td>
<td>0.62</td>
</tr>
<tr>
<td>16</td>
<td>0.70</td>
<td>0.66</td>
<td>33</td>
<td>0.83</td>
</tr>
<tr>
<td>17</td>
<td>0.70</td>
<td>0.54</td>
<td>34</td>
<td>0.88</td>
</tr>
<tr>
<td>Mean</td>
<td>0.74</td>
<td>0.58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Peers and peer weights

<table>
<thead>
<tr>
<th>Farm</th>
<th>Peers (peer weights)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 (1)</td>
</tr>
<tr>
<td>2</td>
<td>4 (0.251) 15 (0.366)</td>
</tr>
</tbody>
</table>

The results presented in Table 8 indicate the scope of reducing the output of the nitrogen surplus, which is a pollutant, without reducing the output of milk. It is interesting to note that even the environmentally efficient farms can still reduce their emissions of nitrogen to the environment. However, the efficient farms can easily achieve the reduction of nitrogen inputs since the only have to do minor adjustments to their production technology. The farms with lower environmental efficiencies have to reduce their emission of nitrogen by bigger amounts as this is the reason par excellence why there are environmentally inefficient. For example farms 3, 18 and 30 need to reduce their emission of nitrogen to 333.3 kg to attain environmental efficiency.

Lastly, a look at the inputs slacks is warranted as this where the farmer has room to manoeuvre in trying to achieve environmental efficiency. Inputs slacks indicate the amount by which each input is over-used. Put differently, the slacks indicate the amounts by which each input can be reduced to minimize nitrogen surplus while keeping milk output unchanged. Milk output was included as an ‘input’ in this analysis to ascertain if a reduction in milk output would result in a corresponding reduction nitrogen surplus. The main intention of including milk output as an input was to see if there is any substance to the

12 Figures in parenthesis denote peer weights. These weights are the most favourable ones from the point of view of the target unit. To obtain the efficiencies of the entire set of units it is necessary to solve a linear program focusing on each unit in turn.
prevailing sentiment in environmental protection circles in South Africa that increasing output of agricultural production will invariably lead to environmental degradation. Clearly, it can be seen from the results that there minimal “over-usage” (over production) of milk thus reducing milk output on its own will not lead to improved environmental efficiency. Roughage area allocated to the dairy herd (farm size), number of mature cows (herd size), and quantity of nitrogen fertilizer applied, present the best scope of reducing nitrogen surplus thus improving environmental efficiency of the dairy farms. The area of imported feed also provides some room for reducing nitrogen surplus by relying more on home grown feed.

In summary, milk production would have to be reduced by 0.8 litres per hectare; land (roughage area allocated to the dairy herd) by 73.69 ha; number of mature cows by 33 cows, nitrogen fertilizer application by 74.3 kilograms per hectare; and imported feed (mainly concentrates) by 13.4 kilograms of dry matter per hectare. The adjustments that would be required if environmentally inefficient farms were to adopt best practice technology and move towards their environmental production frontiers indicate that the production of pollutants (nitrogen surplus) could be reduced at negligible cost to milk production. The positive correlation between technical and environmental efficiencies indicates that improving environmental efficiency could be associated with improvements in technical efficiency. Thus, policies aimed at improving efficiency, such as educating farmers in best practice technology, could have substantial rewards.

<table>
<thead>
<tr>
<th>Farm</th>
<th>Output slacks</th>
<th>Farm</th>
<th>Output slacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>71.43</td>
<td>29</td>
<td>90.91</td>
</tr>
<tr>
<td>13</td>
<td>83.33</td>
<td>30</td>
<td>333.33</td>
</tr>
<tr>
<td>14</td>
<td>166.67</td>
<td>31</td>
<td>76.92</td>
</tr>
<tr>
<td>15</td>
<td>71.43</td>
<td>32</td>
<td>142.86</td>
</tr>
<tr>
<td>16</td>
<td>71.43</td>
<td>33</td>
<td>90.91</td>
</tr>
<tr>
<td>17</td>
<td>76.92</td>
<td>34</td>
<td>90.91</td>
</tr>
</tbody>
</table>

**Table 8: Input slacks for the 34 dairy farms**

<table>
<thead>
<tr>
<th>Farm</th>
<th>Milk output</th>
<th>Farm size</th>
<th>Herd size</th>
<th>N fertilizer (kg/ha/pa)</th>
<th>Imported feed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.17</td>
<td>7.34</td>
<td>54.91</td>
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</tr>
<tr>
<td>3</td>
<td>0.07</td>
<td>0</td>
<td>76.67</td>
<td>13.04</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
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<td>101.41</td>
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</tr>
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<td>56.43</td>
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<td>11</td>
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<td>76.825</td>
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<td>16</td>
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<tr>
<td>17</td>
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<td>70.931</td>
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<td>18</td>
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<td>159.04</td>
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<td>19</td>
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<td>128.6</td>
<td>67.339</td>
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</tr>
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<td>0</td>
<td>170.6</td>
<td>77.003</td>
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<td>1.626</td>
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<tr>
<td>23</td>
<td>0</td>
<td>51.42</td>
<td>103.38</td>
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</tr>
<tr>
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<td>52.68</td>
<td>75.66</td>
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<td>25</td>
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<td>0</td>
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</tr>
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<td>474.0</td>
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<td>71.73</td>
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</tr>
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<td>31</td>
<td>2.087</td>
<td>35.45</td>
<td>67.941</td>
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<tr>
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<td>18.30</td>
<td>59.562</td>
<td>32.705</td>
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</tr>
<tr>
<td>33</td>
<td>8.35</td>
<td>101.7</td>
<td>356.87</td>
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</tr>
<tr>
<td>34</td>
<td>0.09</td>
<td>5.146</td>
<td>51.07</td>
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<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>0.803</td>
<td>73.69</td>
<td>32.70</td>
<td>74.308</td>
<td>13.349</td>
</tr>
</tbody>
</table>

Table 7: Output slacks for the 34 dairy farms

<table>
<thead>
<tr>
<th>Farm</th>
<th>Output slacks</th>
<th>Farm</th>
<th>Output slacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.04</td>
<td>18</td>
<td>333.33</td>
</tr>
<tr>
<td>2</td>
<td>142.86</td>
<td>19</td>
<td>125.00</td>
</tr>
<tr>
<td>3</td>
<td>333.33</td>
<td>20</td>
<td>90.91</td>
</tr>
<tr>
<td>4</td>
<td>166.67</td>
<td>21</td>
<td>125.00</td>
</tr>
<tr>
<td>5</td>
<td>125.00</td>
<td>22</td>
<td>90.91</td>
</tr>
<tr>
<td>6</td>
<td>125.00</td>
<td>23</td>
<td>111.11</td>
</tr>
<tr>
<td>7</td>
<td>125.00</td>
<td>24</td>
<td>50.00</td>
</tr>
<tr>
<td>8</td>
<td>111.11</td>
<td>25</td>
<td>166.67</td>
</tr>
<tr>
<td>9</td>
<td>166.67</td>
<td>26</td>
<td>125.00</td>
</tr>
<tr>
<td>10</td>
<td>58.82</td>
<td>27</td>
<td>125.00</td>
</tr>
<tr>
<td>11</td>
<td>111.11</td>
<td>28</td>
<td>76.92</td>
</tr>
</tbody>
</table>

Table 8: Input slacks for the 34 dairy farms

**ACKNOWLEDGEMENTS**

The author would like to acknowledge the invaluable contributions made by Professors Colin Thirtle of Imperial College London and Jenifer Piesz of Kings College London during the analyses and earlier write-ups of this manuscript.

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Roberts EM., PB. English, J K. Grether, G C. Windham, L.


Modeling grain yield in Estonian countries: a multistage Data Envelopment Analysis (DEA) approach

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ABSTRACT
This paper investigates the formulation of multi-stage DEA grain yields models in Estonian regions and analyses the results of modeling. The data is a balanced panel of fifteen Estonian counties drawn from various publications of Statistical Office of Estonia observed during the period of 1994 to 2001. In this paper we consider multistage (three-stage) DEA models as an alternative method to estimate and predict technical efficiency of grain producers in Estonia. The three-stage analysis starts with data envelopment analysis (DEA). In this stage we define the inputs and conduct the typical static DEA analysis to estimate efficiency scores and the input and output slacks. The second stage is a stochastic frontier analysis (SFA) to explain the variation in performance measured in the first stage. In this stage, we select a set of environmental variables to explain the variation in the input and output slacks. In the third stage we then estimate scores again using adjusted data from the second stage that has been cleaned from the influence of the operating environment and statistical noise. Thus, the evaluation emerging from stage three DEA is said to represent managerial efficiency only.

Keywords: DEA, multistage modeling, SFA, grain yield

INTRODUCTION
Estonia is one of the new members of the European Union. The EU enlargement means a lot of changes in the agriculture of East European countries. These changes are at the political, economical and technical level. This means that information systems on agriculture (databases, models etc) have to move along with those changes.

Consequently, the economic models in Estonia have either to be created, developed or renewed, and must be harmonized with the European requirements. Improving the competitiveness of agriculture is a priority objective of Estonian agricultural policy. The outcome and impacts of those policy actions will strongly depend on developments of the world agricultural markets.

The grain sector is one of the most important sectors of Estonian agriculture. Consequently, the need to further improve the competitiveness of the Estonian grain sector is obvious. In recent years, DEA has become a central technique in productivity and efficiency analysis, applied in different aspects of economics and management sciences (Coelli et al. 2005; Cooper et al. 2007). Recently, accounting for the impact of the environment in efficiency analysis using multiple stage approaches that involve data envelopment analysis (DEA) has been attracted more attention (see Fried et al. 2002; Muñiz 2002; Drake et al. 2006; Avkiran et al. 2008; Cooper et al. 2007).

We at the Estonian University of Life Sciences have investigated the possibilities of SFA methods and have some experience in using SFA (Põldaru et al. 2004; Põldaru et al. 2009). The objective of this paper is to develop a DEA-based model of producer performance that contains a stochastic element designed to isolate the impact of luck from those of managerial performance and environmental impacts.

The rest of this paper is organized as follows: in the next section we describe the three-stage DEA and SFA methodology to be used. We then describe the data for the empirical analyses. Section 3 presents and discusses the results. Section 4 summarizes and gives conclusions.

MODEL DESCRIPTION
The methodology used in this study is a three-stage process commonly used in the efficiency literature. The tree-stage model of Estonian grain sector’s efficiency model is based on Avkiran et al. (2008) approach.
The method consists of a three-stage analysis that starts with data envelopment analysis (DEA). Avkiran et al (2008) propose a more comprehensive methodology where total input and output slacks are measured simultaneously against the same reference set, facilitated by a non-oriented SBM model that is fully units-invariant (assuming variable returns to scale). The “Slacks Based Model” (SBM) introduced by Tone (2001) is shown below.

We assume that there are decision making units (DMU) (i = 1, ..., m) each using inputs (x) to produce outputs (y). For each DMU under evaluation, the input (output) vectors are denoted as x_i ∈ R^m_i and y_i ∈ R^n_i. The input and output matrices are defined as X = (x_1, ..., x_m) ∈ R^{m×n} and Y = (y_1, ..., y_n) ∈ R^{n×n}, respectively.

The non-oriented SBM model with the variable returns to scale (hereafter as SBM) is formulated as:

\[
\min_{\lambda, \beta, s} \rho_o^* = \frac{1 - \frac{1}{m} \sum_{i=1}^{m} s_i^- / x_{io}}{1 + \frac{1}{s} \sum_{r=1}^{s} s_r^+ / y_{ro}}
\]

Subject to \( x_{io} = \sum_{j=1}^{s} y_j \lambda_j - s_i^- \) (i = 1, ..., m) \( \lambda_j \geq 1 \) \( s_i^- \geq 0 \) (i = 1, ..., m), \( s_r^+ \geq 0 \) (r = 1, ..., s), where \( \lambda \) is an intensity vector of dimension n, and \( s^- \) is the m-dimensional vectors of input slacks, and \( s^+ \) is the s-dimensional vectors of output slacks.

The second stage is a stochastic frontier analysis (SFA) to explain the variation in organizational performance measured in the first stage in terms of the operating environment, statistical noise, and managerial efficiency.

In the second stage we focus on stage 1 input slacks \( s^0 > 0 \), and output slacks \( s^0 > 0 \). Thus, stage 2 analysis leads to an estimate of \( m \times s \) (i.e. inputs plus outputs) SFA regressions.

The general function of the SFA regressions is represented in Eq. (5) for the case of input slacks:

\[
s_{ij}^* = f'(z_j, \beta') + v_{ij} + u_{ij}
\]

and in Eq. (6) for the case of output slacks:

\[
s_{ij}^* = f'(z_j, \beta') + v_{ij} + u_{ij}
\]

(\( r = 1, ..., s; j = 1, ..., n \))

(6)

where \( t \) (\( s_{ij} \) is the stage 1 slack in the \( j \)th input for the \( i \)th unit, \( s_{ij}^0 \) is the stage 1 slack in the \( r \)th output for the \( j \)th unit, \( z_j \) the environmental variables, \( \beta' \) and \( \beta'' \) are the parameter vectors for the feasible slack frontier; \( v_{ij} + u_{ij} \) and \( v_{ij} + u_{ij} \) the composed error structure where \( v_{ij} \) and \( v_{ij} \) are random variables which are assumed to be \( N(0, \sigma^2) \) and represents statistical noise; and \( u_{ij}, u_{ij} \geq 0 \) represents managerial inefficiency.

Following each regression, parameters \( \beta' \) and \( \beta'' \), \( \sigma^2 \) and \( \sigma^2 \), \( \sigma^2 \) and \( \sigma^2 \) are estimated and permitted to vary across \( m \) input slack and \( s \) output regressions.

Parameter estimates obtained from SFA regressions are used to predict input slacks attributable to the operating environment and to statistical noise. Thus, observed inputs can be adjusted for the impact of the environment and statistical noise by:

\[
z_j/\beta = x_j/\lambda + \left[ \max_{j} \left( z_j/\beta \right) - \lambda_j \right] + \left[ \max_{j} \left( v_j + u_j \right) - v_{ij} + u_{ij} \right]
\]

(7)

where \( x_j/\lambda \) is the adjusted quantity of \( j \)th input in \( j \)th unit, \( x_j \) the observed quantity of \( j \)th input in \( j \)th unit.

(- The \( j \)th input slack in \( j \)th unit attributable to environmental factors)

- The \( j \)th input slack in \( j \)th unit attributable to \( v_{ij} \) statistical noise.

Similarly, DMUs suffering from relatively unfavourable operating environments and statistical noise would have their outputs adjusted upwards as shown in Eq. (8) by comparing their slacks against those generated by the DMUs operating in the most favourable environment and the most fortunate situation.

\[
y_j = y_j + \left[ z_j/\beta' + \min_{j} \left( z_j/\beta' \right) \right] + \left[ v_{ij} + u_{ij} \right]
\]

(8)

(\( r = 1, ..., s; j = 1, ..., n \))

where \( y_j \) is the adjusted quantity of \( r \)th output in \( j \)th unit, \( y_j \) the observed quantity of \( r \)th output in \( j \)th unit, \( z_j/\beta' \) the \( r \)th output slack in
the \( j \)th unit attributable to environmental factors, and \( v_{ij} \) the \( j \)th output slack in \( j \)th unit attributable to statistical noise. However, to use Eq. (7), it is necessary to distinguish input-sourced statistical noise (\( v_{ij} \)) from managerial inefficiency (\( u_{ij} \)) in the composed error term of the SFA regressions. Once \( v_{ij} \) has been estimated for each unit, Eq. (7) can be implemented and observed input usage adjusted.

The SFA model parameters are estimated using a software FRONTIER Version 4.1 (Coelli 1996).

The third stage concludes with DEA of organizational performance using adjusted data from the second stage that have been cleaned from the influence of the operating environment and statistical noise.

Stage 3 is a repeat of stage 1 analysis using input and output data adjusted in stage 2. The results of stage 3 analysis represent SBM DEA analysis. In this final stage of the three-stage analysis of managerial efficiency the data are cleaned from the influence of operating environment and statistical noise.

**DATA**

The data used were obtained from various publications of Statistical Office of Estonia. To conduct the analysis we constructed a balanced panel covering eight years (\( T = 8 \)), from 1994 – 2001, for \( N = 15 \) counties of Estonia.

**Input and output variables in the productivity model**

In the current study, four inputs and one output (grain yield) are specified in the first-stage DEA model. The characteristics of the input and output data are reported in Table 1.

<table>
<thead>
<tr>
<th>Definitions of inputs</th>
<th>Measure</th>
<th>Variable</th>
<th>Correl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 ) Sown area of grain</td>
<td>Hectare</td>
<td>SownArea</td>
<td>0.525</td>
</tr>
<tr>
<td>( x_2 ) Total use of mineral fertilizer Kg/ha</td>
<td>TotalFertil</td>
<td>0.346</td>
<td></td>
</tr>
<tr>
<td>( x_3 ) Fraction of fertilized area in total grain sown area</td>
<td>%</td>
<td>FracFertilArea</td>
<td>0.323</td>
</tr>
<tr>
<td>( x_4 ) Fraction of grain sown area in total sown area</td>
<td>%</td>
<td>GrainFraction</td>
<td>0.381</td>
</tr>
</tbody>
</table>

Table 1 provides the coefficients of correlation between inputs (independent variables) and the output (grain yield). In the most cases the coefficients of correlation between the output and inputs are statistically significant.

**Environmental variables**

To investigate the impact of environmental factors (\( z_j \)) that may distort the validity of the initial efficiency analysis, five weather conditions measures, two country-specific factors, and a trend variable are included in the second-stage analysis. The selected environmental factors influence the grain production but they are beyond managerial control. The characteristics of environmental variables are reported in Table 2.

**RESULTS**

**Stage 1: initial SBM results**

In this stage, we conducted the typical static DEA analysis to estimate efficiency scores and the output slacks, using model (1) – (4).

Results of the stage 1 SBM analysis are estimated against a common efficient frontier comprised of all the observations across eight years and fifteen Estonian counties.

More importantly, to compare grain production efficiency scores over time, we are forced to measure efficiency relative to a common frontier. Summary statistics of the results are contained in Table 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>St. dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>0.242</td>
<td>0.449</td>
<td>0.303</td>
<td>0.052</td>
</tr>
<tr>
<td>1995</td>
<td>0.207</td>
<td>0.439</td>
<td>0.302</td>
<td>0.062</td>
</tr>
<tr>
<td>1996</td>
<td>0.358</td>
<td>1.000</td>
<td>0.625</td>
<td>0.225</td>
</tr>
<tr>
<td>1997</td>
<td>0.274</td>
<td>0.591</td>
<td>0.380</td>
<td>0.105</td>
</tr>
<tr>
<td>1998</td>
<td>0.186</td>
<td>1.000</td>
<td>0.320</td>
<td>0.196</td>
</tr>
<tr>
<td>1999</td>
<td>0.175</td>
<td>0.658</td>
<td>0.259</td>
<td>0.116</td>
</tr>
<tr>
<td>2000</td>
<td>0.292</td>
<td>1.000</td>
<td>0.482</td>
<td>0.175</td>
</tr>
<tr>
<td>2001</td>
<td>0.307</td>
<td>0.702</td>
<td>0.426</td>
<td>0.118</td>
</tr>
<tr>
<td>/</td>
<td>0.175</td>
<td>1.000</td>
<td>0.387</td>
<td>0.180</td>
</tr>
</tbody>
</table>

The results in stage 1 suggest very low mean efficiencies and high variability. The minimal sample average efficiency score of 0.259 in the crisis year 1999 and maximal sample average efficiency score of 0.625 in the year 1996 differ substantially.

Fig. 1 shows the dynamics of mean efficiency scores in Estonian counties covering eight years from 1994 – 2001. For comparison the
The possible explanation for the low efficiency scores in the initial stage may be that the efficiency estimates for Estonian counties were probably influenced by environmental factors (weather conditions in different years and county-specific conditions), and statistical noise.

Fig. 2 shows the dynamics of mean grain yield in Estonian counties covering eight years from 1994 – 2001. For comparison the dynamics of grain yield in two Estonian counties are provided. The county E has the maximal mean grain yield in eight years and the county C has minimal mean grain yield in eight years. Fig. 2 shows that all three graphs (curves) behave analogously. They have maximum in 1996 and minimum in 1999. All Estonian farmers know that in 1996 weather conditions were favourable and in 1999 unfavourable for grain production.

Consequently, environmental factors essentially influence the efficiency estimates. In the first stage, we found a volatile pattern in the mean efficiency score, making the multistage method legitimate.

In the stage 1 we use a spreadsheet model (Zhu 2009) to estimate slacks and the initial efficiency scores.

**Stage 2: SFA regression results**

The purpose of stage 2 in our analysis is to decompose the stage 1 total slacks into environmental effects, managerial inefficiency, and statistical noise. Input and output slacks obtained from stage 1 across the eight year period are pooled, and then separately regressed on a set of independent (environmental) variables discussed in Table 2. These SFA regressions allow the influences of the environment, managerial inefficiency, and statistical noise to vary across inputs and outputs.

The empirical results of the SFA analysis estimation are presented in Table 4, which suggest that the input and output slacks are indeed affected by the environmental factors. The estimated coefficients on all significant variables have the right signs.

The results in Table 4 show that the most environmental variables are statistically significant. Between three and six of the environmental variables are significant in the four regressions on input slacks, while five independent variables are statistically significant in the regression on output slacks.

**Stage 3: SMB results on adjusted data**

After adjusting the input and output data using equations (7) and (8), we again run the same SMB model to estimate the efficiency scores.
The results in stage 3 (see Table 5) suggest relatively high mean efficiencies and low variability. The minimal sample average efficiency score of 0.906 in the crisis year 1999 and maximal sample average efficiency score of 0.980 in the year 2001 do not differ practically.

The initial and the final estimates of efficiency scores are summarized together in Table 6 and Figure 3.

### Table 6: Comparison of the initial and final efficiency scores

<table>
<thead>
<tr>
<th>Year</th>
<th>Initial estimates</th>
<th>Final estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.Dev.</td>
</tr>
<tr>
<td>1994</td>
<td>0.303</td>
<td>0.052</td>
</tr>
<tr>
<td>1995</td>
<td>0.302</td>
<td>0.062</td>
</tr>
<tr>
<td>1996</td>
<td>0.625</td>
<td>0.225</td>
</tr>
<tr>
<td>1997</td>
<td>0.380</td>
<td>0.105</td>
</tr>
<tr>
<td>1998</td>
<td>0.320</td>
<td>0.196</td>
</tr>
<tr>
<td>1999</td>
<td>0.259</td>
<td>0.116</td>
</tr>
<tr>
<td>2000</td>
<td>0.482</td>
<td>0.175</td>
</tr>
<tr>
<td>2001</td>
<td>0.426</td>
<td>0.118</td>
</tr>
</tbody>
</table>

Figure 3: Comparison of mean efficiency between initial and final estimates

In sharp contrast, the mean efficiency score measured in the final stage exhibits a more stable upward pattern, seemingly independent of the exogenous influences. Meanwhile, the standard deviation of the efficiency scores have trend to narrow. The combination of the increasing mean and narrowing standard deviation value is consistent to the Arrow (1964) theory of learning-by-doing.

**DISCUSSION AND CONCLUSIONS**

In the initial stage the typical static DEA results are affected by the influences of environmental factors and statistical noise pertaining to the sample period.

This study shows how an integrated DEA and SFA approach can account for environmental effects and statistical noise to measure efficiency of grain production in Estonian counties during the period of 1994 – 2001.

The mean efficiency score measured in the final stage exhibits a more stable upward pattern which is consistent to the Arrow theory of learning-by-doing.

Therefore, we consider final estimates to be more convincing as they reflect events like weather condition and quality of land which the initial estimates failed to do.

The key contribution of this paper is an empirical analysis of the impact of environmental variables and statistical noise to efficiency scores of grain production in Estonian counties.

More specifically, the study is important because the impact of environmental variables to efficiency scores is better understood.

Although we are not investigating the causes of the variability of efficiency of grain production, this paper reveals that environmental factors and statistical noise play a greater role than should be expected.

The methodology starts with non-oriented SBM stage 1, which is used to assess technical efficiency of grain production without considering environmental effects, followed by SFA regressions in stage 2 measuring the impact of the environment on grain production inefficiency. Finally, in stage 3, SBM is repeated with adjusted data.

This method enables us to adjust the panel data for environmental effects and statistical noise.

**REFERENCES**


Agricultural policy support and technical efficiency in French agriculture

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ABSTRACT
This paper investigates how the various changes in the policy supporting agriculture in the European Union, the Common Agricultural Policy (CAP) and various types of subsidies, have affected farms’ technical efficiency in France between 1990 and 2006. Several efficiency scores are computed: using the Free Disposal Hull (FDH) method; using Data Envelopment Analysis (DEA) under constant (CRS) and variable returns to scale (VRS); scale efficiency scores calculated with DEA; and DEA technical efficiency scores corrected for bias both under CRS and VRS with the bootstrap method. We consider four periods: 1) 1990-1994 is the benchmark period; 2) 1995-1999 is the period where the effect of the first CAP reform is investigated 3) 2000-2005 represents the period following the second CAP reform; 4) 2006 is the first year of implementation of the latest CAP reform in France. Three types of farming – field crop, dairy and beef cattle – are considered. Graphs and t-tests indicate a significant reduction in efficiency in the period following the first CAP reform compared to the benchmark period. Econometric regressions do not reveal unambiguous findings, as the effect of a particular type of subsidies was found to be negative or positive depending on the efficiency variable and depending on the type of farming considered.

Keywords: technical efficiency, DEA, bootstrap, agricultural policy, France

INTRODUCTION
This paper aims at contributing to the literature about the impact of public support on farmers’ decision making, and in particular on their technical efficiency. More specifically, it investigates how the various changes in the policy supporting agriculture in the European Union (EU), the Common Agricultural Policy (CAP) and the different farm subsidies, have affected farms’ technical efficiency in France. Over the past two decades the CAP has undergone major changes. Three main reforms have taken place, all with the objective of modifying the way European farms benefit from public support. Firstly, there has been a gradual shift from ‘coupled’ support, that is to say support given per unit of output, to more ‘decoupled’ support, that is to say support not related to production but given on an area or livestock head basis. The first decoupled subsidies, namely area and livestock direct payments were introduced in the first CAP reform in 1992. However, these payments were still linked to production decisions, as they concerned only some land sown with specific cultures or some specific livestock type. Within the next reform in 2000, the so-called Agenda 2000, more types of land and livestock could benefit from the direct payments. And the latest reform of 2003 introduced a more decoupled support instrument, the Single Farm Payment (SFP), provided per hectare of land under any type of production or even without production. Secondly, the reforms have modified the total amount of subsidies received by farms, as more and more land was being concerned by payments, and support was redistributed among farms. For example, support to farms in less favoured areas (LFA) was introduced; also, farmers could benefit from increased subsidies if they satisfied, on a voluntary basis, to environmental criteria, the so-called agri-environmental schemes.

Several papers have analysed econometrically how the level of agricultural subsidies alters the farming sector’s technical efficiency, and in general concluded that the effect was negative, due to reduced effort by farmers or preservation of inefficient farms within the sector (e.g. Giannakas et al., 2001; Zhu et al., 2008; Latruffe et al., 2009). Some papers have also compared farms’ technical efficiency evolution across several periods differing in terms of agricultural policy (e.g. Morrison Paul et al., 2000; Coelli et al., 2006; Carroll et al., 2009). In this paper both methods – an econometric approach and the comparison of periods – are used to analyse the effects of the changes in CAP on French farms’ technical efficiency between 1990 and 2006. We consider four periods: 1) 1990-1994 is the benchmark period; 2) 1995-1999 is the period...
where the effect of the first CAP reform is investigated; the reform was decided in 1992 but its full implementation in France was finished only in 1995; 3) 2000-2005 represents the period following the second CAP reform, namely Agenda 2000; 4) 2006 is the first year of implementation of the 2003 CAP reform in France.

The paper is structured as follows. The second section explains the methodology and the third section describes the data used. The fourth section presents the results, and the last section concludes.

METHODOLOGY

The objective is to draw as robust as possible conclusions on the effect of CAP reforms and subsidies on French farms’ technical efficiency, and for this reason we do not use a single efficiency score. Instead, several technical efficiency scores are calculated with non-parametric methods. Technical efficiency scores are computed using the Free Disposal Hull (FDH) method. Then, Data Envelopment Analysis (DEA) technical efficiency scores under constant returns to scale (CRS) and under variable returns to scale (VRS), and scale efficiency scores are computed. Finally, we use DEA technical efficiency scores corrected for bias, both under CRS and VRS, with the bootstrap method introduced by Simar and Wilson (1998, 2000). All scores are computed using the FEAR package developed by Wilson (2008, 2009) within the R Project (R Development Core Team, 2009). Prior to any efficiency calculation, inconsistent data were removed, and outliers were eliminated using the method introduced by Wilson (1993) with the command available in FEAR. The DEA and FDH models are output-oriented and include one single output – the deflated value of the farm total output –, and four inputs: the farm UAA, the total labour used on farm, the deflated value of fixed assets, and the deflated value of intermediate consumption. Separate frontiers are constructed for different production orientation (see next section), and yearly frontiers are built.

The impact of CAP reforms and subsidies is investigated in two ways: with a graphic illustration and with an econometric analysis. Firstly, graphs picturing the evolution of the samples’ mean efficiency scores over the period are drawn. This enables to shed light on potential falls or increases of efficiency at specific dates. In addition, the significance of the difference between efficiency means across the four different periods of CAP reforms (explained in the previous section) is tested with t-tests. Farms are separated in three size groups with the help of a clustering analysis.

Secondly, econometric regressions are run using the various farms’ individual efficiency scores as dependent variables. As the shares of farms on the frontier (that is to say with efficiency score of unity) are high in the case of FDH efficiency scores (between 36 and 49 percent of the sample), truncated regressions are used for this efficiency variable, while ordinary least squares are used for the other efficiency variables. The explanatory variables consist, in particular, of the level of various types of subsidies received by farms. They are incorporated in the regression models as ratios per hectare of UAA or livestock equivalent unit (depending on the production orientation) in order to avoid size effects. Other variables include a dummy indicating whether the farm is not located in a LFA, proxying the farms’ agro-climatic conditions, an extensification dummy for farms specialised in livestock production (namely the number of hectares of land available per livestock unit), and two dummies classifying farms into size groups. Clustering techniques are used to categorize farms into three groups depending on their size proxied. In agriculture there is no consensus on a single indicator for farm size, as it depends on the productions on the farm. For this reason, clusters are created based on four size indicators: the utilised agricultural area (UAA), the total labour used on the farm, the number of livestock equivalent units on the farm, and the deflated value of fixed assets. Finally, regional dummies are included for control.

DATA

Farm-level data are used for the period 1990-2006. They are extracted from the EU Farm Accountancy Data Network (FADN) database. This database consists of yearly accounting data for professional farms, rotating over a five-year period. Each EU Member-State surveys its own farms and produces a national FADN database; it then provides to the European Commission a standardised database, that is to say including variables that are similarly constructed across all countries. In this paper we separate farms according to their production specialisation, for two reasons. Firstly, technology differs across specialisations (e.g. field crop farming vs. livestock farming), and thus separate efficiency frontiers are constructed. Secondly, the CAP modalities, in particular the types and amount of subsidies and the various reforms, are different depending on the farms’ production specialisation. For example, it is notorious that field crop farming has always been among the most supported specialisation, but it also faced the greatest change in support policy over the period studied. We focus specifically on three
RESULTS

Table 2 presents the efficiency averages over the whole period, while Figure 1 illustrates the yearly evolution of efficiency and Table 3 the efficiency averages during the four periods and the results of the t-tests. All efficiency variables indicate that the field crop sample had the highest efficiency on average in the benchmark period that is to say before any CAP reform was implemented (1990-1994), followed by a decrease in efficiency and the lowest efficiency on average in the latest 2003 CAP reform implementation (2006). This is confirmed by Figure 1. The figure shows, by contrast, that the efficiency evolution following the first 1992 reform implementation was hatch-backed for both livestock samples. For both dairy and beef cattle farm samples, the efficiency decreased on average following the first CAP reform implementation, and then increased after the second CAP reform (Agenda 2000); the effect of the most recent reform is negative for the dairy sample but not significant for the beef cattle sample.

Table 4 presents the results of the second-stage econometric regressions on each of the six efficiency variables in turn for all three samples. Several subsidy variables were tested and the best fitted models were retained for the final specifications. Results indicate that, for all three samples, farms not located in LFA were more efficient than farms in LFA, suggesting the negative effect of poor agro-climatic conditions on farms’ performance. For both livestock samples, being extensive (that is to say having a large UAA per livestock unit) does not favour efficiency. The coefficients for the size clusters clearly indicate that larger farms are more efficient in terms of operational scale, while smaller farms are more efficient in terms of management practices. As for subsidies, results do not show unambiguous effects. Investment subsidies have a positive impact on field crop farms’ efficiency, but the impact on scale efficiency for the dairy sample is negative, and no significant effect is found for the beef cattle sample. The total subsidies on crops had a positive impact on dairy farms’ efficiency and on beef cattle farm’s technical efficiency under CRS assumption and scale efficiency; by contrast, for the latter sample, the effect was negative on technical efficiency under VRS assumption. Among the crop subsidies, the crop area payments and the set aside premiums had a negative effect on the field crop farms’ efficiency, except for FDH on which crop area payments had a positive effect. As regard the livestock subsidies, they had a negative impact on the field crop farms’ efficiency. For dairy farms, subsidies on dairying had a positive effect on efficiency except for scale and FDH efficiency on which the effect was negative. A negative effect is also found for subsidies on beef for the dairy sample, and for the beef cattle sample except for technical efficiency under CRS and scale efficiency for which the effect is found to be positive. Environmental subsidies have a negative effect on efficiency for the field crop farms except for scale efficiency where the effect is positive, on FDH efficiency for the dairy farms (but a positive effect on technical efficiency under CRS for this sample) and on all efficiencies for the beef cattle farms. As for LFA subsidies, they have a negative effect on all efficiencies for the beef cattle farms, and on technical efficiency under CRS and scale efficiency for the dairy sample, but a positive effect on technical efficiency under VRS for the latter sample. Finally, decoupled payments (included SFP) have a negative effect on field crop farms’ efficiency except when FDH is concerned (the effect is positive), and a positive effect on dairy farms’ technical efficiency under CRS and scale efficiency.

CONCLUSION

On a methodological point of view, our analysis has shown that the effects of explanatory variables identified in the second-stage regression of DEA efficiency scores under CRS or VRS assumptions, were confirmed in the regression of DEA scores under CRS or VRS corrected for bias with bootstrapping, suggesting that the sampling error problem was not substantial in our samples. In addition, the regression results pointed out that the impact of explanatory variables was often opposite between pure technical efficiency and scale efficiency, suggesting that analyses based on total technical efficiency (under CRS assumption) may lead to incorrect policy recommendations. Moreover, impacts that are provided evidence for a specific production orientation sample may be opposite for other orientations, suggesting that conclusions drawn from analyses on the whole farming sector may not be generalised to all types of farming.
The main results arising from our analyses are that efficiency for the three samples considered (field cropping, dairying, and beef cattle farming) has decreased following the first CAP reform implementation (1992 reform). As for the effect of subsidies, econometric results did not show unambiguous findings, as the effect of a particular type of subsidies was found to be negative or positive for the same type of farming (depending on the efficiency variable), or depending on the type of farming.

REFERENCES

Presented at the 12th EAAE Congress, Gent, Belgium, August 27-30.

ACKNOWLEDGEMENTS
Support from the European FP7 Research Project FACEPA is acknowledged.

Figure 1: Yearly efficiency averages
Panel 1a: Field crop farms (TF 1)
Panel 1b: Dairy farms (TF 41)
Panel 1b: Beef cattle farms (TF 42)
### Table 1: Descriptive statistics of the data used; whole period 1990-2006

<table>
<thead>
<tr>
<th>Variables used in efficiency models (farm averages)</th>
<th>Field crop farms (TF 1)</th>
<th>Dairy farms (TF 41)</th>
<th>Beef cattle farms (TF 42)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations</td>
<td>32,781</td>
<td>20,410</td>
<td>10,003</td>
</tr>
<tr>
<td>Total output (€)</td>
<td>108,829</td>
<td>86,915</td>
<td>52,696</td>
</tr>
<tr>
<td>UAA (ha)</td>
<td>117.54</td>
<td>62.28</td>
<td>90.05</td>
</tr>
<tr>
<td>Total labour input (AWU)</td>
<td>1.67</td>
<td>1.68</td>
<td>1.47</td>
</tr>
<tr>
<td>Fixed assets (€)</td>
<td>148,960</td>
<td>147,500</td>
<td>165,441</td>
</tr>
<tr>
<td>Intermediate consumption (€)</td>
<td>6,6208</td>
<td>50,830</td>
<td>35,589</td>
</tr>
<tr>
<td>Variables used in efficiency models (farm averages)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Other variables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Farm averages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Total subsidies on investments (€)</td>
<td>377</td>
<td>1,004</td>
<td>1,022</td>
</tr>
<tr>
<td>2. Total operational subsidies (€)</td>
<td>29,829</td>
<td>9,244</td>
<td>21,825</td>
</tr>
<tr>
<td>2.1. Total subsidies on crops (€)</td>
<td>26,646</td>
<td>4,340</td>
<td>2,907</td>
</tr>
<tr>
<td>2.1.1. Crop area payments (€)</td>
<td>23,074</td>
<td>3,950</td>
<td>2,115</td>
</tr>
<tr>
<td>2.1.2. Set aside premiums (€)</td>
<td>2,906</td>
<td>336</td>
<td>113</td>
</tr>
<tr>
<td>2.2. Total subsidies on livestock (€)</td>
<td>998</td>
<td>1,961</td>
<td>1,4018</td>
</tr>
<tr>
<td>2.2.1. Subsidies on dairying (€)</td>
<td>21</td>
<td>360</td>
<td>4</td>
</tr>
<tr>
<td>2.2.2. Subsidies on beef cattle (€)</td>
<td>833</td>
<td>1,216</td>
<td>12,542</td>
</tr>
<tr>
<td>2.3. Environmental subsidies (€)</td>
<td>285</td>
<td>740</td>
<td>1,873</td>
</tr>
<tr>
<td>2.4. LFA subsidies (€)</td>
<td>36</td>
<td>686</td>
<td>1,256</td>
</tr>
<tr>
<td>2.5 Decoupled payments (including SFP) (€)</td>
<td>1,523</td>
<td>700</td>
<td>749</td>
</tr>
<tr>
<td>Livestock equivalent units</td>
<td>15.15</td>
<td>75.06</td>
<td>101.50</td>
</tr>
<tr>
<td>UAA per livestock unit (ha)</td>
<td>29.86</td>
<td>0.88</td>
<td>0.94</td>
</tr>
<tr>
<td>Dummy variables</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of farms in LFA (%)</td>
<td>21.1</td>
<td>42.4</td>
<td>80.4</td>
</tr>
<tr>
<td>Share of farms not in LFA (%)</td>
<td>78.9</td>
<td>57.6</td>
<td>19.6</td>
</tr>
<tr>
<td>Share of farms in small-size cluster (%)</td>
<td>68.7</td>
<td>58.4</td>
<td>59.2</td>
</tr>
<tr>
<td>Share of farms in medium-size cluster (%)</td>
<td>25.3</td>
<td>32.2</td>
<td>32.1</td>
</tr>
<tr>
<td>Share of farms in large-size cluster (%)</td>
<td>6.1</td>
<td>9.4</td>
<td>8.7</td>
</tr>
</tbody>
</table>
| UAA: utilised agricultural area. AWU: annual working units (1 AWU is equivalent to 2,200 hours labour per year). LFA: less favoured area. Livestock equivalent units: calculated with the standard EU coefficients applied to each type of livestock on the farm. All values are deflated.

### Table 2: Descriptive statistics of the efficiency scores; averages over the whole period 1990-2006

<table>
<thead>
<tr>
<th>Average efficiency score in the period</th>
<th>Field crop farms (TF 1)</th>
<th>Dairy farms (TF 41)</th>
<th>Beef cattle farms (TF 42)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Field crop farms (TF 1)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEA_CRS</td>
<td>0.519</td>
<td>0.682</td>
<td>0.566</td>
</tr>
<tr>
<td>DEA_VRS</td>
<td>0.708</td>
<td>0.788</td>
<td>0.756</td>
</tr>
<tr>
<td>DEAB_CRS</td>
<td>0.485</td>
<td>0.651</td>
<td>0.523</td>
</tr>
<tr>
<td>DEAB_VRS</td>
<td>0.674</td>
<td>0.758</td>
<td>0.717</td>
</tr>
<tr>
<td>DEA_SCALE</td>
<td>0.748</td>
<td>0.875</td>
<td>0.762</td>
</tr>
<tr>
<td>FDH</td>
<td>0.875</td>
<td>0.923</td>
<td>0.903</td>
</tr>
<tr>
<td><strong>Dairy farms (TF 41)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEA_CRS</td>
<td>0.683</td>
<td>0.703</td>
<td>0.676</td>
</tr>
<tr>
<td>DEA_VRS</td>
<td>0.774</td>
<td>0.813</td>
<td>0.823</td>
</tr>
<tr>
<td>DEAB_CRS</td>
<td>0.654</td>
<td>0.674</td>
<td>0.641</td>
</tr>
<tr>
<td>DEAB_VRS</td>
<td>0.741</td>
<td>0.789</td>
<td>0.805</td>
</tr>
<tr>
<td>DEA_SCALE</td>
<td>0.889</td>
<td>0.877</td>
<td>0.842</td>
</tr>
<tr>
<td>FDH</td>
<td>0.914</td>
<td>0.934</td>
<td>0.941</td>
</tr>
</tbody>
</table>

DEA = score calculated with DEA; DEAB = score calculated with DEA and corrected from bias with bootstrap; _CRS and _VRS = DEA calculations under the assumptions of CRS and VRS respectively; DEA_SCALE = scale efficiency calculated with DEA; FDH = efficiency calculated with FDH.

### Table 3: Efficiency averages in the four periods, and t-tests of the significance of the difference between periods

<table>
<thead>
<tr>
<th>Average efficiency score in the period</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Periods 1 and 2</th>
<th>Periods 2 and 3</th>
<th>Periods 3 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field crop farms (TF 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEA_CRS</td>
<td>0.543</td>
<td>0.513</td>
<td>0.514</td>
<td>0.448</td>
<td>***</td>
<td>ns</td>
<td>***</td>
</tr>
<tr>
<td>DEA_VRS</td>
<td>0.726</td>
<td>0.700</td>
<td>0.704</td>
<td>0.692</td>
<td>***</td>
<td>**</td>
<td>***</td>
</tr>
<tr>
<td>DEAB_CRS</td>
<td>0.509</td>
<td>0.479</td>
<td>0.482</td>
<td>0.411</td>
<td>***</td>
<td>ns</td>
<td>***</td>
</tr>
<tr>
<td>DEAB_VRS</td>
<td>0.692</td>
<td>0.664</td>
<td>0.671</td>
<td>0.662</td>
<td>***</td>
<td>***</td>
<td>**</td>
</tr>
<tr>
<td>DEA_SCALE</td>
<td>0.759</td>
<td>0.747</td>
<td>0.749</td>
<td>0.685</td>
<td>***</td>
<td>ns</td>
<td>***</td>
</tr>
<tr>
<td>FDH</td>
<td>0.879</td>
<td>0.870</td>
<td>0.879</td>
<td>0.861</td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>Dairy farms (TF 41)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEA_CRS</td>
<td>0.683</td>
<td>0.655</td>
<td>0.703</td>
<td>0.676</td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>DEA_VRS</td>
<td>0.774</td>
<td>0.769</td>
<td>0.813</td>
<td>0.823</td>
<td>**</td>
<td>***</td>
<td>**</td>
</tr>
<tr>
<td>DEAB_CRS</td>
<td>0.654</td>
<td>0.620</td>
<td>0.674</td>
<td>0.641</td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>DEAB_VRS</td>
<td>0.741</td>
<td>0.734</td>
<td>0.789</td>
<td>0.805</td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>DEA_SCALE</td>
<td>0.889</td>
<td>0.862</td>
<td>0.877</td>
<td>0.842</td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>FDH</td>
<td>0.914</td>
<td>0.918</td>
<td>0.934</td>
<td>0.941</td>
<td>**</td>
<td>***</td>
<td>**</td>
</tr>
</tbody>
</table>
Beef cattle farms (TF 42)

Table 4: Second-stage regression results

<table>
<thead>
<tr>
<th>Field crop farms (TF 1)</th>
<th>DEA_CRS</th>
<th>DEA_VRS</th>
<th>DEA_CR</th>
<th>DEA_VR</th>
<th>DEA_SCALE</th>
<th>FDH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>5.315e-</td>
<td>7.462e-</td>
<td>5.014e-</td>
<td>7.179e-</td>
<td>7.169e-</td>
<td>8.864e-</td>
</tr>
<tr>
<td>Dummy = 1 if farm not in LFA</td>
<td>3.392e-</td>
<td>1.694e-</td>
<td>2.810e-</td>
<td>1.168e-</td>
<td>2.800e-</td>
<td>2.317e-</td>
</tr>
<tr>
<td>Dummy = 1 if period 1995-1999</td>
<td>5.499e-</td>
<td>6.298e-</td>
<td>-</td>
<td>6.581e-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dummy = 1 if period 2000-2005</td>
<td>3.193e-</td>
<td>6.599e-</td>
<td>0.037</td>
<td>8.027e-</td>
<td>-1.744e-</td>
<td>-1.138e-</td>
</tr>
<tr>
<td>Dummy = 1 if period 2006</td>
<td>-7.339e-</td>
<td>-9.083e-</td>
<td>2.716e-</td>
<td>-1.356e-</td>
<td>-1.333e-</td>
<td>-</td>
</tr>
<tr>
<td>Amount received of investment subsidies per hectare of UAA</td>
<td>3.831e-</td>
<td>1.622e-</td>
<td>1.140e-</td>
<td>7.250e-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Amount received of crop area payments per hectare of UAA</td>
<td>-1.435e-</td>
<td>-7.407e-</td>
<td>-</td>
<td>8.426e-</td>
<td>1.305e-</td>
<td>-</td>
</tr>
<tr>
<td>Amount received of set aside premiums per hectare of UAA</td>
<td>-1.264e-</td>
<td>-8.375e-</td>
<td>-</td>
<td>1.208e-</td>
<td>-1.433e-</td>
<td>5.890e-</td>
</tr>
<tr>
<td>Amount received of livestock subsidies per hectare of UAA</td>
<td>7.815e-</td>
<td>-6.693e-</td>
<td>-6.418e-</td>
<td>-5.450e-</td>
<td>-7.221e-</td>
<td>-</td>
</tr>
<tr>
<td>Amount received of environmental subsidies per hectare of UAA</td>
<td>2.020e-</td>
<td>-1.706e-</td>
<td>-4.010e-</td>
<td>2.905e-</td>
<td>-4.112e-</td>
<td>-</td>
</tr>
<tr>
<td>Amount received of decoupled payments per hectare of UAA</td>
<td>-2.574e-</td>
<td>-1.659e-</td>
<td>-1.745e-</td>
<td>-</td>
<td>-1.716e-</td>
<td>-</td>
</tr>
<tr>
<td>Dummy = 1 if farms in medium-size cluster</td>
<td>7.225e-</td>
<td>6.703e-</td>
<td>-1.157e-</td>
<td>2.229e-</td>
<td>-9.654e-</td>
<td>-</td>
</tr>
<tr>
<td>Dummy = 1 if farms in large-size cluster</td>
<td>9.433e-</td>
<td>8.029e-</td>
<td>-1.146e-</td>
<td>2.478e-</td>
<td>-7.468e-</td>
<td>-</td>
</tr>
</tbody>
</table>

| Number of observations | 32,781  | 32,781  | 32,781  | 32,781  | 32,781  | 32,781  |

<table>
<thead>
<tr>
<th>Model statistic</th>
<th>R-squared</th>
<th>Log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.2243</td>
<td>0.1582</td>
</tr>
<tr>
<td>Dummy = 1 if farm not in LFA</td>
<td>0.2336</td>
<td>0.1788</td>
</tr>
<tr>
<td>Dummy = 1 if period 1995-1999</td>
<td>0.3596</td>
<td>0.1651</td>
</tr>
<tr>
<td>Dummy = 1 if period 2000-2005</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dummy = 1 if period 2006</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Amount received of investment subsidies per livestock unit</td>
<td>3.698e-</td>
<td>-</td>
</tr>
<tr>
<td>Amount received of subsidies on crops per livestock unit</td>
<td>1.158e-</td>
<td>1.064e-</td>
</tr>
<tr>
<td>Amount received of subsidies on dairying per livestock unit</td>
<td>1.858e-</td>
<td>5.737e-</td>
</tr>
<tr>
<td>Amount received of subsidies on beef cattle per livestock unit</td>
<td>6.255e-</td>
<td>3.762e-</td>
</tr>
<tr>
<td>Amount received of environmental subsidies per livestock unit</td>
<td>7.029e-</td>
<td>7.019e-</td>
</tr>
<tr>
<td>Amount received of LFA subsidies per   -</td>
<td>1.113e-</td>
<td>-</td>
</tr>
<tr>
<td>Model statistic</td>
<td>DEA_CRS</td>
<td>DEA_VRS</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>Number of observations</td>
<td>20,410</td>
<td>20,410</td>
</tr>
<tr>
<td>Intercept</td>
<td>6.349e- **</td>
<td>7.628e- **</td>
</tr>
<tr>
<td>Dummy = 1 if farms in medium-size cluster</td>
<td>1.510e- **</td>
<td>1.828e- **</td>
</tr>
<tr>
<td>Dummy = 1 if farms in large-size cluster</td>
<td>5.883e- **</td>
<td>2.415e- **</td>
</tr>
<tr>
<td>Amount received of investment subsidies per livestock unit</td>
<td>2.450e- **</td>
<td>6.659e- **</td>
</tr>
<tr>
<td>Dummy = 1 if period 2006</td>
<td>3.647e- **</td>
<td>3.833e- **</td>
</tr>
<tr>
<td>UAA per livestock unit</td>
<td>3.940e- **</td>
<td>4.758e- **</td>
</tr>
<tr>
<td>DEA = score calculated with DEA; DEAB = score calculated with DEA and corrected from bias with bootstrap; _CRS and _VRS = DEA calculations under the assumptions of CRS and VRS respectively; DEA_SCA = scale efficiency calculated with DEA; FDH = efficiency calculated with FDH. *; **; ***: significance at 10, 5, and 1 percent respectively. Results for regional dummies not shown.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Agricultural productivity in Israel, Jordan, Lebanon and Syria: a DEA approach

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ABSTRACT
In this paper agricultural productivity in four countries Israel, Jordan, Lebanon and Syria is examined. A thorough look at previous studies that considered those countries is provided. The data used is drawn from the Food and Agriculture Organization of the United Nations and adjusted, covering the period of 1972 through 2006. The study utilizes Data Envelopment Analysis (DEA) to calculate Malmquist productivity indices. The study looks at the tendencies in agricultural productivity for the four countries throughout the 35 years, in which wars and conflicts took place. The estimates of efficiency change, technical change, and total factor productivity change obtained for the four countries are calculated. Moreover, a model for technical inefficiency effects in a stochastic frontier production function is suggested to provide a possible explanation of the sources of inefficiencies and the effect of each inefficiency variable.

Keywords: Agricultural productivity, Data Envelopment Analysis, Israel, Jordan, Lebanon, Syria

INTRODUCTION
The main objective of the present study is to estimate the Malmquist agricultural productivity index for Israel, Jordan, Lebanon and Syria between the period of 1972 and 2006. The purpose is to look at the different factors affecting the agricultural Total Factor Productivity (TFP) change differences and to estimate the technical efficiency effects. Agricultural output in Israel declined between 1979 and 1985 from six percent of the GDP to 5.1 percent of the GDP (Metz, 1988). According to Metz, this decline was due to the development of manufacturing and from 1966 to 1984 agriculture had been more productive compared to industry. Moreover, Metz (1988) mentioned that agricultural productivity between 1955 and 1983 increased due to the cut back on employed labor, the use of water, fertilizers, and pesticides; as well as research, training, improved crop varieties, and better organization. Capital played an important role and Israel was able to export citrus fruit, eggs, vegetables, poultry, and melons; in the period between 1950 and 1983, citrus exports declined by sixty five percent but processed agricultural products increased by 4,000 percent (Metz, 1988). Major achievements of the agricultural research in Israel are: the highest level of milk production in the world of more than 11,000 liter/cow/year and an increase in productivity of fruit, vegetable and field crops with the use of less chemical fertilizers and pesticides (Loebenstein and Putievsky, 2007). At this level, it has to be mentioned that organized agricultural research in Israel started as early as the 1920s, and in the late 1920s and early 1930s an institutionalized relationship between researchers and agriculturalists started to take place (Katz and Ben-David, 1975).

The contribution of agriculture in Jordan has been declining from 40 percent of GNP in the early 1950s to six percent by the mid 1980s unlike agriculture in Syria which constituted more than 20 percent of GNP in the 1980s (Metz, 1991). Metz also attributed this decline to many factors such as the loss of prime farmland due to the occupation of Israel of the West Bank, labor emigration and migration to cities. It has to be noted, that even though there was a decline in the agriculture share in the GNP, the total production of cereals and beans increased by about 150 percent and a 200 percent increase in the production of vegetables took place during the same period (Metz, 1991). Moreover, during the period of 1967 and 1970, there was a sharp increase in population and percent of the total area dedicated for fruits and vegetables. Those increases are attributed to two factors: an increase in the investments in the East Ghor canal in the Jordan Valley, and a larger effect was due to the production of fruits and vegetables outside the Jordan Valley (Doan, 1995). Additionally, between 1981 and 1992, the cultivated area increased by 131 percent, and it decreased between 1992 and 1996 while the cultivated area in 1996 was just 76% of the cultivated area of 1992 (Ali Hussain, 2001).

In Lebanon, the situation of agriculture is different. According to a study done by Collelo (1987), the constant warfare has been an obstacle to the full exploitation of the Lebanese land, therefore the livestock production declined especially after the Israeli...
invasion in 1982. Moreover, Collelo notes that Lebanon has the highest portion of cultivable land (almost 1/4) compared to other Arab countries. During the war between 1975 and 1990, and due to the incapability of the government to control different areas and to the high profitability of hashish and opium, a portion of the land devoted to the production of wheat has been used to grow hashish and opium. In the second half of the 1990s, the government destroyed most if not all agricultural lands devoted to the plantation of drug-producing crops. No studies estimated the impact of war on agriculture in Lebanon.

In Syria, the percentage of labor force employed in agriculture declined from fifty to thirty percent between the 1970s and the mid 1980s and in the mid 1980s, the government took measures to rejuvenate agriculture (Collelo, 1987). Moreover, Collelo mentions that about 2/3 of the agricultural output depends on rainfall, which might limit the production to a single crop a year. Additionally, Collelo states that some economists consider that the pricing of the farm produce by the government contributed to the stagnation in agriculture over the 1970s and 1980s. Even though there was stagnation, the national cereal production had a high degree of variability and it was due to the fluctuating rainfalls and the possible effect of technical change and agricultural policy (Nguyen, 1989). Moreover, Keilany (1980) argues that the land reform might have negatively affected the performance of the agricultural sector in the second half of the 1960s. In addition to that, a considerably recent work shows that the reduction of fallow periods and the absence of the application of fertilizers and manures in the Im Milia area have lead to the decrease in yield (Zobisch, 2001).

A few previous studies looked closely at agricultural productivity in the four countries mentioned above (Table 1). Belhaj Hassine and Kandil (2009) discuss the link between trade openness, agricultural productivity growth and poverty reduction in some Mediterranean countries including Israel, Jordan, Lebanon, and Syria. Their results show positive effects of trade openness on farming efficiency and productivity. Another recent study shows that these four countries can be both innovative and efficient and Jordan and Lebanon had the highest Agricultural TFP growth, 4.2 percent (3 percent growth in efficiency and 1.2 percent growth in technical change) and 3.4 percent respectively (Belloumi and Matoussi, 2009). During the 1980s, Syria had a negative 3 percent TFP which pushed the government to adopt more liberal agricultural sector policies and at the end of the 1990s Syria became a net exporter of many agricultural products (Belloumi and Matoussi, 2009). Furthermore, it’s worth mentioning that the Israeli 1.6 percent average growth in TFP was due to a 1.3 percent growth in technical change (TC) and 0.3 percent growth in efficiency change. This paper offers an estimation of the Malmquist productivity index using panel data and an estimation of a model for technical inefficiency effects in a stochastic production function.

THE MALMQUIST TFP INDEX

The explanation below is based on the work of Färe et al. (1994). The Malmquist output-oriented TFP index calculates the TFP change between two data points and between two periods s (base period) and t, and can be written as

\[ m^t_s(q_s, x_t, x_s) = \frac{d^t_s(q_s, x_t)}{d^t_s(q_s, x_s)} \]

where \( d^t_s(q_s, x_t) \) is the distance from the period s observation to the period t technology, \( q_s \) and \( x_t, x_s \) are output and input at time s. Additionally, the Malmquist TFP change index can be written as the product of a technical efficiency change index and an index of technical change

\[ m^t_s(q_s, x_t, x_s, t, _s) = \frac{d^t_s(q_s, x_s)}{d^t_s(q_s, x_t)} \times \left( \frac{d^t_s(q_s, x_s)}{d^t_s(q_s, x_t)} \right)^{1/2} \]

(Coelli, et al., 2005). The ratio outside the square bracket in the above equation measures the technical efficiency change between period s and t and the term inside the square bracket refers to the measure of technical change (Coelli, et al. 2005).

DATA ENVENOPMENT ANALYSIS

The method used to calculate the Malmquist TFP index is the DEA proposed by Färe et al. (1994). Consequently, there is a need to calculate four distance functions to measure the TFP change between two periods for each country. This involves solving the four linear programming problems mentioned in (Coelli, et al., 2005) and presented below:

\[ \left[ d^t_s(q_s, x_t) \right]^{-1} = \max_{\phi, \lambda} \phi, \]

s.t. \(-\phi q_s + Q, \lambda \geq 0, x_t - \lambda X \geq 0 \) and \( \lambda \geq 0 \),

\[ \left[ d^t_s(q_s, x_s) \right]^{-1} = \max_{\phi, \lambda} \phi, \]

s.t. \(-\phi q_s + Q, \lambda \geq 0, x_t - \lambda X \geq 0 \) and \( \lambda \geq 0 \),

\[ \left[ d^t_s(q_s, x_t) \right]^{-1} = \max_{\phi, \lambda} \phi, \]

s.t. \(-\phi q_s + Q, \lambda \geq 0, x_t - \lambda X \geq 0 \) and \( \lambda \geq 0 \),

\[ \left[ d^t_s(q_s, x_s) \right]^{-1} = \max_{\phi, \lambda} \phi, \]

s.t. \(-\phi q_s + Q, \lambda \geq 0, x_t - \lambda X \geq 0 \) and \( \lambda \geq 0 \)

where \( X \) is the \( N \times I \) input matrix, \( Q \) is the \( M \times I \) output matrix representing the data for all I firms, \( \phi \) is a scalar, \( \lambda \) is a \( 1 \times 1 \) vector of
constants. \((X\lambda, Q\lambda)\) is the projected point produced by the radial contraction of the input vector \(\chi_i\).

**EMPIRICAL APPLICATION**

The data used in the present study is based on the data used by Fuglie (2008). Fuglie made adjustments for the inputs and output variables data provided by the Food and Agriculture Organization of the United Nations (FAO). The inputs are: land which is Fuglie’s estimate of quality-adjusted agricultural land and which is done by using different weights from irrigated land, unirrigated crop land and pasture with different weights varying by global region, measured in 1000 ha; machinery, proxied by tractors in use measured in 1000 of tractors; labor as the sum of active male and female population measured in thousands; fertilizers which are measured in tons, and presented as the sum of nitrogen, phosphate and potash contained in the commercial fertilizers consumed, and livestock as the aggregate sum of animals in cattle equivalents weighted with respect to the method of Hayami and Ruttan (1985). The output is the agricultural gross production measured in US $1000 smoothed by Fuglie using the Hodrick-Prescott filter setting \(\lambda = 6.25\) to remove annual fluctuations due to weather. A summary statistics of the data is provided in Table 2. The software used to calculate the Malmquist TFP index is the DEAP Version 2.1 computer program written by Tim Coelli.

Furthermore, a model for technical inefficiency effects in a stochastic frontier function is estimated to provide a better explanation of the fluctuations of the results through time. The stochastic frontier production function is based on the work of Battese and Coelli (1995) and a translog stochastic frontier production function for the four above countries can be specified as:

\[
\begin{align*}
\ln Y_i = & \beta_0 + \beta_1 \ln A_i + \beta_2 \ln B_i + \beta_3 \ln C_i + \\
& \beta_4 \ln D_i + \beta_5 \ln E_i + \lambda T_i + \frac{1}{2} \beta_{\lambda A} (\ln A_i)^2 + \beta_{\lambda B} \ln A_i \ln B_i + \\
& \beta_{\lambda C} \ln A_i \ln C_i + \beta_{\lambda D} \ln A_i \ln D_i + \beta_{\lambda E} \ln A_i \ln E_i + \lambda_{\beta A} (\ln A_i) T_i + \\
& \frac{1}{2} \beta_{\beta A} (\ln B_i)^2 + \beta_{\beta B} \ln B_i \ln D_i + \\
& \beta_{\beta C} \ln B_i \ln E_i + \lambda_{T \beta} (\ln E_i) T_i + \frac{1}{2} \beta_{\beta E} (\ln E_i)^2 + \beta_{\sigma} (\ln A_i) D_i + \\
& \beta_{\sigma A} \ln A_i E_i + \lambda_{\sigma} (\ln C_i) T_i + \frac{1}{2} \beta_{\sigma D} (\ln D_i)^2 + \\
& \beta_{\sigma E} (\ln E_i)^2 + \lambda_{\eta \sigma} (\ln E_i) T_i + \frac{1}{2} \lambda_{\eta \eta} (\ln T_i)^2 + (V_{it} - U_{it}), \ i = 1, \ldots, N, \ t = 1, \ldots, T \end{align*}
\]

\(\eta_{it}\) is the production of the i-th time period: \(x_{it} = (A_{it}, B_{it}, C_{it}, D_{it}, E_{it})\) is a \(k \times 1\) vector of input quantities of the i-th country in the t-th time period. \(T\) represents time, \(\beta_0\) is the intercept, \(\beta_{\lambda A}, \beta_{\lambda B}, \beta_{\lambda C}, \beta_{\lambda D}, \beta_{\lambda E}, \lambda_{\beta A}, \lambda_{T \beta}, \lambda_{\sigma A}, \lambda_{\sigma D}, \lambda_{\sigma E}, \lambda_{\eta \sigma}, \lambda_{\eta \eta}\) are own second derivatives, \(V_{it}\) are random variables assumed to be iid. \(N(0, \sigma^2)\), and independent of the \(U_{it}\) which are non-negative random variables. \(U_{it}\) account for technical inefficiency and can be written as \(U_{it} = z_i \delta + W_i\) and is independently distributed as truncations at zero of the \(N(z_i \delta, \sigma^2)\) distribution. \(\text{Ln}\) denotes the natural logarithm. A, B, C, D, and E represent the same inputs used in the DEA analysis before: A refers to fertilizers, B refers to Labor, C refers to land, D refers to livestock, and E refers to machinery (for more details please refer to the beginning of the “Empirical application” part).

The institutional and socio-political differences between countries are represented by the following variables: political freedom data which is downloaded from the Freedom House database, the previous colony data obtained from the Central Intelligence Agency World Fact book, and war data from the Centre of the Study of Civil War dataset. Illiteracy, considered as an input quality variable for labor, is obtained from the World Development Indicators dataset and UNESCO Statistical yearbook dataset.

The inefficiency model is defined as:

\[
U_{it} = \delta_1 + \delta_2(\text{Free}_{it}) + \delta_3(\text{Partially Free}_{it}) + \\
\delta_4(\text{Not Free}_{it}) + \delta_5(\text{Former UK colony}_{it}) + \\
\delta_6(\text{Former French colony}_{it}) + \delta_7(\text{years since independence}_{it}) + \\
\delta_8(\text{Illiteracy rate}_{it}) + \\
\delta_9(\text{War}_{it}) + \delta_{10}(1 \text{ type } 2 \text{ conflict}_{it}) + \delta_{11}(2 \text{ type } 2 \text{ conflict}_{it}) + \\
\delta_{12}(1 \text{ type } 3 \text{ conflict}_{it}) + \delta_{13}(2 \text{ type } 3 \text{ conflict}_{it}) + \delta_{14}(4 \text{ type } 3 \text{ conflict}_{it}) + \delta_{15}T_i + W_{it}.
\]

The data of the inputs were mean corrected before the estimation. Consequently, the first-order parameters are interpreted as the elasticities at the sample means. Frontier Version 4.1 written by Coelli is the software that is used to calculate the maximum-likelihood estimates of the parameters of the translog stochastic frontier production function. The time squared variable allows for non-monotonic technical change and the time interacted with the logarithm of the input variable allows for non-neutral technical change.

**RESULTS AND CONCLUSIONS**

**DEA Results**

Figure 1 plots the results of the TFP change for every country and every year while table 3 presents the results of the average annual performance of each country over the entire period of 1972-2006. The highest TFP Change is in Syria in the year 2000. A value of TFP Change or any of its components less than one means a regress in performance and a value
higher than one indicates improvements in the relevant performance. The lower part of table 3 indicates that on average, productivity increased slightly over the 1972-2006 period for the four countries in the sample: the average change in the Malmquist productivity index was 1 percent per year for the entire sample. On average, this growth was due to innovation (Technical change) rather than improvements in efficiency (Efficiency Change). A look at the country by country results shows that Israel has the highest TFP Change in the sample at 2.9 percent per year, on average and this is due to innovation. Based on the constant return to scale technology, Lebanon average TFP Change was slightly higher than the sample average (2.1 percent compared to 1 percent).

Figure 3: Average TFP change in the four countries during 1973-2006

The results of technical change for the four countries for the entire period are plot in figure 2. The average results of technical change presented in table 3 do not allow the identification of the countries that shift the frontier over time. Färe, Grosskopf, and Lovell (1994) states that the technical change doesn’t reveal which country affects the shift, it just tells us what happened to the frontier at the input level and mix of each country.

Figure 4: Technical change in the four countries during 1973-2006

Figure 3 illustrates the averages of the productivity growth rates for each country. Only Jordan shows a negative average productivity growth rate. Additionally, as expected, Israel has the highest average of the productivity growth rate.

The Annual percentage change measures of technical efficiency change, technical change, and TFP change are averaged across countries and then converted into cumulative percentage change measures and then plot in figure 4. As can be seen from the plot, after the 1990 the cumulative percentage change measure of TFP change has been positive. Additionally, from the results, the direct average of annual TFP Change value is 61.7/35= 1.763%. A smoothed measure of this average is calculated by running a regression of TFP change upon a time trend. The estimated time trend coefficient is 1.854, indicating an average annual TFP change of 1.854% per year.

Figure 5: Averages of the productivity growth rates during 1973-2006

Figure 6: Cumulative Percentage Change Measures of TEC, TC, TFP change using DEA

INEFFICIENCY MODEL RESULTS

The Maximum Likelihood Estimates of the stochastic frontier model are presented in table 4. With the exception of the negative estimate of labor, the signs of the coefficients of the stochastic frontier production function are as expected. The negative elasticity for labor could be due to the fact that labor is already used extensively and it shall be replaced by machinery. The estimated coefficients for the land and livestock variables, 1.016 and 0.4, respectively, are highly significant, while the coefficient of fertilizers is relatively small but significant. The coefficient of time indicates that the value of output tended to increase by a small but statistically significant rate over the 35 years. The estimated coefficients in the inefficiency model have a great importance. The coefficients of “Former French Colony,” “War,” “1 type 2 conflict,” “1 type 3 conflict,”
“2 type 3 conflict,” and “type 4 conflict” are non-significant at the 5% level. The coefficients of “2 type 2 conflict,” “Free,” Parly Free,” “Not Free,” are negative and the estimate for the coefficient of “Free” is the highest which imply that during the periods in which countries are classified as free, those countries tend to be less inefficient. Additionally, the positive estimate of the coefficient of “Illiteracy” indicates that as the illiteracy rate gets higher in a country, the more inefficient the country becomes. The estimate for the variance parameter, $\gamma$ is equal to 1 indicating that the inefficiency effects are likely to be highly significant in the analysis of the value of output of the countries.

**IMPLICATIONS FOR THE FUTURE**

The focus of this study is agricultural growth of four countries using pooled data in order to construct the output Malmquist indexes. A model for technical inefficiency effects that incorporates input quality, political freedom, institutional environment, conflicts and wars is estimated and will help to provide better explanations to the main factors affecting the inefficiency. This study is expected to be very helpful to agricultural policy makers and decision makers. Other variables related to rule of law, expenditure in agricultural research and development could not be found for the four countries for the whole period. Those two variables are to be considered in future research as they may play an important role in the inefficiency model.

$^4$ The author adopts a similar point of view to the one adopted by the Food and Agriculture Organization (FAO) of the United Nations “The designation employed and the presentation of materials in FAOSTAT do not imply the expression of any opinion whatsoever concerning the status of any country, territory, city or area or its authorities, or concerning the delimitation of its frontiers or boundaries” (FAO).

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**REFERENCES**


APPENDICES

Table 7: Studies of inter-country agricultural productivity that include the 4 countries of interest, 1993-2009

<table>
<thead>
<tr>
<th>Paper</th>
<th>Method</th>
<th>Original and Main source of data used</th>
<th>Countries of interest included separately</th>
<th>Years covered</th>
<th>Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Craig et al. (1997)</td>
<td>CD</td>
<td>FAO</td>
<td>Israel and Syria</td>
<td>1961-1990</td>
<td>98</td>
</tr>
</tbody>
</table>

* Monthly statistical Bulletin, Ministry of Agriculture, Ministry of Labor, Statistical Yearbook, National Account Studies of the United Nations Economic and Social Commission of Western Asia region and other individual sources. a Cobb-Douglas b Data Envelopment Analysis c Translog d United States Department of Agriculture e World Development Indicators.
### Table 8: Summary statistics for the data in the data set

<table>
<thead>
<tr>
<th>Conventional inputs &amp; output</th>
<th>Unit</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>St. deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>US$1000</td>
<td>147463.50</td>
<td>168739.97</td>
<td>505679.48</td>
<td>1219943.01</td>
</tr>
<tr>
<td>Fertilizer</td>
<td>metric tons</td>
<td>96839.21</td>
<td>2500.00</td>
<td>417100</td>
<td>106291.36</td>
</tr>
<tr>
<td>Labor</td>
<td>1,000 labors</td>
<td>385.73</td>
<td>33.20</td>
<td>1731.14</td>
<td>511.93</td>
</tr>
<tr>
<td>Land</td>
<td>1,000 hectares</td>
<td>1854.24</td>
<td>267.11</td>
<td>6711.76</td>
<td>2517.99</td>
</tr>
<tr>
<td>Livestock</td>
<td>No. Of Cattle Equivalents</td>
<td>1183986.05</td>
<td>145568.75</td>
<td>4180370</td>
<td>1051752.49</td>
</tr>
<tr>
<td>Machinery</td>
<td>Number of Tractors in use</td>
<td>23534.64</td>
<td>2850</td>
<td>107946</td>
<td>27836.27</td>
</tr>
</tbody>
</table>

### Table 9: The Malmquist index summary of country geometric means

<table>
<thead>
<tr>
<th>Country</th>
<th>Efficiency Change</th>
<th>Technical Change</th>
<th>Total Factor Productivity Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Israel</td>
<td>1</td>
<td>1.029</td>
<td>1.029</td>
</tr>
<tr>
<td>Jordan</td>
<td>1</td>
<td>1.002</td>
<td>1.002</td>
</tr>
<tr>
<td>Lebanon</td>
<td>1</td>
<td>1.021</td>
<td>1.021</td>
</tr>
<tr>
<td>Syria</td>
<td>0.985</td>
<td>1.004</td>
<td>0.989</td>
</tr>
<tr>
<td>Mean</td>
<td>0.996</td>
<td>1.014</td>
<td>1.01</td>
</tr>
</tbody>
</table>

### Table 10: The Maximum-Likelihood Estimates of the Stochastic Frontier Model

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard-error</th>
<th>t-ratio</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_t$</td>
<td>0.166</td>
<td>0.044</td>
<td>3.759</td>
</tr>
<tr>
<td>$\beta_A$</td>
<td>0.076</td>
<td>0.021</td>
<td>3.621</td>
</tr>
<tr>
<td>$\beta_N$</td>
<td>-0.859</td>
<td>0.076</td>
<td>-11.355</td>
</tr>
<tr>
<td>$\beta_T$</td>
<td>1.016</td>
<td>0.062</td>
<td>16.394</td>
</tr>
<tr>
<td>$\beta_M$</td>
<td>0.400</td>
<td>0.040</td>
<td>10.049</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>0.221</td>
<td>0.025</td>
<td>8.852</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.011</td>
<td>0.002</td>
<td>4.603</td>
</tr>
<tr>
<td>$\lambda_{AA}$</td>
<td>0.125</td>
<td>0.025</td>
<td>4.954</td>
</tr>
<tr>
<td>$\lambda_{AB}$</td>
<td>-0.144</td>
<td>0.025</td>
<td>-5.840</td>
</tr>
<tr>
<td>$\lambda_{AC}$</td>
<td>0.116</td>
<td>0.044</td>
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<td>$\lambda_{AD}$</td>
<td>0.102</td>
<td>0.044</td>
<td>2.288</td>
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<tr>
<td>$\lambda_{AE}$</td>
<td>-0.174</td>
<td>0.023</td>
<td>-7.639</td>
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<tr>
<td>$\lambda_{BT}$</td>
<td>0.000</td>
<td>0.002</td>
<td>-0.039</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>-0.580</td>
<td>0.096</td>
<td>-6.015</td>
</tr>
<tr>
<td>$\delta_{AC}$</td>
<td>-0.203</td>
<td>0.113</td>
<td>-1.798</td>
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<tr>
<td>$\delta_{AD}$</td>
<td>-0.507</td>
<td>0.139</td>
<td>-3.648</td>
</tr>
<tr>
<td>$\delta_{AE}$</td>
<td>0.703</td>
<td>0.056</td>
<td>12.481</td>
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<td>$\delta_{BT}$</td>
<td>0.023</td>
<td>0.002</td>
<td>9.324</td>
</tr>
<tr>
<td>$\delta_{AC}$</td>
<td>0.552</td>
<td>0.173</td>
<td>3.194</td>
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<tr>
<td>$\delta_{AD}$</td>
<td>0.151</td>
<td>0.159</td>
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<td>$\delta_{AE}$</td>
<td>-0.506</td>
<td>0.065</td>
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<tr>
<td>$\delta_{BT}$</td>
<td>-0.027</td>
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<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard-error</th>
<th>t-ratio</th>
<th>t-ratio</th>
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<tbody>
<tr>
<td>$\frac{1}{2}\beta_{DD}$</td>
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<td>0.135</td>
<td>6.931</td>
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<td>$\beta_{DE}$</td>
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<td>0.059</td>
<td>14.811</td>
</tr>
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<td>0.008</td>
<td>0.003</td>
<td>2.931</td>
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<tr>
<td>$\frac{1}{2}\lambda_{TT}$</td>
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<td>0.000</td>
<td>12.732</td>
</tr>
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<td>-9.534</td>
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A comparative study on energy use efficiency for Potato farms under different farming technologies using Data Envelopment Analysis

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ABSTRACT

The aims of this study were to determine the amount of input-output energy and exploring a non-parametric data envelopment analysis (DEA) technique which permits efficiency estimation to investigate the efficiency of potato farms under different farming technologies in Hamadan province, Iran. The population investigated was divided into two groups, Group I farms owner of machinery and high level of farming technology and Group II non-owner of machinery and low level of farming technology. Technical and pure technical efficiency of potato production was estimated 0.82 and 0.91. Frequency distribution of technical and pure technical efficiency represented farmers in Group I was more efficient. Separate analysis of groups and technical efficiency also followed same results for Group I, farms with high farming technology are more efficient and waste less source of energy. Present and target use of energy and energy saving of inefficient farms calculated. The results reveal that, on an average, in Group I and II about 34% and 49% of the total input energy could be saved respectively if the farmers follow the input package recommended by the study and farms with high technology can decrease waste of energy and costs and increase productive efficiency. Keywords: Potato, Productive efficiency, Data envelopment analysis, Farming technologies

INTRODUCTION

Potatoes (Solanum tuberosum L.) are grown worldwide under a wider range of altitude, latitude, and climatic conditions than any other major food crop-from sea level to over 4000 m elevation. No other crop can match the potato in its production of food energy and food value per unit area [1]. Potatoes have become increasingly important in the developing world for both sustenance and income. The United Nations called 2008 International Year of the potato in order to attract consideration of governments to plant this important crop which has a significant role to decrease hunger of people all over the world [2]. Recent publications have shown the importance of the potato as a global food crop, ranking fourth among other crops with an overall annual production of nearly 327 million tones and about 19 million hectares planted. In developing countries, production had actually doubled in the past 15 years, so that, for the first time, over 50 percent of the crop was grown in the developing world. Consumption of this crop in European countries has a decreasing trend while in developing countries has an increasing rate, per capita consumption from 10 kg in 1961 reached to 22 kg in 2003, however consumption of potato in developing countries is 25% of its consumption in Europe yet. China, Russian Federation, India, United States, Ukraine and Germany are the main potato producer countries. The potato is also cultivated in Poland, Netherlands, France, United Kingdom, Iran, and Canada [3]. Based on FAO statistics, 350 million tons of potatoes were now consumed worldwide each year, and Iran produced about 5 million tones of potatoes in 176,000 ha in year 2008. The province of Hamadan is the main production area in Iran, producing more than 20% of which is produced in Iran; therefore, it is an important potato production center. Potatoes are the single most important agricultural commodity in Hamadan province [4].

The present study explores Data Envelopment Analysis (DEA) technique which permits efficiency estimation of potato farms without assuming an a priori functional form for frontier production.
MATERIAL AND METHODS

Farms were randomly chosen from the villages in the area of study. The size of sample of any stratification technique, [5]. In this study potato farms divided into two groups which each group has a different level of production technology and also different farm machinery ownership status and each group consists of 25 farms. It also identifies operations where energy savings could be realized by changing current practices in order to reduce energy consumption and increasing efficiency for potato production. The data included hours or amount of inputs used from effective energy sources: human labor, machinery, diesel fuel, chemicals, farmyard manure and the yield as output. The data transformed to energy term by appropriate energy equivalents of inputs and output in agricultural production (Table 1). Table 2 shows the basic statistics of strawberry output and major inputs used. The most energy consuming inputs for potato production in the different farming technologies investigated were fertilizers (50895MJ ha$^{-1}$) and electricity (50596MJ ha$^{-1}$).

Agricultural production relies on finite and scarce resources; therefore the use of input oriented DEA models is more appropriate to reduce inputs consumed in the production process [15, 16]. Here DEA is used for the estimation of resource use efficiency and ranking of farms or production units on the basis of their performances. Production units are termed Decision Making Units (DMUs) in DEA terminology. This technique can also be used to identify the level as well as sources of inefficiency. Technical efficiency (TE) is defined as the DMU’s ability to achieve maximum output from given inputs, while allocative efficiency is defined as the DMU’s success in selecting inputs in optimal proportions with respect to price [16]. The projection of inefficient DMUs on the efficient frontier depends on the model orientation. Since its inception in 1978, a number of DEA models have been developed, including additive models, output-oriented models and ranking techniques [17]. In the agricultural sector a farmer has more control over inputs rather than output levels, which may often be exogenously bounded (e.g., soil fertility, climate, etc.), this study is centered on input-oriented models. For each inefficient DMU, target input and output levels have to be prescribed. These targets are the results of respective slack values added to outputs.

CRS implies that a given increase in inputs would result in a proportionate increase in outputs and the feasible region of the envelopment problem becomes a conical hull. A restriction on efficiency leads to no condition on the allowable returns to scale, also called variable returns to scale (VRS). Under this condition, the performance frontier line is not then restricted to pass through the origin, and an increase in inputs may not result in a proportionate increase in outputs in this case [18]. Due to convexity, the efficient DMUs form a convex hull on which all inefficient points are projected. Because the VRS is more flexible and envelops the data in a tighter way than the CRS, the score or pure TE (VRS) is equal to or greater than the CRS or overall TE score (CRS).

The technical efficiency can be expressed by the ratio of sum of the weighted outputs to sum of weighted inputs [19]. In 1984, Banker, Charnes and Cooper introduced a model in DEA, which was called BCC model to draw out the technical efficiency of DMUs [20]. With respect to technical efficiency (in CCR model), technical efficiency of BCC model, which is called Pure Technical Efficiency, could separate both technical and scale efficiencies.

According to Cooper et al. [banker] the technical efficiency can be defined by:

\[
\text{Technical Efficiency} = \text{Pure Technical Efficiency} \times \text{Scale Efficiency}
\]

This decomposition shows the sources of inefficiency. The pure technical efficiency deals with efficiency of energy use. In other words, it states that how much percent of energy has contributed on the production applied. Scale efficiency shows the effect of DMU size on efficiency of system. Simply, it indicates that some part of inefficiency refers to inappropriate size of DMU, and if DMU moved toward the best size the overall efficiency (technical) can be improved at the same level of technologies (inputs). Technical efficiency and pure technical efficiency have been calculated by using DEA Solver Professional Release 4.1.

RESULTS AND DISCUSSION

Frequency distribution of technical and pure technical efficiency represented farmers in Group II had less efficiency than Group I (Table 3). The data of pure technical efficiency showed farmers in Group I were more BCC-efficient as well, and 19 farms of 25 could shift on BCC frontier. It means, except 6 inefficient farmers, all farmers technologically had efficiency. The data of pure technical efficiency showed just 10 farmers of Group II were efficient. In other words, farmers in Group II had considerable use of energy and production in the yield. Technical and pure
technical efficiency of potato production was estimated 0.82 and 0.91.

Using BCC model, the pure technical efficiency of a DMU is measured relative to an efficient frontier at the same scale size. BCC is modeled by setting the convexity constraint. In this case, the scale efficiency is determined by measuring the divergence between the actual scale size and the most productive scale size.

Separate analysis of groups and technical efficiency also followed same results for Group I, farms with high farming technology are more efficient and waste less source of energy. The average technical efficiency provides information about the potential resource savings that could be achieved while maintaining the same output level. Average technical efficiency score were calculated 96% for group I and 84% for group II.

For each inefficient farm, target input and output levels have to be prescribed. These targets are the results of respective slack values added to outputs. Table 4 shows energy saving from different sources if recommendations of study are followed. Using the information of Table 4, it is possible to advise an inefficient farmer regarding the better operating practices followed by his peers in order to reduce the input energy level to the target values indicated in the analysis while achieving the output level presently achieved by him.

Analysis showed that in Group I, 18 farms were efficient and in Group II, 9 farms were efficient. The most share of energy saving were in electricity 56% and chemicals 51% in Group I, farm yard manure 86% and fertilizer 66% in Group II. Contribution share of each input for Group I and II is compared in Figure 1. Based on Fig. 1 considerable waste of energy in Group II demonstrated the good potential of energy saving in this part of potato producers.

CONCLUSIONS

In this study, the population investigated was divided into two strata based on tractor and farm machinery ownership and levels of farming technology. Group I which was the owners of machinery and practiced high level of farming technology and Group II which was non-owners of machinery and exercised low level of farming technology. Energy use of inputs and output in potato production in Hamadan province of Iran were determined. The most energy consuming inputs for potato production in the different farming technologies investigated were fertilizers (50895MJ ha⁻¹) and electricity (50596MJ ha⁻¹).

Data envelopment analysis was done in order to investigate the efficiency of 2 groups of farms. Total data were tested in both CCR and BCC model, result showed in different efficiency scores, farms in Group I are more efficient. Technical and pure technical efficiency of potato production was estimated 0.82 and 0.91.

Technical efficiency, target energy use and energy saving of inefficient farms in groups calculated separately. Waste energy in Group I & II determined 34% & 49% respectively. The most waste of energy in farms Group II were farm yard manure and fertilizer and electricity and chemicals in farms Group I.

Energy management is an important issue in terms of efficient, sustainable and economic use of energy. It can be expected that all these measurements would be useful not only for reducing negative effects to environment, human health, maintaining sustainability and decreasing production costs, but also for providing higher productive efficiency.

REFERENCES


---

**Figure 1:** Comparison between Group I and II in the case of contribution input to energy saving

**Table 1:** Energy equivalent of inputs and output in agricultural production

<table>
<thead>
<tr>
<th>Particulars</th>
<th>Un</th>
<th>Energy (MJ unit)</th>
<th>Referenc</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Inputs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Human labor</td>
<td>h</td>
<td>1.96</td>
<td>[6]</td>
</tr>
<tr>
<td>2. Machinery</td>
<td>h</td>
<td>62.7</td>
<td>[6]</td>
</tr>
<tr>
<td>3. Diesel fuel</td>
<td>L</td>
<td>56.31</td>
<td>[6]</td>
</tr>
<tr>
<td>(b)</td>
<td>Pota</td>
<td>12.44</td>
<td>[7]</td>
</tr>
<tr>
<td>(c)</td>
<td>Fun</td>
<td>11.15</td>
<td>[8]</td>
</tr>
<tr>
<td>5. Farmyard</td>
<td>kg</td>
<td>0.30</td>
<td>[9]</td>
</tr>
<tr>
<td>6. Electricity</td>
<td>kW</td>
<td>11.93</td>
<td>[10]</td>
</tr>
<tr>
<td>7. Chemicals (a)</td>
<td>Inse</td>
<td>101.20</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Her</td>
<td>238.00</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Fun</td>
<td>216.00</td>
<td></td>
</tr>
<tr>
<td>8. Water</td>
<td>m³</td>
<td>1.02</td>
<td>[12]</td>
</tr>
<tr>
<td>9. Seeds (potato)</td>
<td>kg</td>
<td>3.6</td>
<td>[13]</td>
</tr>
</tbody>
</table>

**B. Outputs**

10. Potato | kg | 3.6 | [14] |

---

**Table 2:** Basic statistics of strawberry output and major inputs used

<table>
<thead>
<tr>
<th>Item</th>
<th>Max</th>
<th>Min</th>
<th>Average</th>
<th>Standard</th>
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<td>Diesel</td>
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<tr>
<td>Water</td>
<td>21481.2</td>
<td>1028.16</td>
<td>7860.79</td>
<td>187.8159</td>
</tr>
</tbody>
</table>

**Output (kg)**

| Yield | 75000.0 | 20000.0 | 42120.0 | 12785.37 |

---

**Table 3:** Frequency distribution of technical and pure efficiency of potato farmers under different groups

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>1</td>
<td>1</td>
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</tr>
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<td>G</td>
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<td>8</td>
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<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>2</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

| CC | 0.8 |
| CC | 0.9 |

---

**Table 3:** Frequency distribution of technical and pure efficiency of potato farmers under different groups

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>4</td>
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<tr>
<td>BC</td>
<td>G</td>
<td>2</td>
<td>8</td>
<td>4</td>
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<tr>
<td></td>
<td>T</td>
<td>2</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

**CC | 0.8**

**BC | 0.9**
Table 4: Energy saving (MJ ha\(^{-1}\)) from different sources if recommendations of study are followed

<table>
<thead>
<tr>
<th>Group</th>
<th>Input</th>
<th>Present Use (MJ ha(^{-1}))</th>
<th>Target Use (MJ ha(^{-1}))</th>
<th>Energy Saving (MJ ha(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Human</td>
<td>1015.82</td>
<td>848.58</td>
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<tr>
<td></td>
<td>Machinery</td>
<td>2598.07</td>
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<td></td>
<td>Diesel</td>
<td>25738.57</td>
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<tr>
<td></td>
<td>Fertilizer</td>
<td>49145.94</td>
<td>27862.34</td>
<td>21283.6</td>
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<td></td>
<td>Farmyard manure</td>
<td>3262.5</td>
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<td>Electricity</td>
<td>100264.1</td>
<td>43673.8</td>
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<td></td>
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<td>15480</td>
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<td>Chemical</td>
<td>808.5</td>
<td>392.9</td>
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<td>10625.85</td>
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<td>Total input energy (MJ ha(^{-1}))</td>
<td>3157755</td>
<td>1590956</td>
<td>1566799</td>
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<table>
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<th>Group II</th>
<th>Input</th>
<th>Present Use (MJ ha(^{-1}))</th>
<th>Target Use (MJ ha(^{-1}))</th>
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<td>77310.41</td>
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<td>Farmyard manure</td>
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<td></td>
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</tr>
<tr>
<td></td>
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<td>7051.847</td>
<td>4508.47</td>
<td>3193.7</td>
</tr>
<tr>
<td></td>
<td>Total input energy (MJ ha(^{-1}))</td>
<td>170551.6</td>
<td>83656.94</td>
<td>86894.7</td>
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Efficiency and environmental pressures of Farrow-to-finish and finishing pig farms in Hungary

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ABSTRACT

Pig farming is one of the strongest polluter of water resources in developed countries, due to its intensive production techniques and slurry rejection. Several countries have already introduced environmental regulations aiming at reducing nitrate and phosphorus waste from pig farming, but not yet Hungary. This paper aims at shedding light on how Hungarian pig farms would be affected if such regulations are to be introduced in the country. We calculate farm technical efficiency separately for farrowing only and farrow-to-finish farms (FAFI farms), and finishing only farms (FI farms), using farm data relating to pig activity in 2001 and Data Envelopment Analysis (DEA). We then investigate whether environmental regulations would affect Hungarian pig farms’ technical efficiency, with the help of a second-stage regression and various environmental pressures. Results indicate that the level of nitrogen produced by livestock per hectare of agricultural land is still much lower than the potential authorised limit that could be introduced, and that farmers have incentives to reduce such pollution in their region in order to increase their efficiency even in the absence of regulation.

Keywords: technical efficiency, DEA, pig farms, nitrate pollution, Hungary

INTRODUCTION

Pig farming is one of the strongest polluter of water resources in developed countries, due to its intensive production techniques and slurry rejection. Several countries have already introduced environmental regulations aiming at reducing nitrate and phosphorus waste from pig farming. For example, in Taiwan the government introduced in 1987 a law aiming at limiting the level of waste from pig farms (Yang et al., 2009). In the European Union (EU) since 1991 the Council Directive 91/676/EEC (referred hereafter to as the EU Nitrate Directive) aims at promoting the protection of waters against pollution caused by nitrates from agricultural sources. Besides other prescriptions, this EU regulation requires that, for each farm, the amount of nitrogen produced by livestock applied to the land each year shall not exceed a specified amount per hectare (i.e. 170 kg N). In France, livestock farms can proceed to land application either on their own land or on land of other farms (Piot-Lepetit and Le Moing, 2007; Larue and Latruffe, 2009). In the Netherlands farms are required to reduce their pollution caused by nitrates from agricultural sources since 1998 and can adopt the Green Label systems, which are certified less nitrogen polluting pig production systems (Oude Lansink and Reinhard, 2004).

Whether such regulations modify pig producers’ decisions regarding their localisation, production scale and input use has been recently investigated in the literature. For example, Larue et al. (2008) show that the EU regulation regarding nitrogen applied to land had a negative impact on pig farms’ concentration in Denmark, and that the dispersion effect is more pronounced in 2004 than in 1999. Piot-Lepetit and Le Moing (2007) provide evidence of a positive relationship between farms’ technical efficiency and environmental regulation in the French pig sector during 1996-2001, suggesting that pig producers changes introduced changes in their production process. Larue and Latruffe (2009) confirm this finding with data from 2004, but suggest in addition that the reverse may be obtained (i.e. a
decrease in technical efficiency) if the environmental regulation is too stringent that it forces pig producers to spread their manure on land that is far from their farm. By contrast, in Taiwan Yang et al. (2008) do not find a clear-cut effect of the 1987 environmental law on pig farms’ technical efficiency in 2003-2004.

This paper investigates how environmental pressures, focusing on nitrate production, may affect the efficiency of pig farms in Hungary. Despite its accession to the EU in 2004, the EU regulation governing livestock waste pollution is not yet enforced in this country. This paper therefore aims at shedding light on how Hungarian pig farms would be affected if such regulations are to be introduced in the country.

The paper is structured as follows. The second section introduces the methodology and data. The third section describes the results, and the last section concludes.

**METHODOLOGY AND DATA**

Nitrates are an undesirable output of pig activity that is to say an output that is socially undesirable due to its negative externalities, in particular air and water pollution (Oude Lansink and Reinhard, 2004). In efficiency analysis, undesirable outputs may be modelled as ‘bad’ outputs that is to say as inputs, under the assumption of either strong, respectively weak, disposability (i.e. assuming either that it is not costly, respectively costly, to reduce them), or they can be included as ‘good’ outputs by using in a first stage a transformation function (Yang et al., 2008). In this paper, we consider nitrogen from pig activity sources as a strongly disposable input.

We use farm-level data extracted from a specific survey of pig producers in Hungary in 2001. The total sample includes 192 farms. Farms are separated into two groups based on their specialisation: farrowing only and farrow-to-finish farms (FAFI farms, 140 farms), and finishing only farms (FI farms, 52 farms). Considering that technologies differ between these two specialisations, a frontier is constructed for each group separately, using Data Envelopment Analysis (DEA). Inputs and outputs used in the DEA model relate only to the farms’ pig activity.

Firstly, technical efficiency is calculated without accounting for waste emissions. For FAFI farms, the two outputs used are the number of piglets sold and the number of pigs fattened on farm, while the five inputs include the number of piglets and pigs purchased the number of sows, the number of labour hours spent on the porcine activity, the values of feed, and other costs. For FI farms, the single output is the number of pigs fattened on farms, while the four inputs are similar to the inputs for FAFI farms except that the number of sows is not included. Next, total technical efficiency is calculated again for both types of farms, with the inclusion of an additional input, namely the quantity of nitrogen produced. Nitrogen production from pig activity is quantified here with the method of measurement applied by French Authorities to enforce the EU Nitrate Directive; each pig head is assigned a nitrate-equivalent production coefficient depending on its type (sow, swine, piglets etc), representing the nitrogen quantity produced per year (CORPEN, 2003). Table 1 provides descriptive statistics of the data used in the DEA models.

The quantity of nitrogen emitted by FAFI farms is much higher on average than the one emitted by FI farms: 3,515 against 283 kg N.

We then investigate the role of several factors on the technical efficiency of both types of pig farms using a second-stage regression with ordinary least squares. A single equation is estimated, that is to say both samples (FAFI and FI farms) are merged together. The explanatory variables are characteristics of the farms in the sample from the specific farm survey, and regional data extracted from various surveys in Hungary including Farm Structures’ Surveys. The level of the regions considered is the NUTS2 level from the European classification of Nomenclature of Units for Territorial Statistics (NUTS). In Hungary there are seven NUTS2 regions. Several variables were tested in the model. Table 2 provides descriptive statistics of the variables retained for the final specification. The farms’ characteristics are their total utilised agricultural area (UAA) in hectares (ha) proxying the land availability to spread the manure (a positive effect is expected); their total number of livestock units proxying the production of the nitrogen (a negative effect is expected); the share of the farms’ revenue stemming from the pig activity to proxy farm specialisation vs. diversification (no expectation on the sign of the impact); and the share of the farm’s revenue stemming from subsidies to proxy the role of public support (a negative impact is expected). Regarding regional data, the agriculture area with cereals proxies the availability of pig feed (a positive effect is expected). The population and the level of GDP per capita are introduced as proxies for environmental pressures, as inhabitants are disturbed by emissions and may press local governments for regulation (thus a positive sign is expected). The number of pig farms in the region where the farm is located is also used as a proxy for environmental pressures.
pressures, as more farms imply more manure production (a positive sign is expected). Finally, the role of environmental pressure on farms’ technical efficiency is also analysed with the introduction of the total level of nitrogen produced by livestock in the region (calculated from CORPEN, 2003) as a ratio per hectare of UAA in the region where the farm is located. The average quantity of nitrogen per UAA hectare is only 17.5 kg, which is still very far from the possible pollution limit that could be introduced in Hungary (for example the limit in France is 170 kg per hectare).

RESULTS
Table 3 shows the averages of efficiency scores obtained with the DEA models. Results indicate that the total technical efficiency (assuming constant returns to scale) is higher on average for FAFI farms than for FI farms: 0.553 against 0.423, the difference being tested significantly different from zero at one percent. This suggests that the farrowing activity alone or combined with the finishing activity is more technically efficient than the finishing activity alone. However, this conclusion holds when pollution from the pig activity is not considered. Indeed, FAFI farms may be more efficient, but, on the other hand, they have a higher number of pig heads, and produce more waste as shown by Table 1. Indeed, when total technical efficiency is calculated again for both types of farms with the inclusion of an additional input, namely the quantity of nitrogen produced, results differ from the efficiency results obtained when not accounting from nitrate pollution. When nitrate waste is considered, the mean total technical efficiency of FAFI farms is only slightly higher than that of FI farms, 0.568 against 0.546, the difference not being significantly different from zero. These figures suggest that both types of farms could reduce their nitrate pollution by more than 40 percent and still produce the same output level. Thus, if environmental regulations are introduced in Hungary, they may not affect the level of pig production in the country, as there is a substantial room for pollution reduction keeping the pig output constant.

However, environmental regulations may affect Hungarian pig farms’ technical efficiency. This issue is investigated with the help of the second-stage regression. Table 4 provides the econometric results of this regression. It is firstly worth noting that none of the farms’ specific characteristics have a significant effect on their technical efficiency, by contrast to regional variables. The agricultural area under cereals has a positive effect on farms’ technical efficiency as expected, indicated that the closeness of the upstream market is important for pig producers. Regarding the environmental proxies, they all have the expected effect. The population and income have a positive significant coefficient, suggesting that highly populated and richer areas put pressures on the producers to become more efficient. This neighbourhood effect has also been given evidence in France by Larue and Latruffe (2009). The regional density of pig farms as well as the regional quantity of nitrogen produced per hectare have a negative impact on farms’ technical efficiency, conform to the expectation. This finding suggests that there may be some congestion effects or some competition for land.

CONCLUSION
This paper has analysed the relationship between the technical efficiency of farms specialised in pig production and the environmental pressures they are facing or may face in the future in Hungary. Pig farms’ technical efficiency was calculated with pig activity data, including the quantity of nitrogen produced by livestock as a strongly disposable undesirable output. It was then regressed on several farms’ characteristics and on variables specific to the region where each farm is located. Results indicated that neighbourhood pressures regarding environmental pollution increased farms’ technical efficiency while congestion problems due to a high pig farm density and a large regional nitrogen production reduced the efficiency.

If the EU regulation governing livestock activities were to be applied in Hungary, pig producers would have to spread their manure on a minimum land area so that the total quantity of nitrogen produced by livestock was less than 170 kg per hectare of UAA (if the pollution limit applied is the same as in France for example). In this context, our findings suggest that Hungarian pig farmers’ technical efficiency would not decrease. Firstly, the level of nitrogen per hectare is still much lower than the potential authorised limit (namely 17.5 kg on average), and therefore there would still be room for manoeuvre. Secondly, the econometric regression revealed a negative effect of the nitrogen quantity per hectare on the farms’ technical efficiency, indicating that

REFERENCES


**Table 1: Descriptive statistics of the farm-level data used in the DEA models (averages)**

<table>
<thead>
<tr>
<th></th>
<th>Farrow-to-finish (FAFI) farms 140 farms</th>
<th>Finishing only (FI) farms 52 farms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Desirable outputs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of piglets sold</td>
<td>3,052</td>
<td>99</td>
</tr>
<tr>
<td>Number of pigs fattened on farm</td>
<td>99</td>
<td>485</td>
</tr>
<tr>
<td><strong>Undesirable output</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantity of nitrogen produced (kg N)</td>
<td>3,515</td>
<td>283</td>
</tr>
<tr>
<td><strong>Inputs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of sows</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>Number of piglets and pigs purchased</td>
<td>1,759</td>
<td>79</td>
</tr>
<tr>
<td>Labour spent on pig activity (hours)</td>
<td>4,445</td>
<td>2,030</td>
</tr>
<tr>
<td>Pig feed (euros)</td>
<td>22,039,746</td>
<td>3,965,781</td>
</tr>
<tr>
<td>Other costs for pig activity (euros)</td>
<td>35,492,699</td>
<td>4,539,934</td>
</tr>
</tbody>
</table>

**Table 2: Descriptive statistics of the data used in the second-stage regression (averages)**

<table>
<thead>
<tr>
<th></th>
<th>Farrow-to-finish and finishing only farms together 192 farms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample’s farms’ UAA (ha)</td>
<td>190.9</td>
</tr>
<tr>
<td>Sample’s farms’ total number of livestock units</td>
<td>2,634</td>
</tr>
<tr>
<td>Sample’s farms’ share of revenue from pig activity in farm revenue (%)</td>
<td>40.7</td>
</tr>
<tr>
<td>Sample’s farms’ share of subsidies in farm revenue (%)</td>
<td>7.9</td>
</tr>
<tr>
<td>Regional area with cereals (ha)</td>
<td>397,060</td>
</tr>
<tr>
<td>Regional population (inhabitants)</td>
<td>1,235,725</td>
</tr>
<tr>
<td>Regional GDP per capita (euros)</td>
<td>1,022</td>
</tr>
<tr>
<td>Number of pig farms in the region</td>
<td>125,710</td>
</tr>
<tr>
<td>Nitrogen quantity per ha of UAA in the region (kg)</td>
<td>17.5</td>
</tr>
</tbody>
</table>

**Table 3: Descriptive statistics of the efficiency scores (averages)**

<table>
<thead>
<tr>
<th></th>
<th>FAFI farm - 140 farms</th>
<th>FI farms- 52 farms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without the undesirable output</td>
<td>0.555</td>
<td>0.423</td>
</tr>
<tr>
<td>With the undesirable output</td>
<td>0.568</td>
<td>0.546</td>
</tr>
</tbody>
</table>

**Table 4: Results of the second-stage regression**

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>P-value and significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-3.61</td>
<td>0.008 ***</td>
</tr>
<tr>
<td>Sample’s farms’ UAA</td>
<td>2.83 E-05</td>
<td>0.373</td>
</tr>
<tr>
<td>Sample’s farms’ total number of livestock units</td>
<td>-1.23 E-06</td>
<td>0.317</td>
</tr>
<tr>
<td>Sample’s farms’ share of revenue from pig activity in farm revenue</td>
<td>1.09 E-04</td>
<td>0.830</td>
</tr>
<tr>
<td>Sample’s farms’ share of subsidies in farm revenue</td>
<td>-1.73 E-03</td>
<td>0.356</td>
</tr>
<tr>
<td>Regional area with cereals</td>
<td>5.31 E-06</td>
<td>0.022 **</td>
</tr>
<tr>
<td>Regional population</td>
<td>6.09 E-06</td>
<td>0.004 ***</td>
</tr>
<tr>
<td>Regional GDP per capita</td>
<td>9.15 E-06</td>
<td>0.009 ***</td>
</tr>
<tr>
<td>Number of pig farms in the region</td>
<td>-3.12 E-05</td>
<td>0.011 ***</td>
</tr>
<tr>
<td>Nitrogen quantity per ha of UAA in the region</td>
<td>-1.40 E-01</td>
<td>0.010 ***</td>
</tr>
<tr>
<td>Number of observations</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.258</td>
<td></td>
</tr>
</tbody>
</table>

**, ***: significance at 5-percent, 1-percent level.
Efficiency analysis if polices against desertification by applying DEA: a case study in the river Guadalentin catchment (Almeria, Spain)

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ABSTRACT

This paper deals with Data Envelopment Analysis (DEA) to evaluate the different policies against desertification carried out during a twenty five year period (1978-2003) in the eight municipalities which comprise the river Guadalentin catchment (Murcia, Spain). The approach is also based on the European Environmental Agency indicators to measure the efficiency of applied environmental policies. In the analysis three different objectives have been considered: I, a multi-objective with erosion, human population and landscape diversity losses; II, a bi-objective of erosion and landscape and III, a unique erosion objective.

Applying DEA, different efficiency indexes were obtained for each municipality, which revealed the best policy carried out in the specify period, as well as, the contribution of each action to achieve the goals. Four different DEA ranking were calculated according to DEA methods: self-evaluated input-oriented DEA, peer-evaluated Average Efficiency, peer-evaluated Weighted Average Efficiency and Benevolent Cross-Weighted Efficiency. As a result, the latter method performed better than others making a reasonable preference ranking of municipal policies in all scenarios I, II and III. These results are very valuable to establish future long-term desertification policies in similar territories.

Keywords: Data Envelopment Analysis, policy analysis, Guadalentin catchment, Spain

THE APPROACH TO ANALYSING THE POLICIES

Numerous environmental measures have been taken by the different authorities to alleviate the problem of desertification in south-eastern Spain [0]. In the past national strategy was aimed at repopulating large areas and making terraces to avoid soil losses, but in recent years policies have shifted towards softer actions as changing or abandoning crops, generally financed by European Union schemes [2].

Desertification is a slow and complex process in which many mitigation actions have been taken for years, but their effects are not visible overnight, commonly blurred in a thick web of drivers and factors involved in the process. Otherwise, it is admitted that desertification is a multidimensional process composed of a physical, a biological and a social component. Essential international studies [1, 2, 0 include as main desertification components: soil erosion, deforestation, loss of landscape diversity and human depopulation.

Monitoring the environmental system and the involved factors required a set of indicators which reflect their changes along time 0.

In this study, “policies against desertification” were analysed in terms of physical variables such as the area reforested, the area subject to soil conservation practices, the reduction in the dryland farming area, the increase in irrigated farmland and the land declared as a protected natural area [5, 6].

Although many physical data were obtained in a small spatial resolution and managed with a geographical information system (GIS), other measure indicators could only be collected at council level. Consequently, the municipality was selected as minimum working unit or decision maker unit (DMU) each one featured by two vectors, one related to the variations of the desertification status indicators or outputs (y) and other related to conducted actions or inputs (x), and the aim was to find the DMUs which achieved the best mixture of taken measures to improve the desertification situation.
LOCATION AND GEOGRAPHY
The study took in the agricultural area around the river Guadalentín, one of six which make up the region of Murcia, covering an area of 3,096.4km² which is divided into eight municipalities: Aguilas, Aledo, Alhama de Murcia, Librilla, Lorca, Mazarrón, Puerto Lumbreras and Totana. The aquifers in the region are extensive, holding large reserves; however a generalised over-exploitation has been applied in the last decades 0.

The soils are little evolved, with few horizons, and are difficult to differentiate. Their depth, physical and chemical features are determined by the type of geological strata [6].

The vegetation in the region consists of non-deteriorated scrub, retreating scrub, pinewoods with juniper. In highlands and foothill areas, holm oaks and Mediterranean scrub may be found. Lorca and Puerto Lumbreras have extensive areas with dryland farming, scrub and fields of esparto grass, with some pinewoods in the hills. There is irrigated farmland on the Guadalentín plain. To the east (Mazarrón) aquatic ecosystems dominate, with wetlands, salt lagoons, salt marshes and salt flats. In the hills of the Sierra de Carrascoy, Mediterranean holm oak woods can be found. Around Totana cabbage palm is plentiful. In the Sierra Espuña stands of gall oak, terebentin, maple and strawberry tree can be seen 0.

SOURCES OF DATA AND INDICATORS
A system of indicators must be based on variables which are easy to measure and interpret [4]. The sources of information used to produce the indicators were the following:

Vegetation: Digitized maps were obtained from the maps of crops and exploitation of 1978 and 2003 [6, 7].

Erosion: Use was made of the information contained in the National Soil Erosion Inventory to obtain two maps of laminar erosion: one dated in 1978 and the other in 2003 [6, 7].

Diversity: The Shannon-Wiener index was taken as an indicator of structural diversity of landscape [7].

Population: The information collected by the years 1978 and 2003 was obtained from the INE population survey [8].

Reforestation: The information about reforestation and soil conservation practices was obtained from the staff of the regional and central governments and the documentation concerning their proposals [6, 7].

In conclusion, the indicators used to measure the state of desertification were the following:

- (y₁) Erosion: Decrease in laminar erosion (ÅTn/Ha)
- (y₂) Landscape diversity: Increase in Shannon index (ΔIndex/Ha)
- (y₃) Populations: Increase in human density (Aperson x 100/Ha)

The indicators of measures taken were the following:

- (x₁) Repopulated forested area (% of forested area)
- (x₂) Land area under corrective soil practices and water management projects not reforested (% of area managed)
- (x₃) Abandonment of dryland farming (% of dryland crops)
- (x₄) Establishment of irrigated farming (% of irrigated crops)
- (x₅) Land area protected as natural areas (% of area protected).

The description of taken measures during 1978 to 2003 and desertification situation by municipalities are summarized in table 1.

DEA APPLIED TO THE ASSESSMENT OF MEASURES TO PREVENT DESERTIFICATION.

DEA (Data Envelopment Analysis) is an application of multi-criterion linear programming [0] which seeks to optimise the ratio between results and resources. First applications of DEA were focused to identify efficient points in industrial cases where the objective function was composed of goals of a “more the better” nature in combination with resources of a “less the better” nature. However, step by step this former method was extending its applications from the former cost-benefit assessments [0] to not-profit organization efficiency assessments such as schools, universities and hospitals [0,0]. Nowadays DEA is a technical paradigm applied in many sectors and aims 0.

In our case-study, the objective was to obtain the efficiency scores of municipal policies and to reach a preference ranking of them, following the approach of Sexton [14], Doyle, Green and Cook [15, 16].

For getting DEA efficiency scores is necessary to maximise the ratio between a linear combination of output vector (y) and input vector (x) with both loading vectors (u) and (v). Output vector is composed by different indicators relative to different aspect of achieved results (y₁, y₂, y₃) and input vector is set up with indicators about features of activities related to conducted policies (x₁, x₂,
x_3, x_4, x_5). All these values reflected the increment achieved during the application period 1978-2003 and were obtained by difference of status indicator values. In addition, all indicators were scaled to the size of the DMU and shifted to be positive.

The basic mathematical formulation (Eq.1) of DEA-CCR can be expressed as:

Max \( \theta = \frac{\sum u_i \cdot y_i}{\sum v_i \cdot x_i} \) \quad \forall i = 1,2,...,n

(subject to \( \frac{\sum u_i \cdot y_i}{\sum v_i \cdot x_i} \leq 1 \)

and \( u_i, v_i \geq 0 \)  \( j = 1,2,...,m; k = 1,2,...,r \))

Where \( m \) is the number of goal indicators, \( r \) is the number of input resources and \( n \) the number of DMUs. The weights or contributions of the different indicators of the goals achieved are denoted by \( u_i \) and the weights of the inputs or resources are denoted by \( v_i \), both can be considered as “virtual prices” of goals and inputs, respectively.

The solution from the basic system provides the maximum efficiency value, the optimal values for the weights of goals and resources, the cost reductions, the dummy variables, which will give the distance between a given policy and the optimum, and the dual prices. The main results are \( \theta \) calculated for each DMU that summarize the degree of efficiency in each municipal area. The closer to 1 it is, the greater the efficiency. Municipal areas with an efficiency of 1 will be the ideal. In addition, the product of the weighting of resources by the level of use also reveals which input resources are the ones that affect efficiency and which ones make no difference.

However, it is known that the above linear programming system has not a unique solution, so two approaches can be applied whether maximize goals or minimize inputs. In our case-study the second one called input oriented DEA-CCR was chosen [0]. The formulation applied (Eq.2) was:

Max \( \sum u_i \cdot y_i = \theta \) \quad \forall i = 1,2,...,n

(subject to \( \sum v_i \cdot x_i = 1 \)

\(- \sum v_i \cdot x_i + \sum u_i \cdot y_i \leq 0 \)

\( u_i, v_i \geq 0 \)  \( j = 1,2,...,m; k = 1,2,...,r \))

In the basic input oriented DEA-CCR, called self-rated evaluation, each decision-maker is left to choose what vector of weightings prefers, as a consequence many municipalities rate themselves at the maximum level of efficiency producing a numerous set of efficient DMUs and a small number of inefficient DMUs with \( \theta < 1 \). For analysing the inefficient DMUs DEA-CCR model provides the slack variables which determine the inefficient causes and the range for improving. Other information provided by this method is the contribution made by each resource to achieving a standard goal [10, 11, 12, 17].

Unfortunately, when DMUs are few and decision variables are many, it is likely that most of the DMUs were efficient \( \theta = 1 \) and non-ranking can be taken among them due to a large number of ties. In these cases different approaches have been made for discriminating DMUs according to improved efficient levels. Cross-evaluation efficiency assessment [0, 15, 0, 0] is one of these approaches being particularly appropriated when applied to a short number of DMUs as our case-study [0]. In cross-evaluation each decision maker evaluates each other with its self-rated weightings, being called peer-rated evaluation, producing a squared matrix of efficiencies from which an average (AE) can be obtained for each DMU.

Nevertheless adding in the same AE values from inefficient and efficient units can shift the score toward the inefficient range. Therefore, it seemed more accurate to apply a weighted average (AWE) where efficient units have more load.

Doyle and Green [15, 16] noted the non-uniqueness of the DEA optimal weightings and proposed to include a secondary objective function into the linear programming system to deal with the non-uniqueness. Following this two-phase objective approaches new methods have been established, in which cross-evaluated DEA is formulated and interpreted according to the making decision context [15, 18, 20]. For instance, in a competition context a decision maker might seek to maximize its efficiency and also minimize competitor efficiency (Aggressive cross-weighted evaluation) whereas in a collaborative context decision makers can share objectives and seek to maximize an overall efficiency (Benevolent cross-weighted evaluation, BCE).

Considering the aims and the kind of decision makers involved in counter desertification policies, the most reasonable context could be to apply a BCE [0], which has the following formulation:

Max \( \sum u_i \left( \sum y_i \right) \)

(subject to \( \sum v_i \left( \sum x_i \right) = 1 \)

\(- \theta \sum v_i \cdot x_i + \sum u_i \cdot y_i \leq 0 \)

\(- \sum v_i \cdot x_i + \sum u_i \cdot y_i \leq 0 \)  \( i = 1,...,n;  j \neq k \)

\( u_i, v_i \geq 0 \)  \( j = 1,2,...,m; k = 1,2,...,r \)
Finally, three scenarios were established and, for each DMU its self-rated efficiency, its peer-rated average efficiency (AE), the peer-rated weighted average (AWE) and the cross-weighted benevolent evaluation (BCE) were calculated.

ESTABLISHING SCENARIOS

Three scenarios with different goals but equal resources were established.

Scenario I
It was considered a multi-criterion function with three objectives of (y1) mitigating soil erosion, (y2) increasing diversity landscape and (y3) increasing population, as input resources were taken (x1) reforestation, (x2) soil conservation practices, (x3) abandonment of dryland farming, (x4) irrigation crops and (x5) increases in protected land or natural areas.

Scenario II
In scenario II the objective of increasing population was eliminated considering that Spanish coastal municipality population are more dependent on tourism than agro-forestry activities.

Scenario III
In the last scenario, the unique objective was erosion (y1), which historically has been the main reason for reforesting in this part of Spain.

RESULTS

General results
The input-output matrix shows the mean resource distribution, 11% reforestation, 22% soil conservation practices, 28% abandonment of crops and 39% increment of protected areas.

The self-rated input-oriented CCR DEA solution discriminated only two municipalities as inefficient in the one-output scenario III: Alhama \( \theta_{III} = 0.44 \) and Totana \( \theta_{III} = 0.55 \). In scenarios I and II, Alhama was the unique inefficient DMU, \( \theta_{I} = 0.60 \) and \( \theta_{II} = 0.62 \). The analysis of lack variables in self-rated input-oriented CCR-DEA allowed determining the deficient inputs. In our case-study Alhama (DMU-3) and Totana (DMU-8) presented abandon crops (x3) and irrigationed crops (x4) as causes of inefficiency.

A common pattern appeared in results, the longer the output vector is, the more efficient the units are. Conversely, the more efficient the units are, the more difficult to make a preference order is.

The rankings from AE and WAE made good discrimination among DMUs but very limited among different scenarios. In this sense, WAE presented almost the same preference ranking for all scenarios, possibly due to a static peer-evaluated weight matrix. Nevertheless, when BCE were applied then the rankings discriminated among scenarios and DMUs in a reasonable way.

Scenario I: (y1) Erosion, (y2) Diversity, (y3) Population
According to the BCE efficiency score, the preference order was: Aledo and Puerto Lumbreras, Mazarrón, Lorca, Librilla, Aguilas, Totana and Alhama (Table 2). Aledo’s efficiency rating was due to a high weight of the erosion decrement while the most useful resource was the amount of reforestation lands. On the other hand, Puerto Lumbreras was efficient due to increment of diversity and human population, and soil conservation practices were the main input. The worst DMU presents an output vector based on a balanced erosion and landscape diversity with a strong weight in irrigation and land abandons.

Scenario II: (y1) Erosion, (y2) Diversity
According to the self-rated DEA all the policies were optimal except Alhama’s which presented a strong input in irrigation and reforestation to obtain a short result in erosion. When analyzing BCE results the ranking of policies (Table 2) was: Aledo and Puerto Lumbreras, Mazarrón, Lorca and Librilla, Aguila, Totana and Alhama. Aledo’s efficiency rating was due to improvements in erosion and the most important resource was reforestation actions. Otherwise, Puerto Lumbreras improved its diversity with soil conservation practices and a little reforestation. The resources with the highest weights were the abandonment of dryland farming and the declaration of protected areas. In Aguilas, Totana and Alhama extensive soil conservation practices were made to reduce erosion, but efficiency scores were poor. The reforestation resource was poor ratings in Librilla and Puerto Lumbreras where were strongly valued the soil conservation practices.

Scenario III: (y1) Erosion
The last scenario considered and the historically most important was the one-output erosion scenario III. When applying self-rated input-oriented CCR-DEA all the policies are optimal except Alhama and Totana. However, considering BCE method the best policies were conducted in Aledo and Librilla. Efficiencies in Aledo and Librilla were based on reforestation and soil conservation practices. The worst rated policies were those of Alhama and Totana; the former based on
expanding protected areas, while the second was based on extensive soil conservation but with not stopping erosion.

CONCLUSIONS

The DEA methods facilitate the interpretation of data and results of counter desertification policies, being highly expressive when inputs are scarce or very balanced, revealing the more effective inputs and the expected improving outputs. The analysis of the weight matrices (u, w) provides the values per unit of element, so that is a key tool for controlling, analysing and planning enhancement in desertification policies.

DEA peer-rated evaluation methods provide stable preference rankings that determine the worst DMU in order to mend its deficiencies. Among the DEA peer-rated evaluation applied methods, Benevolent Cross-evaluated (BCE) performed the best making differences among DMUs and scenarios. However, WAE did not discriminate the different scenarios, probably due to an excessive weight of efficient DMUs and its input weights.

In our case-study, the best policy was implemented in Aledo, with a high level of reforestation, almost without soil conservation practices and scarce abandonment of dryland farming, irrigation crops and protected areas. Nevertheless other two very efficient municipalities were Puerto Lumbreras, Librilla and Mazarrón where the main efforts were mainly made in crop abandonments, irrigation and soil conservation practices.

As average in all scenarios the highest contributions to the efficiency were made by reforestation and soil conservation practices. In consequence, future counter desertification policies in this area should prioritize the reforestation and soil conservation practices. Additionally, the abandonment of drylands and the expansion of protected areas should be a complement to the formers actions.

In summary, DEA methods have proved to be useful for controlling and planning counter desertification policies.

ACKNOWLEDGMENTS

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REFERENCES


APPENDIX

Table 1: Input – output matrix

<table>
<thead>
<tr>
<th>DMU</th>
<th>y1: Erosion (∆Tn/Ha)</th>
<th>y2: Landscape diversity (∆Index/Ha)</th>
<th>y3: Human population (person x 100/Ha)</th>
<th>y4: Reforestation (% of area managed)</th>
<th>x1: Reforestation practices (% of area managed)</th>
<th>x2: Soil conservation (% of area managed)</th>
<th>x3: Land abandonment (% of dryland crops)</th>
<th>x4: Irrigation lands (% of irrigated crops)</th>
<th>x5: Protected wild lands (% of area protected)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU 1: Aguilas</td>
<td>0.24</td>
<td>19.45</td>
<td>69.62</td>
<td>0.80</td>
<td>84.10</td>
<td>-0.60</td>
<td>19.50</td>
<td>7.50</td>
<td></td>
</tr>
<tr>
<td>DMU 2: Aledo</td>
<td>0.16</td>
<td>5.48</td>
<td>-0.64</td>
<td>9.02</td>
<td>-8.97</td>
<td>0.30</td>
<td>0.30</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>DMU 3: Alhama</td>
<td>-0.01</td>
<td>-13.91</td>
<td>31.84</td>
<td>0.12</td>
<td>21.69</td>
<td>6.70</td>
<td>4.40</td>
<td>28.40</td>
<td></td>
</tr>
<tr>
<td>DMU 4: Librilla</td>
<td>0.07</td>
<td>-30.13</td>
<td>4.09</td>
<td>-1.16</td>
<td>1.16</td>
<td>2.10</td>
<td>4.10</td>
<td>14.80</td>
<td></td>
</tr>
<tr>
<td>DMU 5: Lorca</td>
<td>0.13</td>
<td>3.48</td>
<td>155.98</td>
<td>4.67</td>
<td>2.05</td>
<td>8.60</td>
<td>3.10</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>DMU 6: Mazarrón</td>
<td>0.24</td>
<td>11.92</td>
<td>105.71</td>
<td>0.50</td>
<td>18.50</td>
<td>8.10</td>
<td>10.30</td>
<td>5.30</td>
<td></td>
</tr>
<tr>
<td>DMU 7: P.Lumbreras</td>
<td>0.07</td>
<td>15.03</td>
<td>27.71</td>
<td>-1.30</td>
<td>4.51</td>
<td>12.20</td>
<td>6.90</td>
<td>6.00</td>
<td></td>
</tr>
<tr>
<td>DMU 8: Totana</td>
<td>0.01</td>
<td>2.84</td>
<td>61.20</td>
<td>5.98</td>
<td>41.90</td>
<td>1.00</td>
<td>0.90</td>
<td>1.90</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Efficiency scores by scenarios and DEA methods: Average Efficiency (AE); Weighted Average Efficiency (WAE); Benevolent Cross-evaluated Efficiency (BCE)

<table>
<thead>
<tr>
<th>DMU</th>
<th>SCENARIO- I</th>
<th>SCENARIO- II</th>
<th>SCENARIO- III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AE</td>
<td>WAE</td>
<td>BCE</td>
</tr>
<tr>
<td>DMU 1: Aguilas</td>
<td>0.57</td>
<td>0.22</td>
<td>0.47</td>
</tr>
<tr>
<td>DMU 2: Aledo</td>
<td>0.92</td>
<td>1.00</td>
<td>0.93</td>
</tr>
<tr>
<td>DMU 3: Alhama</td>
<td>0.37</td>
<td>0.17</td>
<td>0.29</td>
</tr>
<tr>
<td>DMU 4: Librilla</td>
<td>0.67</td>
<td>0.53</td>
<td>0.58</td>
</tr>
<tr>
<td>DMU 5: Lorca</td>
<td>0.85</td>
<td>0.70</td>
<td>0.90</td>
</tr>
<tr>
<td>DMU 6: Mazarrón</td>
<td>0.82</td>
<td>0.64</td>
<td>0.92</td>
</tr>
<tr>
<td>DMU 7: P.Lumbreras</td>
<td>0.85</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>DMU 8: Totana</td>
<td>0.50</td>
<td>0.16</td>
<td>0.38</td>
</tr>
</tbody>
</table>
Industrial and Manufacturing Applications Stream
Performance evaluation of international airports in Turkey: a cross efficiency approach based on Goal Programming

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ABSTRACT
Although the cross efficiency method has a widespread usage, it has some deficiencies arising from the classical Data Envelopment Analysis (DEA). Since the optimal input-output weights obtained by classical DEA are usually not unique, the cross efficiency scores depending on these weights may not be unique either. The purpose of this study is to examine efficiencies Turkey’s international airports and determine the reasonable ranks of airports in cross efficiency evaluation based on goal programming. In this study, efficiency of international airports in Turkey is examined by using DEA from different perspectives. The inefficiencies of each factor are investigated for inefficient airports. Moreover, a new selection of weights method in cross evaluation based on goal programming is proposed for efficient airports.

Keywords: Data Envelopment Analysis, cross efficiency, goal programming, airports.

INTRODUCTION
Data Envelopment Analysis (DEA) is a multi-factor productivity analysis model for measuring the relative efficiencies of homogeneous set of decision making units (DMUs). DEA is the most popular approximation which developed very fast and has widespread applications especially in the operations research and management science areas. As the applications of this so powerful technique are improved, many new problems have arisen (Sexton et al. (1986), Doyle and Green (1994)). These problems which were dependent of each others and known for a long time are the issue of unrealistic weights distribution, the weak discrimination power and having multiple optimal solutions to weights for the efficient DMUs. The problem of having multiple solutions to optimal weights occurs as an issue especially in the cross efficiency method in which the efficiencies of DMUs are evaluated by means of optimal weights of any given DMU.

The cross efficiency method is a useful technique developed by Sexton et al. (1986) so as to rate the DMUs by using the cross evaluation scores computed as related to all DMUs and hence identify the best DMUs (Anderson et al. (2002). The basic idea of cross evaluation is to use DEA as machinery in peer evaluation instead of self evaluation. Peer evaluation refers to the assigned score for each DMU that obtained by using the optimal weights of other DMUs. The advantages of cross efficiency method are the ability of rating DMUs and being a useful tool without feeling the need of any expert opinion or prerequisites to work out the unenviable cases such as multiple solutions, solutions with extreme or zero values for the weights in DEA.

The purpose of this study is to determine the reasonable ranks of DMUs in cross efficiency evaluation based on goal programming. In this study, a new selection of weights method in cross evaluation based on goal programming is proposed for efficient units. Our new approach is similar to the Cooper et al. (2007) model. Cooper et al. (2007) propose a two-step procedure to be used for the selection of the weights from the alternative optimal solution set. In their studies, a two-step procedure is proposed for the selection of weights that is based on two general criteria of selection and is implemented by means of two mixed integer linear programming problems.

DATA ENVELOPMENT ANALYSIS
Assuming that there are $n$ DMUs each with $m$ inputs and $s$ outputs the relative efficiency of a particular DMU$_0$ is obtained by solving the following linear programming problem (Cooper et al., 2000):

$$
\text{Max} \sum_{i=1}^{n} u_i y_{i0} \\
\text{st.}
$$
\[
\sum_{i=1}^{n} v_i x_{i0} = 1
\]  
(1)

\[
\sum_{j=1}^{m} u_j y_{j0} - \sum_{i=1}^{n} v_i x_{ij} \leq 0
\]

\[j = 1, 2, \ldots, n\]

\[u_j \geq 0, \quad j = 1, 2, \ldots, s\]

\[v_i \geq 0, \quad i = 1, 2, \ldots, m\]

where \( j \) is the DMU index, \( j = 1, \ldots, n; \) \( r \) is the output index, \( r = 1, \ldots, s; \) \( i \) is the input index, \( i = 1, \ldots, m; \) \( y_{ji} \) is the value of the \( r^{th} \) output for the \( j^{th} \) DMU, \( x_{ij} \) is the value of the \( i^{th} \) input for the \( j^{th} \) DMU, \( u_j \) is the weight given to the \( r^{th} \) output; \( v_i \) is the weight given to the \( i^{th} \) input, and \( w_r \) is the relative efficiency of DMU \( j \), the DMU under evaluation. In this model, DMU \( j \) is efficient if and only if objective function value = 1.

A DMU is considered individually in determining its relative efficiency. This DMU is referred to as the target DMU. The target DMU effectively selects weights that maximize its output to input ratio, subject to the constraints that the output to input ratios of all the \( n \) DMUs with these weights are \( \leq 1 \). A relative efficiency score of 1 indicates that the DMU under consideration is efficient whereas a score less than 1 implies that it is inefficient.

The dual model of model (1) is expressed with a real variable \( \theta \) and non-negative variables \( \lambda_1, \ldots, \lambda_n \) of variables as follows:

\[
\text{Min } \theta - \varepsilon \left( \sum_{j=1}^{n} \lambda_j + \sum_{i=1}^{m} s_i^+ \right)
\]

s.t.

\[
\theta x_{i0} - \sum_{j=1}^{n} x_{ij} \lambda_j - s_i^+ = 0
\]

\[
\sum_{j=1}^{n} y_{ji} \lambda_j - s_i^+ = y_{i0}
\]

\[\lambda_j \geq 0, \quad j = 1, 2, \ldots, n\]

\[s_i^+ \geq 0, \quad r = 1, 2, \ldots, s\]

\[s_i^+ \geq 0, \quad i = 1, 2, \ldots, m\]

where \( \varepsilon \) is the infinitesimal non-Archimedean constant, which ensures that no input or output is allocated zero weight; \( s_i^+ \) and \( s_i^- \) are the slack vectors of the outputs and inputs, respectively; \( \theta \) is the scalar variable that represents the possible radial reduction to be applied to all inputs so as to obtain the projected input values; \( \lambda \) is the vector whose optimal values form a combination of units that make up the performance of the DMU under analysis, and establish a direction in which to identify the sources of inefficiency in this DMU (Cooper et al., 2000).

COOPER’S ET AL. TWO-STAGE MODEL

In Cooper et al. (2007) a two-step procedure is proposed for the selection of weights that is based on two general criteria of selection and is implemented by means of two mixed integer linear programming (MILP) problems. We briefly describe this procedure in the following. First we focus on only the “efficient DMUs”. For members of this class, \( E_f \) of efficient DMUs, the following MILP problem is solved in the first step.

**First step:** The following mixed integer linear programming (MILP) problem selects between the optimal weights obtained for DMU \( j \), when solving model (1), those associated with the hyperplanes that are supported by the maximum possible number of efficient units.

\[
\text{Min } I_s = \sum_{j \in E_f} I_j
\]

s.t.

\[
\sum_{i=1}^{m} v_i x_{i0} = 1
\]

\[
\sum_{j=1}^{n} u_j y_{j0} = 1
\]

\[
\sum_{j=1}^{n} u_j y_{j0} - \sum_{i=1}^{m} v_i x_{ij} + t_j = 0
\]

\[j \in E_f\]

\[t_j - M I_j \leq 0, \quad j \in E_f\]

\[I_j = \{0,1\}, \quad j \in E_f\]

\[u_j \geq 0, \quad r = 1, \ldots, s\]

\[v_i \geq 0, \quad i = 1, \ldots, m\]

\[t_j \geq 0, \quad j \in E_f\]

\(E_f\) is set of efficient units.

\( M \) is a big positive number.

**Second step:** The following MILP problem selects between the alternative optima (if any) provided by model (1), by maximizing the minimum value of the virtual variables.

\[
\text{Max } z_o
\]

s.t.

\[
\sum_{i=1}^{m} v_i x_{i0} = 1
\]
\[
\sum_{j=1}^{s} u_j y_{ro} = 1
\]  
(4)

\[
\sum_{j=1}^{s} u_j y_{ij} - \sum_{j=1}^{s} v_j x_j + t_j = 0,
\]
\[j \in Ef\]

\[
t_j - M I_j \leq 0,
\]
\[j \in Ef\]

\[
\sum_{j=1}^{s} I_j = I'_o,
\]

\[
u_j, y_{ro} \geq z_o,
\]
\[r = 1, \ldots, s\]

\[
v_j, x_{ij} \geq z_o,
\]
\[i = 1, \ldots, m\]

\[
I_j = \{0,1\},
\]
\[j \in Ef\]

\[
u_j \geq 0,
\]
\[j \in Ef\]

\[
t_j \geq 0,
\]
\[j \in Ef\]

\[
z_o \geq 0
\]

Ef is set of efficient units.

\[M\] is a big positive number.

In this paper, we have used Cooper’s et al. (2007) two-step idea to select more suitable weights from multiple optimal solution set for efficient units. We have reduced to simple Cooper’s two-stage model using pre-emptive goal programming. Moreover, these suitable weights have been used cross efficiency evaluation. We have described our goal programming procedure in the following.

A NEW GOAL PROGRAMMING APPROACH TO CROSS EFFICIENCY EVALUATION

A new selection of weights goal programming method can be used in cross evaluation is in the following:

\[
\text{Min} - \left\{ \left( n_1 + p_1 \right) + \left( n_2 + p_2 \right) + \sum_{j=1}^{s} \left( n_j + p_j \right) + \sum_{j=1}^{s} p_{o_j} M \sum_{j=1}^{s} I_j - M z_o \right\}
\]

s.t.

\[
\sum_{j=1}^{s} u_j y_{ro} + n_1 - p_1 = 1
\]

\[
\sum_{j=1}^{s} v_j x_{ro} + n_2 - p_2 = 1
\]

\[
\sum_{j=1}^{s} u_j y_{oj} - \sum_{j=1}^{s} v_j x_{ij} + I_j + n_{o_j} - p_{o_j} = 0,
\]
\[j \in Ef\]

\[
t_j - M I_j + n_{o_j} - p_{o_j} = 0,\]
\[j \in Ef\]

\[
u_j, y_{ro} \geq z_o,
\]
\[r = 1, \ldots, s\]

\[
v_j, x_{ij} \geq z_o,
\]
\[i = 1, \ldots, m\]

\[
u_j \geq 0,
\]
\[j \in Ef\]

\[
t_j \geq 0,
\]
\[j \in Ef\]

\[
z_o \geq 0
\]

Ef is set of efficient units.

\[M\] is a big positive number.

\[M_1\] and \[M_2\] are big positive number such that \[M_1 > M_2\].

Goal programming approach selects those which maximize the relative value of the variable with minimum value for the corresponding “virtual” input or output that is represented by \[u_j y_{ro} \text{ and } v_j x_{ro}\]. In this method, we look for weights that have associated programs of performance in which the inputs and outputs globally maximize their relative “importance” as possible (Bal et al., 2010).

Whereof our aim is to minimize the sum of unwanted deviations \[n_1, p_1, n_2, p_2, \sum_{j=1}^{s} \left( n_{o_j} + p_{o_j} \right) \text{ and } p_{o_j} \] in the first priority.

Under this first priority, our second priority is to minimize \[\sum_{j=1}^{s} I_j\], and under the first two priorities our third priority is to minimize \[-z_o\] (or maximize \[z_o\]).

EFFICIENCY EVALUATION OF TURKEY’S INTERNATIONAL AIRPORTS

This study focuses on 12 international airports of Turkey. Initially, a definition must be arrived at of what features best describe airport performance; i.e. what outputs are most important, and what combinations of the production factors are available to the airport.

We have utilized from the prior work for the selection of weights (Pacheco and Fernandes, 2003; Sarkis and Talluri, 2004).

This study uses the four outputs and for inputs listed below:

Outputs:

\[Y_1\] : Annual average flight;

\[Y_2\] : Number of passengers;

\[Y_3\] : Cargo plus mail, embarked plus disembarked, in tonnes;

\[Y_4\] : Operating revenue, million TL.;

Inputs:

\[X_1\] : Average number of employees;
\( x_2 \): Operating expenses, million TL;
\( x_3 \): Annual airplane capacity;
\( x_4 \): Annual passenger capacity.

Data set belongs to year of 2008 and extracted from www.dhmi.gov.tr. Management of Turkish airports and mission of regulation and control of Turkish airspace are performed by General Directorate of State Airports Authority (DHMI). A normalized data obtained by dividing each input-output with their highest value is used.

Table 1-3 reports the results. Atatürk, Antalya, and Adana airports are the efficient international airports, Esenboğa, Adnan Menderes, Dalaman, Milas Bodrum, Trabzon, Sİleyman Demirel, Nevşehir Kapodokya, Erzurum and Gaziantep airports are the inefficient international airports. Table 1 documents the multiple optimal solutions from the CCR, optimal weights of Cooper’s two-stage model and optimal weights of our goal programming approach for efficient units. Moreover, in this table, the rank values of airports (DMUs) by these approaches for each efficient airport are given. It is noted that goal programming model has smaller number of iterations for the solutions than Cooper’s et al. two-stage model.

Cross efficiency values obtained from multiple optimal weights of the CCR model very are confusing. Cross efficiency values obtained from Cooper’s et al. (2007) model and our goal programming model are more suitable than cross efficiency CCR for ranking DMUs. The remaining 9 airports were considered relatively inefficient, as \( \theta^* < 1 \). These airports are listed in Table 2 and Table 3, together with the values of the four variables used as inputs and outputs, the values of \( \theta^* \) and the inefficiencies of each factor. These inefficiencies for the input variables are calculated by the difference between the values projected and those observed. They may be expressed by \( I_x = (\theta x_i - \delta^i) - x_i \). It can be seen that Milas Bodrum the most efficient of the inefficient airports, given that 0.91% less of the observed combination of inputs could be used to attain the output level achieved. The output inefficiencies can be determined by \( I_y = (y_o + \delta^o) - y_o \).

**CONCLUSION**

Since the optimal input-output weights obtained by classical DEA are usually not unique, the cross efficiency scores depending on these weights may not be unique either. The purpose of this study is to remove the problem of multiple optimal weights and determine the reasonable ranks of DMUs in cross efficiency evaluation. As a result, a goal programming model is proposed for the cross efficiency evaluation. New approach provides more suitable weights than classical DEA models.

**REFERENCES**


www.dhmi.gov.tr.

**ACKNOWLEDGMENTS**

This research was partly supported by the Scientific and Technological Research Council of Turkey (project no: 109T337).
Table 1: Results of efficient airports

<table>
<thead>
<tr>
<th>Efficient Airports</th>
<th>Optimal Solutions of Models</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( u_4 )</th>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( v_3 )</th>
<th>( v_4 )</th>
<th>Cross Efficiency</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Atatürk</strong></td>
<td>CCR</td>
<td>0.32</td>
<td>0.68</td>
<td>0.39</td>
<td>0.732</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.790</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Cooper’s Two-Step</td>
<td>0.36</td>
<td>0.59</td>
<td>0.22</td>
<td>0.079</td>
<td>0.25</td>
<td>0.04</td>
<td>0.08</td>
<td>0.002</td>
<td>0.759</td>
<td>3</td>
</tr>
<tr>
<td><strong>Antalya</strong></td>
<td>CCR</td>
<td>0.15</td>
<td>0.54</td>
<td>0.42</td>
<td>0.196</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.901</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Cooper’s Two-Step</td>
<td>0.95</td>
<td>0.33</td>
<td>0.23</td>
<td>0.179</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.915</td>
<td>3</td>
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<tr>
<td><strong>Adana</strong></td>
<td>CCR</td>
<td>10.51</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td></td>
<td>Cooper’s Two-Step</td>
<td>4.94</td>
<td>3.89</td>
<td>2.69</td>
<td>0.92</td>
<td>0.81</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.00</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Standardized inputs, slacks, proportional decrease and inefficiency factors for inefficient airports (\( \theta < 1 \))

<table>
<thead>
<tr>
<th>Inefficient Airports</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( \theta )</th>
<th>( s_1^- )</th>
<th>( s_2^- )</th>
<th>( s_3^- )</th>
<th>( s_4^- )</th>
<th>( I_{X_1}^- )</th>
<th>( I_{X_2}^- )</th>
<th>( I_{X_3}^- )</th>
<th>( I_{X_4}^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Esenboğa</td>
<td>1</td>
<td>0.75</td>
<td>0.50</td>
<td>0.36</td>
<td>0.61</td>
<td>0.35</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
<td>-0.74</td>
<td>-0.45</td>
<td>-0.19</td>
<td>-0.14</td>
</tr>
<tr>
<td>A. Menderes</td>
<td>0.57</td>
<td>0.59</td>
<td>0.25</td>
<td>0.33</td>
<td>0.76</td>
<td>0.29</td>
<td>0.26</td>
<td>0</td>
<td>0</td>
<td>-0.42</td>
<td>-0.40</td>
<td>-0.06</td>
<td>-0.14</td>
</tr>
<tr>
<td>Dalaman</td>
<td>0.31</td>
<td>0.24</td>
<td>0.40</td>
<td>0.29</td>
<td>0.42</td>
<td>0.05</td>
<td>0</td>
<td>0.04</td>
<td>0.03</td>
<td>-0.23</td>
<td>-0.14</td>
<td>-0.27</td>
<td>-0.20</td>
</tr>
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<td>Milas-Bodrum</td>
<td>0.26</td>
<td>0.29</td>
<td>0.40</td>
<td>0.09</td>
<td>0.91</td>
<td>0.16</td>
<td>0.16</td>
<td>0</td>
<td>0.25</td>
<td>0.18</td>
<td>0.19</td>
<td>-0.28</td>
<td>-0.01</td>
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<td>Trabzon</td>
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<td>0.19</td>
<td>0.15</td>
<td>0.06</td>
<td>0.89</td>
<td>0.05</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
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<td>-0.01</td>
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<td>S. Demirsel</td>
<td>0.06</td>
<td>0.05</td>
<td>0.13</td>
<td>0.02</td>
<td>0.24</td>
<td>0</td>
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<td>0.01</td>
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<td>0.22</td>
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<td>-0.03</td>
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</table>

Table 3: Standardized outputs and slacks for inefficient airports (\( \theta^* < 1 \))

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<tr>
<th>Inefficient Airports</th>
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<th>( y_2 )</th>
<th>( y_3 )</th>
<th>( y_4 )</th>
<th>( s_1^+ )</th>
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<td>0.057073</td>
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<td>0.047230</td>
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<td>0.002005</td>
<td>0.006115</td>
<td>0.006683</td>
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Productivity analysis of Iranian manufacturing industries by Data Envelopment Analysis

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ABSTRACT

One of the legal commitments for executive systems in the Law of the Fourth Development Plan of Iran is to have a 2.5 percent gain in the total factor productivity. This Law expressly enforces the executive systems to take the productivity circle, which measuring productivity indexes is one of the most important parts of it. In this article, we evaluate the efficiency of manufacturing industries by data envelopment analysis and after that we use the DEA Malmquist Index to measure the efficiency change, technology change and productivity growth of Iran’s manufacturing industries, during the third developing plan. Also, we use the super efficiency technique for ranking the efficient units. The results show that, coke and refined petroleum industry and non metallic mineral industry are efficient in the all five years of the third developing plan and have the first and second rank in these years.

Keywords: Productivity, Data Envelopment Analysis, Malmquist Productivity Index.

INTRODUCTION

In today’s world, one of the concerns of developing countries is the economic growth and industry dynamics of the country. These countries try to increase production and exports of industrial goods by adopting advanced production techniques, saving production costs and increasing total factor productivity. Since productivity and efficiency play an important role in economic growth, development, improvement and welfare, they have enjoyed a great deal of interest among economists in recent decades. Therefore, one of the basic parameters of industrial and economical development is focusing on the efficiency and productivity issues in all levels of society activities, particularly in industrial and manufacturing sectors in the community.

Industry segment is a set of enterprises, active in different fields of industry. Manufacturing industry, as a part of industry, is an economic firm that makes physical or chemical changes in the form of materials and components and converts them to the goods and materials processed or completed. The scope and range of activities is included by several classifications. One of the most important classifications is based upon homogeneous subsectors which are classified using the International Standard Industrial Classification (ISIC) codes. Different parts of industrial activities, which are divergences of this section, have different characteristics in terms of economic variable status, technology usage, development, opportunities and challenges ahead. Therefore, in addition to general conditions of the industrial sector, precise and comprehensive understanding of this section requires survey of characteristics of every branch of this segment (Economic Report of the year 1382 and monitoring the performance of the first four-year of development plan, 1383). Study the status of active firms in an industry can specify the possibility and barriers of development in different parts of the industry section. Therefore, it can help in giving a precise and explicit perspective of this section. Giving any perspective of an industry in the future of an economy, needs a comprehensive and general study of the past evolutions and present situation of this sector in the country’s economy. Thus, the first step in this study is preparing a perspective of the country’s industry and perception of performance of industrial firms. Inasmuch as evaluating the performance of a firm refers to measuring productivity and efficiency of it, the purpose of this study is measuring the efficiency and productivity of the industry segment.

Measuring the efficiency has been of a high significance among scientists since long time ago. However, modern efficiency measurement begins with the work of Farrell (1957) who drew upon the work of Debru (1951) and Koopmans (1951) to define a simple measure of firm efficiency which could account for multiple inputs (Coelli, 1995). He proposed that the efficiency of a firm consists of the two components; technical efficiency and allocative
efficiency. Farrell applied his model in measuring the efficiency of American agriculture section, to illustrate his newly defined model (Farrell, 1957). According to the limitative assumptions the Farrell model had, it didn’t become a useful method in measuring efficiency until the year 1978 that Charnes, Cooper and Rhodes introduced their new method applying linear programming models in measuring the efficiency.

Data envelopment analysis (DEA) is a set of linear programming method applied in determining the efficiency of similar decision making units (DMUs) with several inputs and outputs. DEA has been widely used by many recent studies during these years, because of having fewer restrictions in choosing inputs and outputs, comparing with other methods as well as the flexibility in choosing DMUs, which can be chosen from a variety of applications such as universities, banks (Isik & Hassan, 2002; Rezvanian & Medians, 2002; Satheye, 2001), airlines (Ray & Hu, 1997), hospitals (Steinmann & Zweifel, 2003; Fare et al., 1989) industries (Chen, 2003; Ray & kim, 1995), institutions, companies, etc.

Total Factor Productivity (TFP) growth indicates the growth in the productivity of the firm during a particular time period. Malmquist index is an index for measuring the TFP growth of firms during time periods. Malmquist index was first introduced by Malmquist (1953) as a quantitative index for estimating the consumption of inputs and was used as a growth index for the first time in production theory (Cayes et al., 1982). Introducing DEA method in 1978 led the evaluation of productivity growth to using this mathematical programming technique. Therefore, Fare et al. (1992) used DEA Malmquist Index model in measuring the productivity change in Swedish pharmacies. Since then, it has been used in many studies such as banks (Mukherjee & Miller, 2001), tourism industry (Barros & Alves, 2004), agriculture (Coelli & Rao, 2005), insurance industry (Donni & Fecher, 1997). Chen (2003) used DEA Malmquist index in evaluating the productivity growth of tree main industries in China during four periods using investment and labour as inputs of the model and value added as the only output. Chen and Iqbal Ali (2004) used this method in evaluating the productivity growth of computer companies during 1991 to 1997 with tree inputs: assets, labour and investment and a single output of income.

Ranking the decision making units plays an important part in recognizing better units. Data Envelopment Analysis evaluates the relative efficiency of DMUs, but does not allow for a ranking of the efficient units themselves. Andersen and Petersen (1993) developed a new procedure for ranking efficient units. The methodology enables an efficient unit to achieve an efficiency score greater than one by removing the corresponding constraint in the original DEA models (Adler et. al, 2002). This procedure provides a framework for ranking the efficient units and is called super-efficiency DEA model or Andersen-Petersen Model. Chen (2004) used super efficiency model in ranking the 21st largest Japanese companies and also ranking 15 US cities.

The remainder of this paper is organized as follows. The next section deals with the methodology issue. We first describe the data envelopment analysis and measuring efficiency. Then we introduce DEA Malmquist productivity index and its ability to estimate the efficiency and technology growth to measure the efficiency growth over time. At the end of this part we introduce the Andersen-Peterson model for ranking the DMUs. In the next section, we present a case study of measuring the efficiency and productivity of Iranian manufacturing plants over the third development plan. Conclusion remarks are given at the next section. At the Last section we offer implications for the future studies.

METHODOLOGY

DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis has been recognized as an excellent method for analyzing performance and modeling organizations and operational processes especially when market prices are not available. DEA method is generally a method for determining a boundary instead of central regression and unlike the statistical regression methods that tries to fit a regression plane through the center of the data, DEA floats a piecewise linear surface to rest in top of the data by linear programming techniques (Sieford & Thrall, 1990).

DEA method compared to parametric methods used for efficiency evaluation has many advantages. This method allows certain flexibility in the treatment of inputs and outputs as well as in the conversion of multiple inputs and outputs into an easy and comprehensible efficiency measure. In addition, not only does this method estimates and identifies efficient units, but also indicates for those inefficient units, what needs to be done in order to become efficient without fixing a priori relation between the variables. The first model of DEA, made by Charnes, Cooper and Rhodes (1978), is defined as the weighted proportion of outputs to the weighted
proportion of inputs. The efficiency of DMU₀ is given by:

\[
\max \sum_{i} \frac{u_i y_{i0}}{v_i x_{i0}}
\]

The terms \(x_{i0}\) and \(y_{r0}\) stand for the ith input and rth output of DMU₀ and \(u_i\) and \(v_i\) are variables of the model. As we can see, this model is indefinite. In order to solve this problem and avoid the zero weights, researchers have added some constraints to the original model. The following model is called the input oriented CCR model and is the model used in this study to evaluate the efficiency of Iranian manufacturing industries.

\[
\min \{\theta; s.t. \sum_{j=1}^{n} x_{ij} \lambda_j + s^- = \theta x_{i0}, \sum_{j=1}^{n} y_{rj} \lambda_j - s^+ = y_{r0}, \lambda_j, s^-, s^+ \geq 0, \forall i, j, r\}
\]

**DEA MALMQUIST PRODUCTIVITY INDEX**

The Malmquist Productivity Index measures total factor productivity change between two periods of activities, fulfilled by a specific set of DMUs, by using the distance functions. This index decomposes the total factor productivity change into two components, one measuring the efficiency change and the other measuring the technology change between the given periods. This separation of the total factor productivity change is important because it studies the performance of firms from different aspects and reveals the specific cause of the reduction of productivity for each firm.

Suppose we have n DMUs, each DUMj (j = 1, 2, ..., n) with the inputs \(x_{ij} = (x_{1ij}, ..., x_{mij})\) and the outputs \(y_{rj} = (y_{r1j}, ..., y_{r3j})\) at each time period t, t=1, ..., T. The input-oriented Malmquist productivity index is defined as (Fare & Grosskopf, 1992):

\[
M_0 = \left( \frac{D_0^1(x_0^{t+1}, y_0^{t+1})}{D_0^1(x_0^{t}, y_0^{t+1})} \right) \times \left( \frac{D_0^1(x_0^{t+1}, y_0^{t+1})}{D_0^1(x_0^{t}, y_0^{t})} \right)^{1/2}
\]

In this case \(M_0 > 1\) indicates productivity gain; \(M_0 = 1\) indicates productivity loss and \(M_0 < 1\) shows there was no change in productivity from time t to t+1 (Fare et al., 1992). Fare et al. (1992) decomposed their Malmquist productivity index into two components as follows:

\[
M_0 = \left( \frac{D_0^1(x_0^{t+1}, y_0^{t+1})}{D_0^1(x_0^{t}, y_0^{t+1})} \right) \times \left( \frac{D_0^1(x_0^{t+1}, y_0^{t+1})}{D_0^1(x_0^{t}, y_0^{t})} \right)^{1/2}
\]

The first component measures the efficiency change \(EC_0 = D_0^1(x_0^{t+1}, y_0^{t+1})/D_0^1(x_0^{t}, y_0^{t})\) and the second one measures the technology change \(TC_0\) by the following formula during the time period t and t+1:

\[
TC_0 = \left[ \frac{D_0^1(x_0^{t+1}, y_0^{t+1})}{D_0^1(x_0^{t}, y_0^{t+1})} \right] \times \left[ \frac{D_0^1(x_0^{t+1}, y_0^{t+1})}{D_0^1(x_0^{t}, y_0^{t})} \right]^{1/2}
\]

that shows the aggregated change in technology of a DMU form time t to t+1 can also be viewed as technology frontier shift between time period t and t+1. Fare et al. (1992, 1994) proved that \(TC_0 > 1\) indicates a positive shift in the production function or technical progress, \(TC_0 < 1\) indicates a negative shift or technical regress and \(TC_0 = 1\) indicates no shift in technology frontier (Chen & Iqbal, 2004).

As the above equations indicate measuring the Malmquist productivity index leads us calculating four distance functions (two single period and two mixed period functions). The two single period functions can be obtained by using the CCR-DEA model (Chen & Iqbal, 2004; charnes et.al, 1978)

\[
D_0^1(x_0^{t}, y_0^{t}) = \min \{\theta; s.t. \sum_{j=1}^{n} \lambda_j x_{ij}^{t} \leq \theta x_{i0}, \sum_{j=1}^{n} y_{rj}^{t} \lambda_j \geq y_{r0}, \lambda_j \geq 0 \}
\]

Also the first mixed period measure is obtained by solving the following linear programming problem:

\[
D_0^1(x_0^{t+1}, y_0^{t}) = \min \{\theta; s.t. \sum_{j=1}^{n} \lambda_j x_{ij}^{t} \leq \theta x_{i0}, \sum_{j=1}^{n} y_{rj}^{t} \lambda_j \geq y_{r0}, \lambda_j \geq 0 \}
\]

The other mixed period measure is the optimal value of the following linear programming problem:

\[
D_0^1(x_0^{t+1}, y_0^{t+1}) = \min \{\theta; s.t. \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq \theta x_{i0}, \sum_{j=1}^{n} y_{rj}^{t+1} \lambda_j \geq y_{r0}, \lambda_j \geq 0 \}
\]

By solving these three linear programming problems we obtain the efficiency change, technology change and therefore the total factor productivity change during each time period.

**SUPER EFFICIENCY**

Data Envelopment Analysis groups the DMUs into two sets; those that are efficient and define the Pareto frontier and those that are inefficient. DEA models assign an efficiency score less than one to inefficient DMUs, from which the ranking can be derived, however all efficient DMUs have the efficiency score one so that for these units no ranking can be given. A model for ranking efficient units was proposed by Andersen and Petersen which is called super efficiency model. In this model the constraint related to each efficient DMU is omitted from the primal DEA model (Andersen & Petersen, 1993).

**AN APPLICATION TO IRANIAN MANUFACTURING INDUSTRIES**

In this section the efficiency and productivity of Iranian manufacturing plants are evaluated
during the third development plan. At the end of this section, ranking of manufacturing industries is presented in each year during these five years.

Data and DMU Selection

According to the policy of Iranian government all companies are forced to increase their productivity during the fourth development plan. So we should first estimate the productivity of all firms in the past years to investigate the appropriate changes needed for increasing the efficiency and productivity of these industries in the fourth development plan. Therefore we evaluate the productivity and efficiency of all Iranian manufacturing industries during the third development plan, from 1379 to 1383; each year in Persian calendar starts from March 21 and year 1379 is corresponded to year 2000. The data used in the current study is the annual data reported from Statistics Centre of Iran with the ISIC classification of the industries. This classification contains 23 manufacturing industries. The two industries; manufacture of tobacco products and recycling, were excluded from the sample, since the information required was not accessible in the current years. Therefore, we have 21 DMUs in each year (list of the DMUs is given in Table 6).

In order to keep the study consistency, we select four inputs and a single output. The four inputs used in this research are as follows; labour (the number of workers and staffs working full time or part time in a firm), investment (the amount of variations in the assets during the statistical period (one year)), service amends (wages, salaries and other payments (money, stuffs, ...) given to labours in an industry), and data value (value of the staples, wrapping materials, consumed tools, fuel, water, electricity and industry service).

We use value added, which stands for the value of net staffs and services produced during the statistical period (one year), as the only output for evaluating the efficiency and productivity of manufacturing industries during these years.

RESULTS

Table 1 reports the efficiency of manufacturing industries during five years of third development plan. We used the CCR input-oriented model for evaluating the efficiency of DMUs. The results show that the two industries with the ISIC codes 23 (manufacture of coke, refined petroleum products and nuclear fuel) and 26 (manufacture of other non-metallic mineral products), are efficient in all these years and the industry with the code 20 (manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials) becomes efficient in the year 1381 and preserves its being efficient in the next three years. The last row of table 1 shows the average efficiency off all industries in each year. The industry average efficiency is increasing in all these years except the year 1382, which has decreased compared with a year before. In table 2, efficiency change of all industries is given during four time periods of the third development plan. As it is shown in this table, the efficiency changes of the two industries 23 and 26, which were efficient in all years, hasn’t changed in any of the time periods. This is the result of being efficient in two consecutive years. However, when a DMU is efficient (is a frontier DMU) in two consecutive years, which yields to TC=1 in that period, this doesn’t necessarily mean that an industry with TC>1 has a better performance in improving its efficiency. As an example, comparing two industries with code 30 (manufacture of office, accounting and computing machinery), which is a frontier DMU in the years 1379 and 1380, that yields to have EC=1 during these two years, and the industry with code 31 (manufacture of electrical machinery and apparatus n.e.c.), which has EC= 1.047. Although the industry with code 31 has an improvement in this period and in the industry with code 30 there is no improvement in the efficiency, doesn’t necessarily mean that industry 31 has a better performance improvement in this period.

For the industry average, technical efficiency improves about 0.7% during the year 1379 to 1380, improves about 4% from 1380 to 1381, declines about 1% from 1381 to 1382 and has a 4.3% improvement from 1382 to 1383. The amount of technology change (or frontier shift) is given in the table 3. It can be seen that some industries didn’t have any technology change in some periods during the four periods of third development plan. The industries with code 23 and 26 didn’t have any technology change during these four periods.

Table 1: Efficiency

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<tr>
<th>ISIC Code</th>
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<th>1380</th>
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<td>0.917</td>
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<td>0.955</td>
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<tr>
<td>23</td>
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<td>0.819</td>
<td>0.866</td>
<td>0.88</td>
<td>0.869</td>
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</table>
From 1379 to 1383, as it is seen in the table, the manufacturing industries in every year from 1379 to 1383 to 1382 have a 1% decline in the average industry productivity experiences a amount of improvement occurred again in the years. It can be seen that from 1379 to 1380 industry improvement in the productivity of this period. These indicate that the reason of 4.4% decline in efficiency change, but 6% improvement in technology change during the years 1379 and 1380, which results in a 1.4% improvement in the productivity during this period. These indicate that the reason of improvement in productivity was the technological changes that this industry had during this time period. In the fourth time period (82-83) the foresaid industry has both 0.6% and 1.8% efficiency and technology gain respectively, which result in a 2.4% improvement in the productivity of this industry. The last row of table 4 shows productivity improvement in the first two periods and productivity loss in the last two years. It can be seen that from 1379 to 1380 the average industry has improved 1.3%, this amount of improvement occurred again in the period 1380 to 1381. But, from 1381 to 1382 the average industry productivity experiences a 0.1% loss in the productivity change and from 1382 to 1383 it has a 1% decline in productivity.

Table 5 indicates the ranking of all manufacturing industries in every year from 1379 to 1383. As it is seen in the table the industry with code 30 (manufacture of office,
accounting and computing machinery) has got the first place among other industries in 1379 and 1380, and the industry 23 (manufacture of coke, refined petroleum products and nuclear fuel) is in the second place in the first two years. The industry 23 moves to the first place in the next three years (1381 to 1383). The table shows that the industry 26 (manufacture of other non-metallic mineral products) which is in the third place in the first two years improves and moves to the second in the next year (1381) and keeps this place for the next year, but in the 1383 declines and moves to the fourth place.

Table 5: Ranking

<table>
<thead>
<tr>
<th>Rank</th>
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<th>1381</th>
<th>1382</th>
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</table>

CONCLUSION

The results of this study show that the manufacture of coke, refined petroleum products and nuclear fuel (ISIC Code 23), is efficient in all years of the third developing plan and is in the first or second step in ranking manufacturing industries, during all five years. However, as the productivity measurement shows, this industry was stable without any growth or decline in productivity during all periods.

The results of the tables show that the manufacture of textiles (code 17), is inefficient in all the 5 years of evaluation and is in the last two steps in ranking in all years. But, the results show that it has had productivity improvement in first, second and third period. As the tables show in the first and third periods (79-80 and 81-82 respectively) the technology improvement leads into productivity growth, but in the second period (80-81) the technology declined and the reason of productivity improvement was the efficiency gain. Therefore, although this industry is productive in three periods, it has been known as the worst industry in ranking among all manufacturing industries.

REFERENCES


Economic Report of the Year 1382 and Monitoring the Performance of the First Four-year of Development Plan of Iran (1383), Management and Planning Organization 3(2).


APPENDIX

The International Standard Industrial Classification of manufacturing section released by United Nations Statistics Division (ISIC Rev.3) and DMU selection is given in the appendices table below:

---

**Table 6: International Standard Industrial Classification for manufacturing Plants**


<table>
<thead>
<tr>
<th>DMU</th>
<th>ISIC Code</th>
<th>Activity Group</th>
</tr>
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<tbody>
<tr>
<td>DMU1</td>
<td>15</td>
<td>manufacture of food products and beverages</td>
</tr>
<tr>
<td>DMU2</td>
<td>17</td>
<td>manufacture of textiles</td>
</tr>
<tr>
<td>DMU3</td>
<td>18</td>
<td>manufacture of wearing apparel; dressing and dyeing of fur</td>
</tr>
<tr>
<td>DMU4</td>
<td>19</td>
<td>tanning and dressing of leather; manufacture of luggage, handbags, saddlery, harness and footwear</td>
</tr>
<tr>
<td>DMU5</td>
<td>20</td>
<td>manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting</td>
</tr>
<tr>
<td>DMU6</td>
<td>21</td>
<td>manufacture of paper and paper products</td>
</tr>
<tr>
<td>DMU7</td>
<td>22</td>
<td>publishing, printing and reproduction of recorded media</td>
</tr>
<tr>
<td>DMU8</td>
<td>23</td>
<td>manufacture of coke, refined petroleum products and nuclear fuel</td>
</tr>
<tr>
<td>DMU9</td>
<td>24</td>
<td>manufacture of chemicals and chemical products</td>
</tr>
<tr>
<td>DMU10</td>
<td>25</td>
<td>manufacture of rubber and plastic products</td>
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<td>DMU11</td>
<td>26</td>
<td>manufacture of other non-metallic mineral products</td>
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<td>DMU12</td>
<td>27</td>
<td>manufacture of basic metals</td>
</tr>
<tr>
<td>DMU13</td>
<td>28</td>
<td>manufacture of fabricated metal products, except machinery and equipment</td>
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<td>DMU14</td>
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<td>manufacture of machinery and equipment n.e.c.</td>
</tr>
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<td>DMU15</td>
<td>30</td>
<td>manufacture of office, accounting and computing machinery</td>
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<td>DMU16</td>
<td>31</td>
<td>manufacture of electrical machinery and apparatus n.e.c.</td>
</tr>
<tr>
<td>DMU17</td>
<td>32</td>
<td>manufacture of radio, television and communication equipment and apparatus</td>
</tr>
<tr>
<td>DMU18</td>
<td>33</td>
<td>manufacture of medical, precision and optical instruments, watches and clocks</td>
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<tr>
<td>DMU19</td>
<td>34</td>
<td>manufacture of motor vehicles, trailers and semi-trailers</td>
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<tr>
<td>DMU20</td>
<td>35</td>
<td>manufacture of other transport equipment</td>
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<tr>
<td>DMU21</td>
<td>36</td>
<td>manufacture of other transport equipment</td>
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</table>
Measuring Syrian construction firms performance using Data Envelopment Analysis (DEA)

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ABSTRACT
Our attempt at this paper was to develop a performance benchmarking model for Syrian construction firms. This identified one of the first steps in an improvement process. We utilized the Data Envelopment Analysis model (DEA), which is a recognized modern approach to the assessment of performance of organizations and their functional units. It accomplishes this by identifying overall performance through benchmarking economical, technical, environmental and social performance for construction firms. The benchmarking model was developed by using field data collected from 37 Syrian construction firms. The analysis using the DEA software appeared to show that 29 of the 37 firms are functionally inefficient, and provided each firm with projected values. This model can be used as an improvement tool to help guide firms in understanding how to adjust their policies and practices to improve their overall performance.

Keywords: Performance Benchmarking, linear programming, Data envelopment analysis.

INTRODUCTION
It's widely recognized that in the long term, the success of both individual construction firms and industry will depend on improving performance by continually acquiring and applying new knowledge. This will require a more comprehensive understanding of how existing practices can get better. Construction firms therefore recognized the need for benchmarking tools that provide perspectives on both their current practices and existing shortcomings versus industry leaders.

This led us to research existing benchmarking models in attempt to develop a new performance benchmarking model. A model: that is both convenient and can be used as an improvement tool for managers with their current practices. In addition to help improve future performance and be capable of guiding the Syrian construction industry towards more efficient and effective performance.

BENCHMARKING MODELS IN CONSTRUCTION
Adding to the complexity of the benchmarking task is the current industry structure and fact that a number of organizations get involved in the design and construction of a single project.

Due to the above stated facts, information on the performance of the construction industry is relatively scarce. There have been some initiatives concerned with performance measurement systems for benchmarking in different countries, these are Fisher (1995), Hudson (1997) & CII(2000),CBBP(1998), SISIND(2000). As stated by (EL-MASHALEH, 2007), the existing construction benchmarking models fall short in main respects:

- The existing benchmarking models report project-level industry norms of some performance metrics (i.e., cost, schedule, etc.). This doesn't allow detecting overall performance of the firm. We have no means to answer the following question: Where does a certain firm stand compared to the other firms when considering overall performance (i.e., all metrics simultaneously)?

This limitation furthermore makes it difficult to identify practices that lead to superior firm overall performance because with existing models we only have individual metrics. In order to know the overall performance we need to assign weights which may be arbitrary and subjective. Thus we will not get accurate values.

With previous models, the relationship between expenses and the performance is absent, as two firms that arrive at the same performance are considered to be similarly efficient. And from economical point of view, we have to consider the firm that commits fewer resources to arrive at a certain
performance as a better performer than another firm.

So Existing models have serious limitations to guide the industry into both benchmarking and identifying practices of superior performance. As we can consider the performance superior, when it is measured as certain overall value, here we guarantee that the improvement on one metric is not on expense of other one.

Thus, a new benchmarking model was proposed by (EL-MASHALEH, 2007). The proposed model is robust enough to address the limitations of the previous benchmarking models. The model provides an overall firm level performance measure and supports trade-off analysis among the several metrics of performance. Additionally, the proposed benchmarking model is deployed using the technique of Data Envelopment Analysis (DEA) that evaluates efficiency, which refers to the relationship between scarce factor inputs (project management expenses) and outputs (schedule, cost, customer satisfaction, safety and profit).

PROPOSED BENCHMARKING MODEL:

EL-MASHALEH developed Technical Economic Benchmarking model that provides the firms overall performance. We have worked to develop EL-MASHALEH model to be more efficient because due to a several recent studies new directions have to be considered in performance measurement, such as environmental and social efficiency. (Hendrickson and Horvath 2000) stated that construction projects pose enormous challenges not only to finish within an owner’s schedule and budget, but also to eliminate and minimize harmful impacts to the environment.

Although the topic of environmental management systems is fairly new to the construction industry, recent literature supports the need for construction firms to consider developing and implementing such systems. An increasing number of construction firms are becoming certified to international standards worldwide, especially the International Organization for Standardization (ISO) 14001 series, which provide guidelines for implementing an EMS.

There are over 36,000 organizations in 112 countries that have received ISO 14001 certification (ISO 2001). Japan is leading the world with over 8,000 certifications. Construction firms are realizing that environmental management is a primary key to their success. They understand that it is imperative to eliminate or minimize harmful environmental impacts from construction. This led us to consider the environmental impact in benchmarking performance. Another important benchmarking indicator we studied is the social benefit for the construction sector in the Syrian government. The government stated, at its tenth fifth plan, its future vision for the construction sector: That within the next two decades the construction sector has to follow the national policy to achieve sustainable development and achieve its long term targets. The quantitative targets as mentioned at tenth fifth plan for the Syrian government are:

To achieve average yearly growth by 12%
To contribute to GDP (gross domestic production) to 2.6%.
From the above targets it’s clearly seen that Syrian construction companies are required to share the government the social responsibility by contributing to creating working opportunities.

As such, a developed benchmarking model is proposed in the following which will cover all the required aspects of performance that are needed in the highly current competitive environment. The model aimed at benchmark economical, technical, environmental and social performance for construction firms.

DEVELOPMENT OF PERFORMANCE BENCHMARKING MODEL

The Data Envelopment Analysis model proved to be an appropriate tool to benchmark performance, so we chose it to utilize our research and to benchmark the performance of Syrian construction firms. To choose the metrics of the benchmarking model, we reviewed the existing literature and included the relevant metrics. We found that construction literature primarily concentrates through performance measurement on basic key indicators as: schedule, cost, quality, safety, profit which are usually considered to develop the metrics of the firms overall performance.

Since there are new international requirements of performance standards, and since we have some special circumstances in Syria as we previously referred to, we have added additional indicators to measure performance which are Changes in scope of work, environmental performance and social benefit, that we obtain a benchmarking model to measure economical, technical, environmental and social performance.

We previously explained the importance of environmental performance as it has become
an essential requirement during recent construction and also the importance of social benefit for public companies in Syria.

We found according to our previous research (2003), that there is usually a big ratio of changes occurring through the construction process and that these changes usually lead to many problems and delays. We must be able to measure it so that we can try and put policies together in attempt to eliminate the delays.

Below, we display the metrics which we have selected for performance measurement and their calculations. We measured benchmark performance of construction firms for the last two years.

1- Schedule performance (SP) = (Number of projects delivered on/ahead of schedule / Total number of projects)* 100%.
2- Cost performance (CP) = (Number of projects delivered on/under budget / Total number of projects) *100%.
3- Costumer satisfaction (CSP) = percentage of private customers that come back for a repeated business with the firm + percentage of public customer satisfaction.
4- Safety performance (SAP) = number of recordable accidents / number of employees.
5- Environmental performance (EP) = percentage of commitment to ISO 14000 standards.
6- Change in scope of work (CWP) = cost of change / planed cost
7- Profit (pp) = (Net profit/ total expenditure )*100.
8- Social benefit (Sb1P) = the number of permanent employees which have health and security assurance.
Sb2P = the value of annual executed projects by S.P.
9- PM expenses: the project management personnel salaries and expenses.

Customer satisfaction is measured in terms of the percentage of repeated business customers for the private sector, but for public customers there are not repeat customers because their contracts are usually bids so we evaluated their satisfaction depending on their opinions at the end of project.

Safety performance is usually measured using OSHA incident rates which are based on the Occupational Safety and Health Act (1970), that requires employers to record and report detailed accident information.

As most construction firms do not have accurate safety information, we calculated it as a previous formula, but it's preferable, when information is available, to follow the OSHA guidelines.

Environmental standards are defined in many publications like (ISO 2002). The ISO 14001 standard defines an EMS as “a management tool enabling an organization of any size or type to control the impact of its activities, products or services on the environment.” The ISO 14001 establishes a framework for management through the development of formal processes, procedures and the environmental aspects of an organization. The standard contains 17 key elements grouped into five major areas: environmental policy, planning, implementation and operation, checking and corrective action, and management review. A unique aspect of the system is that it is designed to be appropriate for any company, regardless of industry, size, location, and the level of their environmental responsibilities (ISO 1996).

We selected a metric that measures the percentage of commitment upon these requirements for construction firms in our proposed model.

As for social benefit, all of us noticed that in the last fifteen years we have witnessed the growth of large interest on public investment criteria for benefit-cost analysis for projects and programs in both developed and less developed countries. The criteria have been adjusted to allow for surplus labor, savings constraint and reinvestment benefit.

Usually the social benefit concerns are in regard to income distribution among different categories in society; therefore, we can explain the social sides as follows:

The effect on the creation of new job opportunities, number of job opportunities, percentage of employees

The effect of income distribution to the limited income categories

If the outcome is to serve the low income category

In our case we have to be concerned, as we previously mentioned, with the Syrian governments strategy to allow for surplus labor in construction sector. So it is necessary to take this point into consideration when evaluating the company's outputs. Companies that employ permanent employees have important social outcomes including, but not limited to: workplace health and safety, employee retention, labor rights, human rights, wages and working conditions, naturally will perform better in the scope of social benefit.

By including the social benefit metric, we align the measures of performance to the strategies which have been pointed out in the literature: a key point in the development, implementation and use of performance
measurement systems. This new indicator adds value to our developed model from previous proposed models as it considers the changing nature of work and the Changing organizational roles.

Because rates of return on social investment are hard to calculate and they include many standards which are out of the scope of this research, we considered only two of its standards, they include: the number of permanent employees which have health and security insurance and the value of annual executed projects by S.P as this indirectly benefits the largest number of people.

RESULTS ANALYSIS

To measure performance, we designed a survey questionnaire that measure performance on the selected metrics explained above to quantify the performance of construction companies. We then applied it to a chosen random sample of Syrian firms which execute different types of projects.

The data for this research was collected through a survey questionnaire of personal interview of respondents from construction firms in Damascus and Lattakia during 2007.

In figure -1- In appendix A we show the firms general information. There are 37 firms that execute yearly projects for more than 12,519 Million SP.

Only 1 firm is making profit over 50 million SP, 2 are between 25-50 million SP, 6 are between 5-25 million SP, 15 are between 1-5 million SP, 5 less than 1million SP, and 8 gain no profit which has special positions as most of them are public firms, and in accordance to Syrian government strategies. As mentioned above, they have a social responsibility in provide working opportunities and allowing for surplus labor, so all their revenues come back to surplus employees. It is also worth mentioning that most of the respondents are not convinced by these ratios of profit, due to non systematic ways implemented in Syrian contracts, also they are not satisfied in the way the contractor is chosen and the obligation to decrease the price of labor in order to get the bid. This means that naturally there will be a decrease in the profit, which will be reflected negatively on the quality of the projects, as said by respondents.

We applied DEA model introduced by Charnes, Cooper and Rhodes (1978), using DEA-Solver learning version LV3.0/ CCR (CCR-I) attached by (Cooper; Seiford; Tone 2006) book, and which is input oriented model as we collected data about the project management expenses as input. The following analysis represents the data and benchmark firms. The analysis supplies inefficient firms with projected values for the metrics of performance.

The table -1- in appendix A shows the descriptive statistics about performance metrics. Table 2 gives descriptive statistics about the results of model, the number of DMUs is 37 included 8 efficient and 29 inefficient ,the average of scores is 0.63 with standard deviation of 0.26 with maximum score is 1 and minimum score is 0.21.

<table>
<thead>
<tr>
<th>No. of DMUs</th>
<th>DMU</th>
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<tr>
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<tr>
<td>Minimum</td>
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Table 2: descriptive statistics about the results of CCR model

Table-3- in appendix A shows the results of the CCR model using CCR-I model of DEA (we displayed part of them). For each DMU, the results show the score data, projection, difference between projection and data, and percentage difference between projection and data. The firms with score 1 are efficient like firm B, but for inefficient DMUs The projected values have great importance, as by arriving at the projected values they have an opportunity to become 100 % efficient, and the projected value help guiding how to reduce inputs and improve outputs . For example, firm C efficiency score is 0.77 and projected values lead it to raise its score and become 100% efficient by reducing project management expenses by 22.47% and it has to increase its cost performance by 386.45% which means to manage the cost of projects to increase the number of projects delivered on budget , and increase Safety and environmental performance 1/SA, E performance by 999.9% which means to decrease the number of recordable accidents by this ratio and put polices which raise performance of environmental aspects to increase the percentage of commitment to ISO 14000 standards by 999.9% more than current ones. Also it has to increase Change in scope of work 1/Cw performance by 136.39% which means to reduce cost of change by this ratio. In addition to increase the social benefit 1 performance by 149.68% which means increasing the number of permanent employees who have health and security assurance ,and social benefit 2 by 22. 74% that means to increase the value of annual executed projects and with no change in profit, schedule, and customer satisfaction values.

Also the firm A which has 0.52 efficiency score has to reduce project management
expenses by 47.59% and has to increase its cost and schedule performance by 619.84% and 0.69% respectively, and it has to increase environmental performance E performance by 999.9%. Also has to increase performance in Change in scope of work 1/Cw by 194.03% and customer satisfaction performance by 45.5% and social benefit performance by 999.9% with no change on safety, profit, and social benefit 2 performance.

We also tried to rerun the DEA model with the absolute data for the selected input which is project management expenses and one of outputs which is profit. They are the only available absolute data besides to safety and social benefit metrics, as the other metrics was given as ratio data by respondents companies. Table 4 in appendix A shows the descriptive statistics about performance metrics with absolute values. Table 5 shows descriptive statistics about the results of DEA model, the number of DMUs is 37 included 5 efficient and 32 inefficient, the average of scores is 0.52.

**Tables 5: descriptive statistics about the results of CCR model with the absolute data for selected input & output**

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<th>No. of DMUs in Data</th>
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<td>Average of scores</td>
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<td>No. of efficient DMUs</td>
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<tr>
<td>No. of inefficient DMUs</td>
<td>32</td>
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<tr>
<td>No. of over iteration DMUs</td>
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</tr>
</tbody>
</table>

Table-6 in appendix A shows the results of the CCR model using CCR-I model of DEA with absolute values for some metrics (we displayed part of them). We can see a little change in firms' scores, for firm A the score was 0.52 with ratio values to 0.5 when using absolute values. Also firm b which was efficient with score 1 to inefficient with score 0.99 when increasing number of absolute metrics.

The number of efficient firms decreased from 8 to 5 when using absolute values.

On reference to (Emrouznejad & Amin, 2009) it is advised to use the DEA model with the absolute data when they are available for the selected metrics.

**CONCLUSION & RECOMMENDATIONS:**

The benchmarking model was developed using field data collected from 37 Syrian construction firms. The analysis using the DEA software appeared to show that 29 of the 37 firms are functionally inefficient and they became 32 when using absolute Data for some metrics of performance. The software also provided each firm with projected values that they must meet in order to become efficient.

This developed benchmark model addresses the limitations that have been previously identified in other benchmarking models. The ability of this benchmark model to better access economical, technical, environmental and social performance for construction firms is what makes it superior. The model currently used industry-relevant metrics most frequently identified in the literature, and measures the overall efficiency of a construction company by adding new performance metrics that are required in this currently highly competitive market. The newly developed model allows Syrian construction firms to be evaluated for overall performance and identify specific areas for improvement. It also allows for individual firms to become as efficient as the most efficient firms in industry. This model is distinguished by its ability to analyze large number of firms (DMUS) and metrics (inputs and outputs).

We recommend the use of the developed benchmarking model as the first step in improving the performance of Syrian construction firms. Also to help in the expansion of research by encouraging construction firms to apply this model to benchmark performance in more types of industry.

**REFERENCES**


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<td>8.124</td>
<td>1647.666</td>
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</table>

Table 1: descriptive statistics about the results of CCR model

Table 3: results of the CCR-I model

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Figure 1 - shows the respondents position by annual profit

---

180
Table 4: descriptive statistics about the results of CCR model with absolute values for PM EX and P PER

<table>
<thead>
<tr>
<th></th>
<th>PM EX</th>
<th>S PER</th>
<th>C PER</th>
<th>O PER</th>
<th>1/S PER</th>
<th>I PER</th>
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<td>5.34146</td>
<td>41335593</td>
<td>1647.6589</td>
<td>713716512</td>
</tr>
</tbody>
</table>

Table 6: results of the CCR-I model with selected absolute values

<table>
<thead>
<tr>
<th>No.</th>
<th>DMU</th>
<th>Score Data</th>
<th>Projection</th>
<th>Difference</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>PM EX</td>
<td>7600000</td>
<td>7600000</td>
<td>-0.00%</td>
</tr>
<tr>
<td></td>
<td>S PER</td>
<td>100</td>
<td>470</td>
<td>1470000</td>
<td>1470000%</td>
</tr>
<tr>
<td></td>
<td>C PER</td>
<td>850</td>
<td>640</td>
<td>2100000</td>
<td>2100000%</td>
</tr>
<tr>
<td></td>
<td>O PER</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>1/S PER</td>
<td>1000</td>
<td>992.14</td>
<td>0.99214</td>
<td>0.99214%</td>
</tr>
<tr>
<td></td>
<td>I PER</td>
<td>0.12</td>
<td>0.0</td>
<td>0.02</td>
<td>0.02%</td>
</tr>
<tr>
<td></td>
<td>1/G PER</td>
<td>0.02</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02%</td>
</tr>
<tr>
<td></td>
<td>P PER</td>
<td>30000000</td>
<td>30000000</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>SP PER</td>
<td>10</td>
<td>115</td>
<td>105</td>
<td>105%</td>
</tr>
<tr>
<td></td>
<td>SP2 PER</td>
<td>20000000</td>
<td>20000000</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>PM EX</td>
<td>74263.297</td>
<td>-7467.7032</td>
<td>-1.00%</td>
</tr>
<tr>
<td></td>
<td>S PER</td>
<td>161</td>
<td>61</td>
<td>100</td>
<td>61.53%</td>
</tr>
<tr>
<td></td>
<td>C PER</td>
<td>70</td>
<td>146.2847</td>
<td>46.2847</td>
<td>67.57%</td>
</tr>
<tr>
<td></td>
<td>O PER</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>1/S PER</td>
<td>1000</td>
<td>923.1923</td>
<td>73.1923</td>
<td>7.232%</td>
</tr>
<tr>
<td></td>
<td>I PER</td>
<td>10</td>
<td>181</td>
<td>181</td>
<td>1.00%</td>
</tr>
<tr>
<td></td>
<td>1/G PER</td>
<td>0.12</td>
<td>0.1468016</td>
<td>0.1468016</td>
<td>0.1468016%</td>
</tr>
<tr>
<td></td>
<td>P PER</td>
<td>12500000</td>
<td>1172828179</td>
<td>90.10%</td>
<td>90.10%</td>
</tr>
<tr>
<td></td>
<td>SP PER</td>
<td>6</td>
<td>3563.2812</td>
<td>3563.2812</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>SP2 PER</td>
<td>25600000</td>
<td>25600000</td>
<td>0</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
Evaluation of electricity distribution sector in Iran using parametric and non-parametric methods

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Management Department, Ferdowsi University of Mashhad, kazemi@um.ac.ir

ABSTRACT
This paper conducts a comparative technical efficiency analysis of 36 Electricity Distribution firms using data of 2007-2008 and two Methods: Deterministic Frontier Production Function (DFPF) and Data Envelopment Analysis (DEA). The results of DEA and DFPF Analysis are compared with correlation method. This comparison indicates a relatively good correlation.

Keywords: Electricity Distribution Sector, Parametric Method, DEA

INTRODUCTION
This study applies two techniques of efficiency measurement in a sample of electricity distribution firms in Iran. The aim of this paper is to compare the relative efficiency of 36 firms working in the Electricity Distribution Sector in Iran. For this purpose DEA and DFPF models are applied to evaluate the efficiencies of the firms. Detailed statistics from Iranian Electricity consumption for the years 2007-2008 has been used as the source of information for the variable of models.

PREVIOUS STUDIES
Cullman et al. (2008) tested the hypothesis that the economic transition toward a market economy increases the efficiency of firms. They studied 32 polish electricity distribution companies between 1997 and 2002, by applying common benchmarking methods to the panel, the nonparametric data envelopment analysis (DEA), the free disposal hull (F.D.H), and, as a parametric approach, the stochastic frontier analysis (S.F.A). They found that the technical efficiency of the companies has indeed increased during the transition, while allocate efficiency has deteriorated.

Hattori (2001) conducted U.S-Japan comparison of performance of electric utilities during 1982 through 1997, but it focuses on electricity distribution and used stochastic Frontier Analysis to estimate technical efficiency of the utilities. Jamasb and Pollitt (2001) reported an efficiency of 63 European electricity distribution utilities to assess the potential of and issues involved in the use of cross-country analysis as input incentive regulation process. The sample includes utilities from the UK, Norway, Netherlands, Portugal, Italy and Spain. They used DEA, SFA and COLS method with Different models specification to a set of data from 1997-8.

PARAMETRIC METHODS
Cullman et al. (2008) tested the hypothesis that the economic transition toward a market economy increases the efficiency of firms. They studied 32 polish electricity distribution companies between 1997 and 2002, by applying common benchmarking methods to the panel, the nonparametric data envelopment analysis (DEA), the free disposal hull (F.D.H), and, as a parametric approach, the stochastic frontier analysis (S.F.A). They found that the technical efficiency of the companies has indeed increased during the transition, while allocate efficiency has deteriorated.

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MUTILLO-ZAMORANO L.R. et al. (2001) reported that parametric frontier models and non-

parametric methods have monopolized the recent literature on productive efficiency measurement. Park Soo-Uk and I.B. Lesourd (2000) conducted the efficiency of conventional fuel power plants in South Korea by comparison of parametric and non-parametric approaches.

DATA
Using data are three inputs and one output for 36 firms in Electricity distribution sector in Iran. In the models inputs and output are as follows:

\[ X_1 = \text{number of professional employees} \]
\[ X_2 = \text{low voltage line (km)} \]
\[ X_3 = \text{capacity of transformers (MVA)} \]
\[ Y = \text{Total Electricity sales (MWh)} \]

MODELS
The Parametric and Non-Parametric methods have been applied to measure efficiency in electricity distribution sector by researchers. In this study, the CCR input oriented model is used to measure relative technical efficiency with constant return to scale (CRS). In modeling deterministic frontier production function (DFPF) for electricity distribution sector, it was assumed that total Electricity sales (y) are produced by three inputs: X_1, X_2, X_3. A Cobb-Douglas functional form was first applied as follows:

\[ y^* = A_0(X_1^{a_1})(X_2^{a_2})(X_3^{a_3}) \]

The deterministic frontier production that has not a random error function is estimated by using the Linear Programming Model as follows (Aigner et al., 1977):

\[ \min Z = a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 \]

\[ \text{s.t.:} \]
\[ a_0 + a_1 (\ln X_{j1}) + a_2 (\ln X_{j2}) + a_3 (\ln X_{j3}) \geq \ln (y_j) \quad j = 1 - 36 \]
The estimated model is as follows:
\[ y^* = A_0 (X_1^{a_1})(X_2^{a_2})(X_3^{a_3}) \]
where, \( A_0 = e^{a_0} = 8645.5 \)
\[ a_0 = \ln A_0 \]

The coefficient of professional employees and low voltage line are approximately zero, therefore in the final model, only capacity of transformers included.

In parametric model, the efficiencies are computed as follows:
Efficiency of \( j \) firm = \( y_j / y_j^* \)

Where:
\( y_j \) = the actual output of \( j \) firm
\( y_j^* \) = the function output of \( j \) firm
\( j = 1, 2 \ldots 36 \)

RESULTS

Results in terms of both Non-parametric (CCR) and parametric (deterministic frontier production function) efficiencies are given in Table (1) and (2).

The comparison of correlation coefficient (\( r = 0.824 \) in table (3)) indicates a good correlation between two approaches. In this case a null hypothesis (no difference in efficiencies average as evidenced by two approaches) cannot be rejected at the significant level of 0.05. The results of this hypothesis test are given in table (4).

The efficiency measures generated by the different approaches have similar means.

REFERENCES


<table>
<thead>
<tr>
<th>#</th>
<th>EFF1</th>
<th>EFF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.867</td>
<td>0.860</td>
</tr>
<tr>
<td>2</td>
<td>0.678</td>
<td>0.670</td>
</tr>
<tr>
<td>3</td>
<td>0.687</td>
<td>0.720</td>
</tr>
<tr>
<td>4</td>
<td>0.630</td>
<td>0.570</td>
</tr>
<tr>
<td>5</td>
<td>0.830</td>
<td>0.870</td>
</tr>
<tr>
<td>6</td>
<td>0.722</td>
<td>0.840</td>
</tr>
<tr>
<td>7</td>
<td>0.640</td>
<td>0.550</td>
</tr>
<tr>
<td>8</td>
<td>0.815</td>
<td>0.750</td>
</tr>
<tr>
<td>9</td>
<td>0.609</td>
<td>0.630</td>
</tr>
<tr>
<td>10</td>
<td>0.568</td>
<td>0.570</td>
</tr>
<tr>
<td>11</td>
<td>0.781</td>
<td>1.0</td>
</tr>
<tr>
<td>12</td>
<td>0.679</td>
<td>0.630</td>
</tr>
<tr>
<td>13</td>
<td>0.872</td>
<td>0.930</td>
</tr>
<tr>
<td>14</td>
<td>0.904</td>
<td>0.994</td>
</tr>
<tr>
<td>15</td>
<td>0.884</td>
<td>0.750</td>
</tr>
<tr>
<td>16</td>
<td>0.624</td>
<td>0.540</td>
</tr>
</tbody>
</table>
# | EFF1 | EFF2 | # | EFF1 | EFF2
---|---|---|---|---|---
17 | 1.0 | 0.740 | 35 | 1.0 | 0.760
18 | 0.797 | 0.760 | 36 | 0.905 | 0.900

#: Number of firms

EFF1: CCR efficiency (with constant returns to scale)
EFF2: Parametric efficiency (deterministic frontier production function)

<table>
<thead>
<tr>
<th>Pair</th>
<th>Mean</th>
<th>N</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFF1</td>
<td>0.7234</td>
<td>36</td>
<td>0.15117</td>
<td>0.02520</td>
</tr>
<tr>
<td>EFF2</td>
<td>0.6997</td>
<td>36</td>
<td>0.15947</td>
<td>0.02658</td>
</tr>
</tbody>
</table>

Table (2): Comparison: T-test results, paired samples statistics

<table>
<thead>
<tr>
<th>N</th>
<th>Correlation</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair EFF1 &amp; EFF2</td>
<td>36</td>
<td>0.824</td>
</tr>
</tbody>
</table>

Table (3): Paired Samples Correlations

<table>
<thead>
<tr>
<th>Paired Differences</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>St. Error Mean</th>
<th>95% confidence Interval</th>
<th>t</th>
<th>df</th>
<th>Sig.(2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair EFF1-EFF2</td>
<td>0.02372</td>
<td>0.09241</td>
<td>0.1540</td>
<td>-0.00754</td>
<td>0.05499</td>
<td>1.154</td>
<td>35</td>
</tr>
</tbody>
</table>

EFF1: CCR efficiency
EFF2: Parametric efficiency
Manufacturing performance measurement using Data Envelopment Analysis

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bagherpour@iust.ac.ir

ABSTRACT
In today highly competitive markets, manufacturing firms should analyze production efficiency in order to identify and improve their weakness. Manufacturing performance measurement thus can be efficiently used in order to detect efficient period-, products, in manufacturing processes. In this paper, a data envelopment analysis (DEA) is applied in a production planning problem in order to analyze production performance. In this regard, the production planning problem firstly is considered through Earned value analysis to detect production performance indexes from cost and schedule points of view. Further, the obtained results have considered within a DEA model. The research has addressed the application of DEA in manufacturing systems while the other related researches had been carried out in the other areas such as organizational units. The approach is successfully implemented and the results are given.

Keywords: Data Envelopment analysis, performance measurement, production planning and control

INTRODUCTION:
Data envelopment analysis is the most important techniques used in order to measurement system performance according to corresponding decision making units (DMU). The results will be clearly shown the performance of each DMU. Further, the best DMU can be identified which to be taken as benchmark for the other DMUs. DEA usually applies based on linear programming model. It also has frequently used in organizational units such as hospital, bank and etc. DEA; not only consider outputs but also covers both inputs during performance analysis.

The importance of using DEA comes from the fact that it simultaneously considered both inputs and outputs while it is really difficult to measure performance in such multi inputs – multi outputs systems. DEA applications can be mostly found in services rather than industry. However, in this paper, DEA is used in order to measure manufacturing performance. The typical DEA model can be stated as follows: [1]

\[
\begin{align*}
\text{Max} & \phi = \varepsilon (\sum s_i^- + \sum s_i^+) \\
\text{S.t.} & \sum x_{ij} \lambda_j - s_i^- = x_{io} \\
& \sum y_{rj} \lambda_j - s_i^+ \leq \phi y_{io} \\
& \lambda_j \geq 0
\end{align*}
\]

In this model:
\( \lambda \) s are variables of dual model of model 1.
\( s^+, s^- \) are slack variables.

Above model should be solved for each DMU once in order to calculate the efficiency degree.

As an example, DEA is used to estimate the Bank services costs and to present an efficient and reliable method for bank branches [2]. DEA is also used to evaluate the failure mood of equipments [3]. Product comparison and their performances, is other field of DEA applications [4]. A combination of DEA and other algorithms (such as fuzzy algorithm) is used to estimate the performance of decision making units (DMUs) [5], [6]. The main objective of this research is to review the application of this method in estimating the performance of production systems. MPMP is used as production system in this research. The MPMP is a system of producing multiple products in multiple periods. The influential
elements of this system are: Demand, production costs, equipments capacity, cycle time, holding and slack costs. Besides, these elements can be brought in to picture more complicated or simple according to the planner requirements. The main data used in this research is from Baker and Byrne article with some points added for better results [7].

**Production planning problems**

A “Multi Period-Multi Product (MPMP) production planning consists of some machine centers. Every product has several tasks. Each task has to be processed in a specific machine center within a deterministic processing time. Obviously, in a machine center no more than one task can be carried out in the same time. Each product must be processed in a given sequence order. Since the machine centers facing limited capacities constraints, it is required to adapt the production level of products according to the variation of demands for a given number of future periods, while the constraints such as machine centers capacity limits or the precedence relationships between the tasks of each product have been considered.

**The importance of applying DEA in production planning problems**

In today highly competitive market, manufacturing firms are looking for competitive advantages in which helps them to develop their efforts through generating continual improvement. However, main problem hereby is to consider efficient DMU and making a benchmark around those DMU that are working properly. In this regard, since production planning problems should be performed in each period – each product level, it is thus logical to consider each period – each product as a DMU. At the end of running DEA model, it is clearly shown that which period – product has been efficiently performed. Moreover, DMU ranking, sensitivity analysis and the other advantages of applying DEA model can be therefore used within manufacturing environment.

**APPLICATION OF DEA IN PRODUCTION PLANNING**

The aim of DEA usage in this Sample will be the Comparison of a production system in different period and for different product to find the most effective systems and determine the aim (pattern) for non effective system or semi effective system. In this Sample, for the use of DEA at first we must make its modeling.

**MODELING**

Before the use of DEA, this system must be analyzed very carefully. This is very necessary for (doing) a suitable modeling. With analyzing this system, all the selections for the selecting of decision making units, inputs and outputs that are the main character of DEA, bring out and are evaluated and the most useful selection will be selected for modeling. In this manner, the decision making units will be describe and determine for each of inputs and outputs.

**DECISION MAKING UNITS (DMUS)**

For modeling is necessary to define DMUs, so number of period and kind of product were employed.

<table>
<thead>
<tr>
<th>Period</th>
<th>Product</th>
<th>DMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>DMU 11</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>DMU 12</td>
</tr>
<tr>
<td></td>
<td>j</td>
<td>DMU 1j</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>DMU 21</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>DMU 22</td>
</tr>
<tr>
<td></td>
<td>j</td>
<td>DMU 2j</td>
</tr>
<tr>
<td>i</td>
<td>1</td>
<td>DMU i1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>DMU i2</td>
</tr>
<tr>
<td></td>
<td>j</td>
<td>DMU ij</td>
</tr>
</tbody>
</table>

According to table 1, product number j, produced in period i create DMU ij. So will exist ij DMUs.

**Inputs and outputs**

After selecting the DMUs we must select the needing inputs and outputs. The selecting of proper inputs and outputs for modeling is very important because of the basis of DEA will depend on these inputs and outputs. Wrong selection will have direct and non desirable effects on its conclusions. So the general view of modeling was showed in figure 1.

**Mathematical model**

In it is project, we use from xIDEA software version 2.0 to solve the model and to evaluate the units. In this paper, we use (VRS) BBC model as output oriented. BCC model that use in this paper, is found with added one following constraint ( \( \sum \lambda_j = 1 \) ) to model 2:
$$Max \phi = \varepsilon (\sum s_i^- + \sum s_j^+)$$

Subject to:

$$\sum x_{ij}\lambda_j + s_i^- = x_{io}$$
$$\sum y_{ij}\lambda_j - s_j^+ \leq \phi y_{io}$$
$$\lambda_j \geq 0$$
$$\sum \lambda_j = 1$$

(3)

Case study

In sample of this paper, 3 kind of products, produce in 3 separate periods, so the production in this system, perform in 9 processes (refer to table 2).

Table 2: Define DMUs for the case

<table>
<thead>
<tr>
<th>Period</th>
<th>Product</th>
<th>DMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>DMU1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>DMU2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>DMU3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>DMU4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>DMU5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>DMU6</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>DMU7</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>DMU8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>DMU9</td>
</tr>
</tbody>
</table>

With attention to the selection of units, the application of each of these units assessed in production and their effectiveness will be compared. In this sample, after the assessing probable selections for Inputs and outputs the sum of planned cost (SC) select as Input and the ratio of real production to demand (P/D) and CPI (Cost Performance Index) factor select as outputs of the problem. The main aim of this problem, is the determine of production system in different periods that the demand degree and the satisfied demand degree have important roles, so these 2 items will be the main points of this problem that consider in P/D ratio. Besides this we must consider the system cost, because of this we consider the inputs and outputs that including cost factors, such as production cost, inventory holding cost and etc. Thus to prevention from problem complexity and improving the resulted answers, surrender the inputs and outputs that including time fetors.

Besides the actual production degree, that selected proportional to demand degree as an output, because the above mentioned for inputs and also because of the improvement of problem answers in outputs selection, we consider the outputs have cost factor (same inputs) and surrender from the other outputs. It is important to say that in showing samples by Bakir, haven’t any actual cost, but in this problem, because of the cost study in Comparison to units function and for configure the CPI factor, this case added to the model with hypothetical degrees. So among the outputs that consists of cost factor, CPI as a good element that involve all the system costs (actual cost, planned cost, etc), selected as a second output. Inputs and outputs of this sample were explained in table 3.

Table 3: Amounts of inputs & outputs for each DMU

<table>
<thead>
<tr>
<th>DMU</th>
<th>SC</th>
<th>CPI</th>
<th>P/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.833</td>
<td>0.909</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>0.833</td>
<td>0.042</td>
<td>0.12</td>
</tr>
<tr>
<td>13</td>
<td>0.8</td>
<td>0.16</td>
<td>0.455</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>0.962</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>0.556</td>
<td>0.088</td>
<td>0.233</td>
</tr>
<tr>
<td>23</td>
<td>0.606</td>
<td>0.132</td>
<td>0.394</td>
</tr>
<tr>
<td>31</td>
<td>0.781</td>
<td>0.941</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>0.556</td>
<td>0.004</td>
<td>0.013</td>
</tr>
<tr>
<td>33</td>
<td>0.8</td>
<td>0.149</td>
<td>0.431</td>
</tr>
</tbody>
</table>

The name of DMUs contains number of periods (first character) and number of products (second character). It is important to say that although DEA don’t need to data with the same degree, but for improving the results, this data was normalized by use of normalization method and then it use on this model. The other point is that with respect to shortage of DMU numbers and the correlation between some of data, it had used of the smaller number input and output (totally 3 inputs and outputs). Because of the use of more inputs and outputs have a bad effect on accuracy and accurate. General view of this case is illustrated in figure 2.

Figure 8: General view of sample case

MODEL SOLVING:

In table (4) its conclusion (results) founded by use of software. (According to equ. 3)

Table 4: Result of the model using sIDEA software

<table>
<thead>
<tr>
<th>DMU</th>
<th>Peers</th>
<th>Inputs/Outputs</th>
<th>Slacks</th>
<th>Score</th>
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<td></td>
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<td>22 31 SC CPI P/D</td>
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</table>

In this table the Peers columns contains information about the efficient peers of each DMU and the relevant weights. The main use of the slacks results however is to check that a DMU with efficiency score 1 is indeed fully
efficient. This is the case if all its slacks are zero.

**RESEARCH FINDINGS**

**Performance evaluation**

Between studded units, four unit reach to 1 score. These units are as follows: 11, 21, 22 and 31. Peer units select from them. If we select the efficiency score above 0.9 as good, under 0.5 as weak and the other score select as middle, we find out that between 9 units, select 1 unit as middle efficiency, 3 units select as weak efficiency and 5 unit select as good efficiency. So, it says that this system has a proper efficiency.

Between 4 units with 1 efficiency score, unit 11 is a weak efficiency unit and can’t be a peer because of its slack in CPI as output, then only units 21, 22, and 31 will be the peer between all the units. Between efficiency peers, unit 31 is peer for 5 units, so this unit is the most usage peer. Refer to table (4). Table (5) shows the potential improvement for all DMUs.

<table>
<thead>
<tr>
<th>Table 5: virtual amount of the inputs &amp; outputs</th>
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</table>

These results of the Virtual inputs and outputs table show the potential improvement for all DMUs. The first column in each input/output contains the target value and the second column shows the corresponding percent decrease/increase.

**STUDY OF SOME SELECTED UNITS**

**Unit 11:**

To be efficient this unit, don’t need to many changes, of course this is a normal problem because the efficiency of this unit equal to 1 and only has a slack that must be removed them. And this slack removes by sampling from units 21 and 31. As the table (4) is showing the changes are very little. With refer to amount of slacks in table (4) determine that all 4 percent result in changeable in this input.

**Unit 12**

To reach this unit to efficiency, need many changes. The most important changes for this unit showing on table (5).

Increase of 2152% CPI and reach it to 0.95.
Increase of 733% P/D and reach it to 1.

The highest greatness of changes percent are blous of the sample do that it seems that untrue but it can be show more attention on the unit.

**SENSITIVITY ANALYSIS**

With the study of these changes on the units we obtain these results:

The most changes implemented on unit 32 and at whole the greatest need are on the second product. The extents of these changes are very high in comparison with the other units that need to special decision making. In this diagram, the greatest need on changes observes in second production.

The most changes, implemented on unit 33 inputs that shows its inputs more attention. And at whole it can be say that the third production in second and third periods and second production in first and second periods need to the most changes in their inputs.

For studding the effect of each of costs, we can consider them as an input individually or simultaneously. For example with respect to inventory holding cost (IHC) instead of the sum of scheduling cost as input, in this input and CPI as output, look very extent slacks (table 6).

<table>
<thead>
<tr>
<th>Table 6: Result of replacement of IHC with SC</th>
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<tbody>
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<td>Virtual inputs / outputs</td>
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<td>33</td>
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</table>

This table shows needed change in production system, with replacement IHC as study base input. This table shows remarkable decrease in IHC that demonstrates these cost are very high undue. Of course this item is justifiable with respect to that in this sample there aren’t any predation excess in any period. Also, very high shortage cost in second production is the most important obtained result from the replacement of shortage cost (lost sale cost (LSC)) with the sum of scheduling cost (table 7).

<table>
<thead>
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<th>Table 7: Result of replacement of LSC with SC</th>
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</table>
This table shows some slack in LSC of product 2 and also in outputs, changes shift from first period to two last periods.

### CONCLUSION

The result of this study shows that, DEA can be a useful tool for managers and decision makers to analyze efficiency of production systems. It also can be used as powerful technique for creating competitive advantage (especially cost reduction) in production systems. Recognition and deleting the excess SC have very useful benefit to a company.

A case in simple MPMP production system is given for analyzing manufacturing performance measurement which much more complex production systems will be further developed.

Further research may be carried out in the area of parallel production systems.

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Efficient utilization of information and communications technology and transparency in government: a nonparametric analysis

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ABSTRACT

The state of development of Information and Communications Technology (ICT) has rapidly emerged as an important determinant of the level of overall economic and social development of a nation. Also, there is general agreement that availability and utilization of ICT brings both the government and the corporate sector under closer scrutiny by the public and thereby contributes to both financial accountability and overall transparency in government. In this paper we conceptualize a production process where the different components of the information capital at the disposal of a nation are used to produce transparency in governance.

**Keywords:** Information and communications technology, data envelopment analysis,

INTRODUCTION

The Economist Intelligence Unit (EIU) of *The Economist* magazine publishes an annual report of what is described as the level of “e-readiness” of different countries. At the same time, Transparency International, a highly respected watchdog group monitoring levels of corruption across countries publishes a report card on Corruption Perception Index (CPI) every year. Despite some measure of overlap, the scores reported by the two agencies are constructed from different surveys and provide complementary information.

In the report, EIU argues that when a country uses ICT to conduct more of its activities, the economy can become more transparent and efficient. Further, it also claimed that their e-readiness ranking allows the governments to gauge the success of their technology initiatives against those of other countries. However, their overall “e-readiness” score is a weighted sum of various scores measuring different components of the information infrastructure of a country and is, in essence, a measure of the available information capital input. To measure the extent to which a country has been able to properly utilize this input, one must consider some output produced with it. The volume of output that is actually produced from some bundle of inputs depends on the efficiency of the producer. Contrary to what is implicitly suggested in the EIU Report, a higher e-readiness score of a country does not automatically translate into a higher level of transparency. For example, in 2009 South Korea ranked ahead of Japan both in the overall e-readiness score and in respect of several important components of information capital (like connectivity), nevertheless Japan ranked 17th in the transparency ranking (with a CPI score of 7.7) while South Korea ranked 39th (with a score of 5.5). Measurement of the technical efficiency of a nation permits one to determine whether a poor transparency score is the result of inadequate development of information capital or is due to inefficiency in properly utilizing the available ICT.

DEFINITION AND MEASUREMENT OF INPUTS AND OUTPUT:

For efficiency measurement we consider a 1-output 6-input variable returns to scale output-oriented radial DEA model. The single output (y) is measured by the country specific score of Corruption Perception Index constructed by Transparency International. For the different inputs, we consider 6 individual components of ICT reported by EIU in their e-readiness report. The various inputs are indexes of: connectivity (x₁), business environment (x₂), social and cultural environment (x₃), legal environment (x₄), government policy and vision (x₅), and consumer and business applications (x₆). The different input measures are based on the following:

**Connectivity:** broadband penetration; broadband affordability; mobile phone penetration; mobile phone affordability; internet user penetration; international internet bandwidth; internet security.

**Business Environment:** overall political environment; macroeconomic environment; market opportunities; policy towards private
enterprise; foreign investment policy; foreign trade and exchange rate regimes; tax regime; financing; and labor market.

Social and Cultural Environment: education (school life expectancy and gross enrolment); internet literacy; degree of entrepreneurship; technical skills of the workforce; degree of innovation (generation of patents and trademarks and RD spending).

Legal Environment: effectiveness of traditional legal framework; laws covering the internet; levels of censorship; ease of registering an new business; electronic ID.

Government Policy and Vision: per capita government spending of ICT; digital development strategy; e-government strategy; online procurement; availability of public services for citizens and business online; e-participation (UN index of e-participation).

Consumer and Business Adoption: consumer spending per capita on ICT; level of e-business development; use of internet by consumers; use of online public services by citizens and business.

EIU takes a weighted average of the scores in the individual categories to construct an overall score of e-readiness of a country. It may be argued, however, that of the 6 different components, only connectivity (x₁) reflects the state of information infrastructure directly. Other factors (like educational enrolment, legal environment, and frequency of consumer purchases online) enhance the productivity of connectivity and serve more like facilitating attributes. Instead of aggregating them into a single input measure, we decided to treat each individual component as a separate input.

THE EMPIRICAL ANALYSIS

The BCC output-oriented DEA model was solved for each of the 4 years (2006, 2007, 2008, and 2009). Countries included in the sample during a specific year were those for which data were available both for the transparency reports and the e-readiness reports. Efficiencies were measured both with reference to annual contemporaneous frontiers for each year.

PRELIMINARY FINDINGS

Based on the initial empirical analysis some tentative conclusions can be drawn:

- The mean technical efficiency improved between 2006 and 2007 from 0.837 to 0.888. In 2008 it showed a slight decline (to 0.861) but increased to 0.871 in 2009. However, it did not reach the level attained in 2007.

- The distribution of efficiency (judged by the location of the first and the third quartiles) shifted downwards over 2006-2008 but recovered in 2009;

- Most of the Nordic countries showed high levels of technical efficiency over all of the years. These included Denmark, Sweden, Switzerland, Finland, and Norway. New Zealand and Netherlands were among the other advanced economies that were highly efficient. Singapore was near the top in terms of efficiency. All of these countries had high e-readiness and ended up with high achievement in terms of transparency;

- The Latin American countries were, in general, inefficient. Countries like Argentina, Mexico, and Venezuela performed poorly. Compared to them, Chile and Peru were far more efficient.

- In North America Canada was fairly efficient. By comparison USA performed far worse.

- Many countries (like Pakistan) had low ICT and low transparency. But there is no evidence that they were inefficient in utilizing whatever IT capital they had.

CONCLUSION

Countries found to be fully efficient include high transparency nations like Denmark and others like Pakistan and India (where corruption is rampant) on the one hand and China and Iran (where political repression is high). Nations like Denmark or Finland are on the frontier because they have greater IT capital and make the best use of what is available. Better utilization of available IT capital and socio-political infrastructures would lead to greater transparency in developed economies like Italy and Greece. In the less developed world, lack of IT capital as well as undeveloped political institutions seems to be the problem.

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## APPENDICES

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DEA Theory Stream
Overcoming the infeasibility of super-efficiency DEA model: A model with generalized orientation

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ABSTRACT

The super-efficiency model is identical to the standard model, except that the unit under evaluation is excluded from the reference set. This model has been used in ranking efficient units, identifying outliers, sensitivity and stability analysis. Under the assumption of variable, non-increasing and non-decreasing returns to scale (VRS, NIRS, NDRS), the super-efficiency model may be infeasible for some efficient DMUs. Based on the necessary and sufficient conditions for the infeasibility of super-efficiency, the current paper develops a DEA model with generalized orientation to overcome infeasibility issues. The DEA model with generalized orientation extends the orientation of DEA model from the traditional input-orientation and output-orientation to non-orientation, modified input-orientation, input-prioritized non-orientation, modified output-orientation and output-prioritized non-orientation. All the extended orientations are always feasible in their super-efficiency models. In addition, the modified input-oriented model and the modified output-oriented model overcome the problem of infeasibility in super-efficiency models and at the same time keep the concordance with the traditional oriented models. The newly developed model is illustrated with a real world dataset.

Keywords: Data envelopment analysis (DEA), Super-efficiency, Infeasibility, Orientation

INTRODUCTION

Data envelopment analysis (DEA), originally developed by Charnes et al (1978), is a linear programming methodology for evaluating the relative technical efficiency for each member of a set of peer decision making units (DMUs) with multiple inputs and multiple outputs. A weakness of DEA is that there typically exist more than one unit which is evaluated as efficient when the number of DMUs is not big enough relative to the number of inputs and outputs. An approach to overcome this weakness is what is called super-efficiency model developed by Banker et al (1989) and Anderson and Petersen (1993). The super-efficiency model is identical to the standard model, except that the unit under evaluation is excluded from the reference set. Super-efficiency score is a measure for efficiency of the evaluated unit relative to the frontier created by the other units. This model has been used in ranking efficient units, identifying outliers, sensitivity and stability analysis. However, super-efficiency model may not have feasible solutions under variable, non-increasing and non-decreasing returns to scale (VRS, NIRS, NDRS) technology. Many efforts have been made for the infeasibility problem. Seiford and Zhu (1999) provide the necessary and sufficient conditions for the infeasibility of super-efficiency. Lovell and Rouse (2003) developed an equivalent standard DEA model to provide super-efficiency scores by scaling up the inputs (scaling down the outputs) of a DMU under evaluation. And the super-efficiency score for an efficient unit without feasible solutions in the standard super-efficiency model is equal to the user-defined scaling factor. Chen (2005) suggests that both the input- and output- oriented super-efficiency models be used to measure the super-efficiency scores when infeasibility occurs, which will be a failure if both the input- and output- oriented super-efficiency models are infeasible for an efficient unit.

In the current paper, we develop a modified model with generalized orientation to resolve the infeasibility problem in VRS, NIRS and NDRS super-efficiency models, and discuss the special cases of the generalized model that equivalent to the oriented models. It is shown
that the new approach yields identical standard efficiency scores and super-efficiency scores to those from its counterparts of traditional model, and yields optimal solutions and scores for efficient DMUs that are infeasible in the traditional model. The current paper presents the modified model for VRS only, but it can be easily extended to NIRS and NDRS.

THE NEW MODEL WITH GENERALIZED ORIENTATION

Infeasibility occurs when an efficient DMU \( k \) cannot reach the frontier formed by the rest of the DMUs by either increasing inputs in the input-oriented model or decreasing outputs in the output-oriented model. In such cases, both input increasing and output decreasing must be made for the efficient DMU \( k \) to reach the frontier. In order to resolve the problem of infeasibility, a non-oriented model must be developed, in which both input increasing and output decreasing are permitted for an efficient DMU to reach the frontier formed by the rest of the DMUs.

Consider the following model for DMU \( k \):

\[
\begin{align*}
\min & \quad \frac{1-\alpha}{1+\beta} \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq (1-\alpha)x_{ik}, \quad i=1,2,\ldots,m \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq (1+\beta)y_{rk}, \quad r=1,2,\ldots,s \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \geq 0, \quad j = 1, 2, \ldots, n
\end{align*}
\]

(1)

where both input decreasing and output increasing are permitted for an inefficient DMU to reach the frontier, and the efficiency score is defined as \( (1-\alpha)/(1+\beta) \).

If DMU \( k \) is efficient in model (1), its super-efficiency model is expressed as

\[
\begin{align*}
\min & \quad \frac{1-w_l^I \alpha}{1+w_l^O \beta} \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq (1-\alpha)x_{ik}, \quad i=1,2,\ldots,m \quad (\text{if } w_l^I > 0) \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik}, \quad i=1,2,\ldots,m \quad (\text{if } w_l^I = 0) \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq (1+\beta)y_{rk}, \quad r=1,2,\ldots,s \quad (\text{if } w_l^O > 0) \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, \quad r=1,2,\ldots,s \quad (\text{if } w_l^O = 0) \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \geq 0, \quad j = 1, 2, \ldots, n
\end{align*}
\]

(3)

where \( w_l^I \) and \( w_l^O \) are user-defined non-negative numbers and at least one of them is positive, and the efficiency score is defined as \( (1-\alpha)/(1+\beta) \).

If DMU \( k \) is efficient in model (3), its super-efficiency model is expressed as

\[
\begin{align*}
\min & \quad \frac{1-w_l^I \alpha}{1+w_l^O \beta} \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq (1-\alpha)x_{ik}, \quad i=1,2,\ldots,m \quad (\text{if } w_l^I > 0) \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik}, \quad i=1,2,\ldots,m \quad (\text{if } w_l^I = 0) \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq (1+\beta)y_{rk}, \quad r=1,2,\ldots,s \quad (\text{if } w_l^O > 0) \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, \quad r=1,2,\ldots,s \quad (\text{if } w_l^O = 0) \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \geq 0, \quad j = 1, 2, \ldots, n
\end{align*}
\]

(4)

where \( w_l^I \) and \( w_l^O \) are user-defined non-negative numbers and at least one of them is positive,
and the super-efficiency score is defined as \((1-\alpha)/(1+\beta)\).

In model (3) and model (4), \(w^1\) and \(w^0\) denotes the priority of orientation. The generalized super-efficiency model (4) is always feasible when both \(w^1\) and \(w^0\) have non-zero values.

Table 1 lists seven special cases of the generalized model and their definitions for (super-) efficiency score. In case 1, the generalized model is equivalent to the traditional input-oriented model, and the (super-) efficiency score \(1 - \alpha^*\) in the generalized model is equal to \(\theta^*\) in the traditional input-oriented model. In case 2, the generalized model is equivalent to the traditional output-oriented model, and the (super-) efficiency score \(1/(1 + \beta^*)\) in the generalized model is equal to \(1/\phi^*\) in the traditional output-oriented model. In case 4, input-orientation is given priority with output-orientation retained. The (super-) efficiency score is defined as \((1 - \alpha^*)\) without the denominator \((1 + \beta^*)\), which means that the efficiency score is measured by movements of inputs only. It is a modified input-orientation with the following properties: the standard efficiency score \((1 - \alpha^*)\) is equal to \(\theta^*\) in the traditional input-oriented model, and the super-efficiency score is equal to \(\theta^*\) in the traditional input-oriented model as well when it is feasible. In addition, when the traditional input-oriented super-efficiency model is infeasible, this modified input-oriented model will still yield an optimal solution. The modified input-oriented model overcomes the problem of infeasibility, and at the same time, it keeps the concordance between the modified model and the traditional model.

The equation of the DEA model with generalized orientation is nonlinear programming, but it can be transformed into the linear programming using a method similar to the Charnes–Cooper transformation (Charnes & Cooper, 1978)\(^{14}\).

**APPLICATION**

We apply the modified output-oriented model and the output-prioritized non-oriented model (special case 6 and case 7 of the generalized model) to a real world dataset used in Chen (2004) which consists of the 20 largest Japanese companies in 1999.

Table 2 compares the results from the traditional output-oriented models with those from the modified output-oriented model and the output-prioritized non-oriented model. The efficiency component \(1 + \beta^*\) in the new models is equal to \(\phi^*\) in the traditional output-oriented model. Both the modified output-oriented model and the output-prioritized non-oriented model yield super-efficiency scores (optimal solutions) for DMU18, which is infeasible under the traditional output-oriented super-efficiency model. But note that the output-prioritized non-oriented model cannot yield equivalent standard efficiency scores to the traditional output-oriented model (DMU 4, 7, 9, 10 and 16).

**CONCLUSIONS**

Based on the necessary and sufficient conditions for the infeasibility of super-efficiency, the current paper develops a DEA model with generalized orientation to overcome infeasibility issues. The DEA model with generalized orientation extends the orientation of DEA model from the traditional input-orientation and output-orientation to modified input-orientation, input-prioritized non-orientation, modified output-orientation and output-prioritized non-orientation. All the extended orientations are always feasible in their super-efficiency models. In addition, the modified input-oriented model and the modified output-oriented model overcomes the problem of infeasibility in super-efficiency models and at the same time keep the concordance with the traditional oriented models.

**REFERENCES**


\(^{14}\) The DEA model with generalized orientation has been included in the DEA software “MaxDEA 4.0” (www.maxdea.cn).


Table 1: Special cases of the generalized model and their definitions for efficiency score

<table>
<thead>
<tr>
<th>Case</th>
<th>Model</th>
<th>Standard efficiency model</th>
<th>Super-efficiency model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$w^I$</td>
<td>$1 - \alpha'$</td>
</tr>
<tr>
<td>1</td>
<td>Input-oriented</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Output-oriented</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Non-oriented</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Input-oriented (modified)</td>
<td>1</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>5</td>
<td>Non-oriented (input-prioritized)</td>
<td>1</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>6</td>
<td>Output-oriented (modified)</td>
<td>$\varepsilon$</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Non-oriented (output-prioritized)</td>
<td>$\varepsilon$</td>
<td>1</td>
</tr>
</tbody>
</table>

$\varepsilon$ is the non-Archimedean infinitesimal (in practice, we use $10^{-5}$).

Table 2: Results of special cases of the generalized model for Japanese companies

<table>
<thead>
<tr>
<th>DMU</th>
<th>Company</th>
<th>$1/\phi^*$</th>
<th>$\alpha'$</th>
<th>$\beta'$</th>
<th>$1 - \alpha'/1 + \beta'_{(case 7)}$</th>
<th>$1 - \beta'/1 + \varepsilon_{(case 6)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MITSUI &amp; CO.</td>
<td>1.0152</td>
<td>0</td>
<td>-0.0150</td>
<td>1.0152</td>
<td>1.0152</td>
</tr>
<tr>
<td>2</td>
<td>ITOCHU CORP.</td>
<td>1.3177</td>
<td>0</td>
<td>-0.2411</td>
<td>1.3177</td>
<td>1.3177</td>
</tr>
<tr>
<td>3</td>
<td>MITSUBISHI CORP.</td>
<td>0.9807</td>
<td>0</td>
<td>0.0197</td>
<td>0.9806</td>
<td>0.9807</td>
</tr>
<tr>
<td>4</td>
<td>TOYOTA MOTOR CORP.</td>
<td>0.9119</td>
<td>0.5483</td>
<td>0.0966</td>
<td>0.4119</td>
<td>0.9119</td>
</tr>
<tr>
<td>5</td>
<td>MARUBeni CORP.</td>
<td>0.8913</td>
<td>0</td>
<td>0.1220</td>
<td>0.8913</td>
<td>0.8913</td>
</tr>
<tr>
<td>6</td>
<td>SUMITOMO CORP.</td>
<td>1.0227</td>
<td>0</td>
<td>-0.0222</td>
<td>1.0227</td>
<td>1.0227</td>
</tr>
<tr>
<td>7</td>
<td>NIPPON TELEGRAPH &amp; TEL.</td>
<td>0.6959</td>
<td>0.6173</td>
<td>0.4369</td>
<td>0.6959</td>
<td>0.4369</td>
</tr>
<tr>
<td>8</td>
<td>NISSHO IWAI CORP.</td>
<td>1.1478</td>
<td>0</td>
<td>-0.1288</td>
<td>1.1478</td>
<td>1.1478</td>
</tr>
<tr>
<td>9</td>
<td>HITACHI LTD.</td>
<td>0.5706</td>
<td>0.3113</td>
<td>0.7525</td>
<td>0.5706</td>
<td>0.5706</td>
</tr>
<tr>
<td>10</td>
<td>MATUSHITA ELECTRIC INDL.</td>
<td>0.5465</td>
<td>0.1605</td>
<td>0.8299</td>
<td>0.5465</td>
<td>0.5465</td>
</tr>
<tr>
<td>11</td>
<td>SONY CORP.</td>
<td>0.5134</td>
<td>0</td>
<td>0.9479</td>
<td>0.5134</td>
<td>0.9479</td>
</tr>
<tr>
<td>12</td>
<td>NISSAN MOTOR</td>
<td>0.4707</td>
<td>0</td>
<td>1.1245</td>
<td>0.4707</td>
<td>0.4707</td>
</tr>
<tr>
<td>13</td>
<td>HONDA MOTOR</td>
<td>0.5903</td>
<td>0</td>
<td>0.6941</td>
<td>0.5903</td>
<td>0.5903</td>
</tr>
<tr>
<td>14</td>
<td>TOSHIBA CORP.</td>
<td>0.4183</td>
<td>0</td>
<td>1.3908</td>
<td>0.4183</td>
<td>0.4183</td>
</tr>
<tr>
<td>15</td>
<td>FUJITSU LTD.</td>
<td>0.4883</td>
<td>0</td>
<td>1.0478</td>
<td>0.4883</td>
<td>0.4883</td>
</tr>
<tr>
<td>16</td>
<td>TOKYO ELECTRIC POWER</td>
<td>0.3640</td>
<td>0.2088</td>
<td>1.7475</td>
<td>0.3640</td>
<td>0.2088</td>
</tr>
<tr>
<td>17</td>
<td>NEC CORP.</td>
<td>0.4567</td>
<td>0</td>
<td>1.1898</td>
<td>0.4567</td>
<td>0.4567</td>
</tr>
<tr>
<td>18</td>
<td>TOMEN CORP.</td>
<td>infeasible</td>
<td>-1.8999</td>
<td>0</td>
<td>2.8999</td>
<td>1.0000</td>
</tr>
<tr>
<td>19</td>
<td>JAPAN TOBACCO</td>
<td>0.9563</td>
<td>0</td>
<td>0.0457</td>
<td>0.9563</td>
<td>0.9563</td>
</tr>
<tr>
<td>20</td>
<td>MITSUBISHI ELECTRIC CORP.</td>
<td>0.4414</td>
<td>0</td>
<td>1.2657</td>
<td>0.4414</td>
<td>0.4414</td>
</tr>
</tbody>
</table>
Super efficiency in Data Envelopment Analysis: a distance-based model

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ABSTRACT

Conventional data envelopment analysis (DEA) assists decision makers in distinguishing between efficient and inefficient decision making units (DMUs) in a homogeneous group. However, DEA does not provide more information about efficient units. Super-efficiency DEA models can be used in ranking the performance of efficient DMUs. This research proposes a methodology to determine a distance-based measure of super-efficiency. Then, the DMUs are ranked according to their super-efficiency score.

Keywords: Super-efficiency, Data Envelopment Analysis

INTRODUCTION

In DEA, the best performers have the full-efficiency status denoted by unity, and from experience we know that plural DMUs have this efficient status. Super-efficiency can be used in order to discriminate between these efficient DMUs. Many attempts have been made to determine a scalar measure of super efficiency in DEA. Andersen and Petersen (1993) proposed a procedure for ranking DEA efficient units. Their procedure was later referred to as the super efficiency method. Super efficiency refers to the DEA efficiency measured by excluding the DMU under evaluation from the production possibility set and has been deeply studied in the literature. See for instances Chen(2004), Li et al.(2007). A more detailed review of ranking methods in DEA was provided by Adler et al.(2002).

In this paper, we will propose a super efficiency measure based on the Euclidean distance. Our Euclidean distance based (EDB) measure of super-efficiency can be used to fully ranking of efficient units. If our EDB measure of super-efficiency is applied in ranking of efficient units, infeasibility does not occur. An illustrative application of the methodology to a sample of gas companies from twenty five regions of Iran is given.

A MEASURE OF SUPER EFFICIENCY

In this section, we discuss the super-efficiency issue under the assumption that the $DMU_o$ is CCR-efficient. The production possibility set $T_{(x_o,y_o)}$ spanned by $(x_j, y_j) : j = 1,2,...,n, j \neq o$ is defined as

$$T_{(x_o,y_o)} = \{(x, y) : x \geq \sum_{j=1}^{o} \lambda_j x_j ,$$

$$y \leq \sum_{j=1}^{o} \lambda_j y_j, \lambda_j \geq 0, j = 1,...,n, j \neq o\}$$

Obviously, $T_{(x_o,y_o)}$ is not empty. Before introducing our super efficiency measure, we provide some definitions.

Definition 1- A surface $H = \{(x, y) : -\alpha' x + \beta' y = 0, \alpha, \beta \geq 0 \} \cap T_{(x_o,y_o)}$ is called an efficient supporting surface of $T_{(x_o,y_o)}$ if for each extreme efficient observation $j$, $-\alpha' x_j + \beta' y_j \leq 0$.

Definition 2- Let $H = \{(x, y) : -\alpha' x + \beta' y = 0 \} \cap T_{(x_o,y_o)}$ be a hyperplane. If $(\hat{x}, \hat{y}) \in H$, the point in $H$ nearest to $(\hat{x}, \hat{y})$ in terms of the Euclidean distance is

$$(\bar{x}, \bar{y}) = (\hat{x}, \hat{y}) - \frac{-\alpha' \hat{x} + \beta' \hat{y}}{\sqrt{\langle \alpha, \beta \rangle}} (-\alpha, \beta).$$

It can be verified that the straight line joining $(\bar{x}, \bar{y})$ and $(\hat{x}, \hat{y})$ is perpendicular to the hyperplane at $(\hat{x}, \hat{y})$.

Definition 3- (Reference supporting surface) For a $DMU_o$, a reference supporting surface is an efficient surface of $T_{(x_o,y_o)}$ that contains the reference units of $DMU_o$.

Definition 1 states that an efficient supporting surface of $T_{(x_o,y_o)}$ has to support at least one
extreme efficient unit. The vector $\alpha$ and $\beta$ are selected such that at least one inequalities of $-\alpha'x_j + \beta'y_j \leq 0$ is binding. Toward this end, we use the slack variables $s_j$ and rewrite the inequalities as $-\alpha'x_j + \beta'y_j + s_j = 0$. Let $E$ be the set of indices corresponding to all extreme efficient units. Constraints $s_j \leq (1-\gamma_j)M$, $\gamma_j \in [0,1]$, for $j \in E$, and $\sum_{j \in E} \gamma_j \geq 1$, force at least one of the $s_j$ at zero level ($M$ is a large positive constant).

Clearly, selecting $\gamma_j = 1$ forces the $s_j = 0$. Hence, to determine a reference supporting surface of $DMU_o$, we solve the following mixed-integer linear program

$$\begin{align*}
\max \; & \pi_o = -\alpha'x_o + \beta'y_o \\
\text{s.t.} \; & -\alpha'x_j + \beta'y_j + s_j = 0, \; j \in E, \; j \neq o, \\
& 1\alpha + 1\beta = 1, \\
& s_j \leq (1-\gamma_j)M, \; j \in E, \; (1) \\
& \sum_{j \in E} \gamma_j \geq 1, \\
& \alpha, \beta \geq 0, \; \gamma_j \in [0,1], \; s_j \geq 0, \; j \in E.
\end{align*}$$

in which $1 = (1,1,\cdots,1)$. It is to be noted that in the first constraints of (6) we exclude inefficient units; this means that $DMU_o$ will be evaluated with efficient units. Let an optimal solution to (1) be $(\hat{\alpha}, \hat{\beta})$. Obviously, $\hat{H} = \{(x, y): -\hat{\alpha}'x + \hat{\beta}'y = 0\} \cap T_{(x_o,y_o)}$ is an efficient surface of $T_{(x_o,y_o)}$. By the definition 2, the orthogonal projection of $(x_o,y_o)$ in $\hat{H}$ is $(\hat{x}, \hat{y}) = (x_o, y_o) + \mu(\hat{\alpha}, -\hat{\beta})$ in which $\mu = \frac{\hat{\alpha}'x_o + \hat{\beta}'y_o}{\lVert \hat{\alpha}, \hat{\beta} \rVert^2}$. Clearly, $\mu \geq 0$, because, $DMU_o$ is CCR-efficient and $(x_o, y_o) \notin T_{(x_o,y_o)}$. Let $E_o = \{i: x_o > 0\}$ and $F_o = \{r: y_o > 0\}$. We define the EDB-super-efficiency index $\pi_o(x_o, y_o)$ as

$$\pi_o(x_o, y_o) = \frac{1}{\text{Card}(E_o)} \sum_{x \in E_o} \frac{x_o + \mu x}{x_o} \frac{x_o}{y_o}$$

In which Card(.) is the cardinality of a set.

We illustrate the EDB-super efficiency measure with a small-scale example consisting of five $DMU$'s. The $DMU$'s use two inputs to produce a single output whose value is normalized to one for each $DMU$. Note that $DMU_i$ has zero elements in the first input, e.g. $x_1 = 0$. This means that $DMU_i$ has no function as to the input 1. Also, $DMU_i$ has a small element in the second input. The CCR model indicates that all $DMU$'s are efficient. It can be seen that AP model is infeasible when $DMU_i$ is under evaluation and this model yields to a large score 11.111 to $DMU_i$. We have calculated the super EDB measure for each $DMU$. The data set, the super efficiency score of AP model and the super EDB measure $\pi_o$ are listed in table 1. Our approach shows that $DMU_i$ is the top-ranked $DMU$ followed by $DMU_j$, $DMU_k$, $DMU_l$ and $DMU_m$.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$y_1$</th>
<th>$\phi_o$</th>
<th>$\pi_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>0.09</td>
<td>11.11</td>
<td>3.99(1)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>1.04(5)</td>
<td>1.05</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>1.33</td>
<td>1.17</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>8</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The number in parentheses represents rank.

AN EMPIRICAL STUDY

This section illustrates the EDB-super efficiency measure discussed in this paper with the analysis of gas companies’ activities. The data set consists of 25 gas companies located in 24 regions in Iran. The data for this analysis are derived from operations during 2005. We have used seven variables from the data set as inputs and outputs. Inputs include capital ($x_1$), number of staff ($x_2$) and operational costs(excluding staff costs) ($x_3$), and outputs include number of subscribers ($y_1$), amount of pipe-laying ($y_2$), length of gas network ($y_3$) and the revenue of sold-out gas ($y_4$). Because of some limitations, we didn’t report the original data.

In table 2 we have recorded different efficiency measures. The 2-th and 3-th columns of the table report the optimal values to CCR and AP models, respectively. The CCR model indicates that ten companies (#1, #2, #6, #7, #13, #16, #17, #21, #22 and #23) are full-efficient(see column 2 in table 2). The fourth column of the table reports the super-EDB measure of efficiency $\pi_o$. As the results indicate, in the proposed approach company
#17 is the top-ranked company followed by #6, #23, #13, #1, #22, #16, #7, #21 and #2. (The number in parentheses represents rank.) It is to be noted that based on the results reported in the third column, in model (3) company #6 is the top-ranked company followed by #17, #2, #22, #23, #13, #1, #7, #16 and #21.

Table 2: Results for empirical study

<table>
<thead>
<tr>
<th>( j )</th>
<th>( e_o )</th>
<th>( \phi_o )</th>
<th>( \delta_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.188(7)</td>
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CONCLUSIONS

In this paper, we have proposed a super-efficiency measure based on the Euclidean distance. If the proposed EDB measure of super-efficiency is applied in ranking of efficient units, infeasibility does not occur. The applicability of the models developed is illustrated in the context of the analysis of gas companies' performance.

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Expanded global Malmquist index

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ABSTRACT

The circle-type (global) Malmquist index, based on Data Envelopment Analysis (DEA) models, is one of the prominent indexes for measuring the relative productivity change of decision-making units (DMUs) in multiple time periods. This index, similar to the traditional approach of measuring the productivity changes, standard (contemporaneous) Malmquist index, breaks down into various components. Therefore, DEA models with different technologies are widely used for measuring the impact of several components on productivity changes. For example, in FGLR decomposition, CRS technology is employed and the Malmquist index breaks down into efficiency change (EC) and technological change (TC). In addition, FGNZ decomposition needs both VRS and CRS technologies to break down the Malmquist index into Pure Efficiency Change (PEC), Scale Efficiency Change (SEC), and Technological Change (TC). However, empirical studies show that there are several rules and regulations, which can affect the result of the productivity changes. Therefore, this paper presents a new insight on the global Malmquist index named expanded global Malmquist index for measuring the impact of the rules and regulations on productivity changes and provides a new decomposition of this index.

Keywords: DEA, Expanded Global Malmquist Index, Regular Efficiency Change

INTRODUCTION

The (contemporaneous-type) classic Malmquist index is the most important index, which is practically used in various situations for measuring the relative productivity change of DMUs in multiple time periods by DEA. First, Caves, Christensen, and Diewert [3] introduced the earliest type of the Malmquist index, and then Fare, Grosskopf, Lindgren, and Roos (FGLR) [5] applied DEA for measuring the Malmquist index. In fact, they assumed constant returns to scale (CRS) and identified the technological change and the change of technical efficiency as two components of the productivity changes over time. Subsequently, Fare, Grosskopf, Norris, and Zhang (FGNZ) [6] considered variable return to scale (VRS) and offered an extended decomposition of the Malmquist index with another important factor capturing change in scale efficiency.

The Malmquist index components in this approach use DEA models with different technologies for computing their distance functions. However, each DEA model makes an efficient frontier with the calculation of a maximal efficiency measure for each DMU relative to other observed measures. Therefore, all components are measured based on these constructed efficient frontiers. However, there is another approach to measure the relative productivity changes, which has been recently proposed by Pastor and Lovell [7]. They defined the global form for Malmquist index measurement. In fact, in their way, all observed DMUs from all available periods are used to construct a best practice frontier (global efficient frontier) for the global measurement and all contemporaneous efficient frontiers are employed to compute the traditional components. However, they presented a new insight on the Malmquist index named the (circle-type) global Malmquist index, which was based on the definition of the global efficient frontier. It is important to note that, this index, similar to the traditional approach of measuring the productivity change, breaks down into various components such as efficiency change, scale efficiency change, and best practice change.

As it was mentioned earlier, DEA models with different technologies are used for measuring the productivity changes in both classic and global forms. For example, in FGLR decomposition, CRS technology is employed and the Malmquist index breaks down into efficiency change (EC) and technological change (TC) or, FGNZ decomposition needs both VRS and CRS technologies to break down the Malmquist index into pure efficiency change (PEC), scale efficiency change (SEC), and technological change (TC). Moreover, in global Malmquist index, VRS and CRS technologies are employed and the global Malmquist index breaks down to pure
efficiency change (PEC), scale efficiency change (SEC) and best practice change (BPC).

However, empirical studies showed that there are some important rules and regulations, which can affect on the productivity over periods. In other words, the result of the Malmquist index might change in presence of some additional rules. Therefore, Alirezaee and Afsharian [1] proposed a new modification of the Malmquist index namely, the expanded Malmquist index by the using of trade-offs (TO). In fact, they employed trade-offs as some rules and regulations and defined the expanded production possibility set (PPS) instead of ordinary PPS. Hence, the PPS that was based on trade-offs technology allowed them to present a new insight into the Malmquist index and its decompositions. However, they then showed using VRS, CRS and TO technologies, the expanded Malmquist index can be broken down into pure efficiency change (PEC), scale efficiency change (SEC), regulation efficiency change (REC), and expanded technological change (ETC).

It is noticeable that, the concept of regular efficiency change using trade-offs technology in this literature has been just developed on contemporaneous-type Malmquist index. It seems that improving the meaning of the global efficiency and using expanded PPS, trade-offs technology—instead of VRS and CRS technology in global Malmquist index—is necessary. In fact, this concept can be applied for defining an expanded global Malmquist index, and it can be broken down into different components.

Hence, the remainder of the paper is organized as follows. Section 2 presents a general view of the Malmquist index in both classic and global forms, based on DEA models. Section 3 after presenting the definition of the regular efficiency change, proposes new insight into the global Malmquist index for measuring the impact of the rules and regulations on productivity changes and provides a new decomposition of this index. Concluding remarks appear in section 4.

CLASSIC AND GLOBAL MI FORMS

Consider observed output vector 
\[ Y_j = (y_{j1}, \ldots, y_{jn}) \in \mathbb{R}^n \] 
and input vector 
\[ X_j = (x_{j1}, \ldots, x_{jm}) \in \mathbb{R}^m, \]
we assume that the inputs and outputs are nonnegative and nonzero for \( DMU_j, j=1, \ldots, n \). The following production possibility sets (PPSs) based on constant returns to scale (CRS) and variable returns to scale (VRS) technologies are defined, respectively (for more detail see [4,2]):

\[
PPS_{vrs} = \{(X,Y) \mid Y \leq \bar{Y}, X \geq \bar{X}, \lambda \in R^+ \}
\]

(1)

\[
PPS_{crs} = \{(X,Y) \mid Y \leq \bar{Y}, X \geq \bar{X}, \lambda, 1 = 1, \lambda \in R^+ \}
\]

(2)

where \( \bar{X} \) is the \( m \times n \) matrix with the columns \( X_j (j=1, \ldots, n) \) and \( \bar{Y} \) is the \( s \times n \) matrix with the columns \( Y_j (j=1, \ldots, n) \) and \( 1, \lambda = 1 \) define the normalization equation ( \( \sum_{j=1}^{n} \lambda_j = 1 \)).

However, regarding the definition of \( PPS_{crs}^t \) in period \( t (t=1,2,\ldots,T) \) and following the model presented by Pastor and Lovell [7], the global production possibility set that satisfies CRS can be defined:

\[
GPPS_{crs}^g = \text{Convex hull} \{PPS_{crs}^1, PPS_{crs}^2, \ldots, PPS_{crs}^T \}
\]

(3)

Where convex hull is the all-convex combination of all points belonging to \( PPS_{crs}^t, t (t=1,2,\ldots,T) \).

Suppose that we have \( n \) DMUs in period \( t \) as well as in period \( t+1 \). The classic Malmquist index (MI) for \( DMU_p (p=1,\ldots,n) \) between \( t \) and \( t+1 \) is defined:

\[
MI = \left[ \frac{\bar{D}_{crs}^{t+1}(x_{p}^{t+1}, y_{p}^{t+1})}{\bar{D}_{crs}^{t}(x_{p}^{t}, y_{p}^{t})} \times \frac{\bar{D}_{crs}^{t}(x_{p}^{t}, y_{p}^{t})}{\bar{D}_{crs}^{t+1}(x_{p}^{t+1}, y_{p}^{t+1})} \right]^{\frac{1}{2}}
\]

(4)

Now, consider the following components:

Pure Efficiency Change
\[
PEC = \frac{\bar{D}_{crs}^{t+1}(x_{p}^{t+1}, y_{p}^{t+1})}{\bar{D}_{crs}^{t}(x_{p}^{t}, y_{p}^{t})}
\]

(5)

Efficiency Change
\[
EC = \frac{\bar{D}_{crs}^{t+1}(x_{p}^{t+1}, y_{p}^{t+1})}{\bar{D}_{crs}^{t}(x_{p}^{t}, y_{p}^{t})}
\]

(6)

Technological Change
\[
EC = \frac{\bar{D}_{crs}^{t+1}(x_{p}^{t+1}, y_{p}^{t+1})}{\bar{D}_{crs}^{t}(x_{p}^{t}, y_{p}^{t})}
\]

(7)
$$TC = \left[ \frac{D_{D}(X'_1, Y'_1) - D_{D}(X'_1, Y''_1)}{D_{D}(X'_1, Y'_1) - D_{D}(X'_1, Y''_1)} \right]^2$$

Scale Efficiency Change

$$SEC = \left[ \frac{D_{D}(X'_p, Y'_p) - D_{D}(X'_1, Y''_1)}{D_{D}(X'_2, Y'_2) - D_{D}(X'_1, Y''_1)} \right]$$

where the distance function can be defined as follows:

$$\tilde{D}_{Ip}(X', Y') = \min \left\{ \delta : (X', Y' / \delta) \in \text{PPS}_w \right\}$$

Moreover, following Pastor and Lovell, the global Malinqui index (GMI) based on the global production possibility set is also defined:

$$GMI = \frac{D_{D}(X'_1, Y''_1)}{D_{D}(X'_1, Y''_1)}$$

In addition, consider the additional components as follows:

$$BPC = \left[ \frac{\tilde{D}_{Ip}(X'_1, Y''_1) - \tilde{D}_{Ip}(X'_2, Y''_1)}{\tilde{D}_{Ip}(X'_1, Y''_1) - \tilde{D}_{Ip}(X'_2, Y''_1)} \right]$$

Similar to the classic MI, we can now generate two important decompositions of the GMI:

$$GMI = EC \times TC$$

$$GMI = PEC \times SEC \times TC$$

Now, regarding these components, we need to solve eight different linear programming problems to obtain the following distance functions:

$$D_{Ip}(X'_1, Y''_1), D_{Ip}(X'_2, Y''_1), D_{Ip}(X'_2, Y''_1)$$

It should be noted that, for determining $\tilde{D}_{Ip}(X'_1, Y''_1)$ and $\tilde{D}_{Ip}(X'_2, Y''_1)$ we need to solve the following problems:

$$\left[ \tilde{D}_{Ip}(X'_1, Y''_1) \right] = \max \theta'_p$$

$$s.t. \sum_{j=1}^{n} x'_j \leq x'_i \quad i = 1,...,m$$

$$\sum_{j=1}^{n} y'_j \leq y'_r, \theta'_p \quad r = 1,...,s$$

$$\theta'_p \text{ free in sign}$$

$$\left[ \tilde{D}_{Ip}(X'_2, Y''_1) \right] = \max \theta'_p$$

$$s.t. \sum_{j=1}^{n} x'_j \leq x'_i \quad i = 1,...,m$$

$$\sum_{j=1}^{n} y'_j \leq y'_r, \theta'_p \quad r = 1,...,s$$

$$\sum_{j=1}^{n} \theta'_i = 1$$

$$\theta'_i \geq 0 \quad i = 1,...,n$$

$$\theta'_p \text{ free in sign}$$

In addition, the computations of $\tilde{D}_{Ip}(X'_1, Y''_1)$ and $\tilde{D}_{Ip}(X'_2, Y''_1)$ are like 17 and 18, respectively, where $t$ is substituted by $t+1$. For the computation of $\tilde{D}_{Ip}(X'_1, Y''_1)$ and $\tilde{D}_{Ip}(X'_2, Y''_1)$, we need to solve two linear programming problems that have a mixed period problem. Moreover, based on the
definition of $CPS^G_{CRS}$ in 3, the following linear programming problem can be employed for determining $\tilde{D}^G_{CRS}(X'_p, Y'_p)$ and $\tilde{D}^G_{CRS}(X'^{i+1}_p, Y'^{i+1}_p)$:

$$\left[\tilde{D}^G_{CRS}(X'_p, Y'_p)\right]^i = \max \theta'_p$$

s.t. \[\sum_{k=1}^{n} \sum_{j=1}^{m} x_{k}^h \leq x'_j, \quad i = 1, \ldots, m\]

$$\sum_{k=1}^{n} \sum_{j=1}^{m} y_{k}^h \geq y'_j \theta'_p, \quad r = 1, \ldots, s$$

$$\lambda^h \geq 0, \quad j = 1, \ldots, n,$$

$$\theta'_p \text{ free in sign}$$

EXPANDED GLOBAL MALMQUIST INDEX

Below first, we review the expanded Malmquist index measurement in its classic form, after that, we propose our method for developing this concept on global form.

Regarding the definition of trade-offs by Podinovski, a trade-off is a judgment about possible variation in some inputs and/or outputs levels, with which DMU can work without changing the other inputs and/or outputs. For example, in the case of two inputs and a single output, the trade-off $(P, Q) = (1, -2, 0)$ indicates that the DMU can work by increasing the first input by one and decreasing the second input by 2 without changing its output (for more detail, see Podinovski [8]).

Now, suppose we have $k$ trade-offs. We shall represent the trade-offs in the following form: $(P_r, Q_r)$. Where $r = 1, \ldots, k$. Also, the vectors $P_r \in R^n$ and $Q_r \in R^m$ modify the inputs and outputs, respectively. Now expanded PPS which is based on trade-offs technology is defined:

$$PPS_{TO} = \{ (X, Y) | Y \leq \tilde{Y}_k + \sum_{r=1}^{k} \pi_r Q_r, \quad X \geq \tilde{X}_k + \sum_{r=1}^{k} \pi_r P_r, \quad \lambda \in R^s \}$$

It is noticeable that when we use trade-offs in our models, the original technology expands to include the new area. In fact, the PPS generated by the traditional technologies such as VRS and CRS, may not include all the producible production points. It means that the PPS made by the DEA models is only the subset of PPS with trade-offs:

$$PPS_{VRS} \subseteq PPS_{CRS} \subseteq PPS_{TO}$$

This relation shows that the DEA model based on CRS technology exhibits more discrimination than the DEA model based on VRS technology, and the DEA model using trade-offs technology exhibits even more discrimination than CRS technology.

Now let us focus on different technologies employed in two well-known decompositions namely FGLR and FGNZ. In FGLR decomposition, CRS technology is used and the classic Malmquist index breaks down into efficiency change (EC) and technological change (TC). Therefore, the basic technology is CRS. FGNZ decomposition also needs both VRS and CRS technologies to break down the classic Malmquist index into pure efficiency change (PEC), scale efficiency change (SEC), and technological change (TC). Indeed, in this decomposition, not only the basic technology is CRS, but also it use VRS as an additional technology with less discrimination than its basic technology. However, Alirezaze and Afsharian [1] employed trade-offs technology as their basic technology instead of CRS technology and based on this modification, defined the expanded Malmquist index (EMI) as below:

$$EMI = \left[ \frac{\tilde{D}^G_{CRS}(X'^{i+1}_p, Y'^{i+1}_p)}{\tilde{D}^G_{CRS}(X'_p, Y'_p)} \right] \left[ \frac{\tilde{D}^G_{CRS}(X'_p, Y'_p)}{\tilde{D}^G_{CRS}(X'^{i+1}_p, Y'^{i+1}_p)} \right]^\frac{1}{2}$$

(22)

They also used CRS and VRS technologies as some additional technologies and presented the following components:

Expanded Efficiency Change

$$EEC = \frac{\tilde{D}^G_{CRS}(X'^{i+1}_p, Y'^{i+1}_p)}{\tilde{D}^G_{CRS}(X'_p, Y'_p)}$$

(23)

Expanded Technological Change

$$ETC = \left[ \frac{\tilde{D}^G_{CRS}(X'_p, Y'_p)}{\tilde{D}^G_{CRS}(X'^{i+1}_p, Y'^{i+1}_p)} \right]^\frac{1}{2}$$

(24)

Regular Efficiency Change

$$REC = \left[ \frac{\tilde{D}^G_{CRS}(X'_p, Y'_p)}{\tilde{D}^G_{CRS}(X'^{i+1}_p, Y'^{i+1}_p)} \right]^\frac{1}{2}$$

(25)

Subsequently, the expanded Malmquist index can be exhibited by three expanded decompositions:

$$EMI = EEC \times ETC$$

(26)

$$EMI = EC \times REC \times ETC$$

(27)
EMI = PEC × SEC × REC × ETC

Now, regarding these components, eight different linear programming problems have to be solved:

\[
\begin{align*}
\tilde{D}_{i_0}^{\text{VRS}}(X^i_p, Y^i_p) &= \max_{\theta^i_p} \theta^i_p, \\
\text{s.t.} & \quad \bar{Y}^i \lambda^i + \sum_{i=1}^{k} \pi_i Q^i \geq \theta^i_p Y^i_p, \\
& \quad \bar{X}^i \lambda^i + \sum_{i=1}^{k} \pi_i P^i \leq X^i_p, \\
& \quad \lambda^i \geq 0, \quad \theta^i_p \text{ sign free}
\end{align*}
\]

\[
\begin{align*}
\tilde{D}_{i_0}^{\text{CRS}}(X^i_p, Y^i_p) &= \max_{\theta^i_p} \theta^i_p, \\
\text{s.t.} & \quad \bar{Y}^i \lambda^i + \sum_{i=1}^{k} \pi_i Q^i \geq \theta^i_p Y^i_p, \\
& \quad \bar{X}^i \lambda^i + \sum_{i=1}^{k} \pi_i P^i \leq X^i_p, \\
& \quad \lambda^i \geq 0, \quad \theta^i_p \text{ sign free}
\end{align*}
\]

As mentioned earlier in 17 and 18, the distance functions with VRS and CRS technologies has been defined, therefore we need to consider the following problems:

\[
\begin{align*}
\tilde{D}_{i_0}^{\text{VRS}}(X^i_p, Y^i_p) &= \max_{\theta^i_p} \theta^i_p, \\
\text{s.t.} & \quad \bar{Y}^i \lambda^i + \sum_{i=1}^{k} \pi_i Q^i \geq \theta^i_p Y^i_p, \\
& \quad \bar{X}^i \lambda^i + \sum_{i=1}^{k} \pi_i P^i \leq X^i_p, \\
& \quad \lambda^i \geq 0, \quad \theta^i_p \text{ sign free}
\end{align*}
\]

\[
\begin{align*}
\tilde{D}_{i_0}^{\text{CRS}}(X^i_p, Y^i_p) &= \max_{\theta^i_p} \theta^i_p, \\
\text{s.t.} & \quad \bar{Y}^i \lambda^i + \sum_{i=1}^{k} \pi_i Q^i \geq \theta^i_p Y^i_p, \\
& \quad \bar{X}^i \lambda^i + \sum_{i=1}^{k} \pi_i P^i \leq X^i_p, \\
& \quad \lambda^i \geq 0, \quad \theta^i_p \text{ sign free}
\end{align*}
\]

In addition, for the computation of \( \tilde{D}_{i_0}^{\text{VRS}}(X^i_p, Y^i_p) \) and \( \tilde{D}_{i_0}^{\text{CRS}}(X^i_p, Y^i_p) \), we need to solve two linear programming problems that have a mixed period problem.

Now, similar to expanded Malmquist index developed on classic form, we can modify the definition of the global Malmquist index. Therefore, we consider the global production possibility set that satisfies TO technology:

\[
GPPS_{i_0} = \text{Convex hull of } \{ PPS_{i_0}, PPS_{i_0}^2, \ldots, PPS_{i_0}^T \}
\]

where convex hull is the all-convex combination of all points belonging to \( PPS_{i_0}, t = (1, 2, \ldots, T) \).

Suppose that we have \( n \) DMUs in period \( t \) as well as in period \( t+1 \). The expanded global Malmquist index (EMI) for DMU\( _p \) (\( p = 1, \ldots, n \)) between \( t \) and \( t+1 \) can be defined:

\[
\begin{align*}
EMI_{i_0} &= \left( \frac{\tilde{D}_{i_0}^{\text{VRS}}(X^i_p, Y^i_p)}{\tilde{D}_{i_0}^{\text{VRS}}(X^i_p, Y^i_p)} \right), \\
\text{where the distance function can be defined as follows:}
\end{align*}
\]

\[
\begin{align*}
\tilde{D}_{i_0}^{\text{VRS}}(X^i_p, Y^i_p) &= \max_{\theta^i_p} \theta^i_p, \\
\text{s.t.} & \quad \bar{Y}^i \lambda^i + \sum_{i=1}^{k} \pi_i Q^i \geq \theta^i_p Y^i_p, \\
& \quad \bar{X}^i \lambda^i + \sum_{i=1}^{k} \pi_i P^i \leq X^i_p, \\
& \quad \lambda^i \geq 0, \quad \theta^i_p \text{ sign free}
\end{align*}
\]

\[
\begin{align*}
\tilde{D}_{i_0}^{\text{CRS}}(X^i_p, Y^i_p) &= \max_{\theta^i_p} \theta^i_p, \\
\text{s.t.} & \quad \bar{Y}^i \lambda^i + \sum_{i=1}^{k} \pi_i Q^i \geq \theta^i_p Y^i_p, \\
& \quad \bar{X}^i \lambda^i + \sum_{i=1}^{k} \pi_i P^i \leq X^i_p, \\
& \quad \lambda^i \geq 0, \quad \theta^i_p \text{ sign free}
\end{align*}
\]

Now, it is sufficient that, we consider the following component:

\[
\begin{align*}
\tilde{D}_{i_0}^{\text{VRS}}(X^i_p, Y^i_p) &= \max_{\theta^i_p} \theta^i_p, \\
\text{s.t.} & \quad \bar{Y}^i \lambda^i + \sum_{i=1}^{k} \pi_i Q^i \geq \theta^i_p Y^i_p, \\
& \quad \bar{X}^i \lambda^i + \sum_{i=1}^{k} \pi_i P^i \leq X^i_p, \\
& \quad \lambda^i \geq 0, \quad \theta^i_p \text{ sign free}
\end{align*}
\]

This allows us to generate three decompositions based on different technologies. However, if we only use TO technology as the basic technology, expanded global Malmquist index can be exhibited as below:

\[
EMI = EEC \times EBPC
\]

Where EEC is expanded efficiency change. If we also use CRS as the additional technology, we can have decomposition:

\[
EMI = EC \times REC \times EBPC
\]

Where REC is regular efficiency change.

Finally, if we use VRS and CRS as the additional technologies, we can define the following decomposition:

\[
EMI = PEC \times SEC \times REC \times EBPC
\]

where PEC and SEC are pure efficiency change and scale efficiency change respectively.

To summarize, the following graph provides a clear picture of the relationship between the different decompositions of the expanded Malmquist index in both classic and global forms.
CONCLUSION

To measure the impact of the rules and regulations on productivity changes, we proposed the new modification of the global Malmquist index named the expanded global Malmquist index. First, the expanded PPS in the presence of trade-offs technology was used, and the expanded global Malmquist index breaks down into expanded efficiency change (EEC) and expanded best practice change (EBPC). Second, we employed both CRS and trade-offs technologies to break down the expanded global Malmquist index into efficiency change (EC), regulation efficiency change (REC), and expanded best practice change (EBPC). Third, by adding VRS technology, the expanded global Malmquist index breaks down into pure efficiency change (PEC), scale efficiency change (SEC), regulation efficiency change (REC), and expanded best practice change (EBPC).

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OpenDEA: An open source framework

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ABSTRACT
OpenDEA is an open source framework with the purpose to provide a DEA-oriented platform for interaction between researchers on the one hand and industrial practitioners on the other hand. Its software (architecture) is structured in modules, and its source code is written in Java. The framework intends to meet the requirements of potential DEA users which are not experts in the field. In its final version, it will support the data collection, the choice of a suitable DEA model, as well as the interpretation of the data by using business-oriented tools. Thereby, interactive knowledge transfer is encouraged, enabling the participants to, e.g., further develop the software and exchange use cases. Access to the framework is possible via the internet platform http://www.opendea.org.

Keywords: OpenData Envelopment Analysis,

MOTIVATION
The paper addresses two main stakeholder groups of DEA, the DEA research community and industrial practitioners as a type of DEA users with no profound knowledge about the instrument. In order to bring these two groups closer together, DEA software can be seen as a key factor.

Against this background, Hentrich and Ahn (2009) have analyzed available DEA software concluding that these products are mostly designed for users with scientific purposes, showing shortcomings when considering industrial practitioners’ requirements on usability. As a consequence, DEA is not yet a typical instrument in the field of management control.

In order to raise acceptance of DEA in this business field, a DEA software tool should meet the special requirements of practitioners, including the in-house IT support staff. Respective guidelines for software usability can be found in the engineering standards DIN EN ISO 9241, for example:

Part 110 – fundamentals of man-machine dialog:
Self-disclosure – comprehensible feedback/help
Guided learning – tutorials for users

Customized usage – adaptability to different user types and their context of work
Fault tolerance – intelligent design of dialog allowing faults to be avoided/corrected easily by the users
Part 11 – requirements to usability; among others:
Effectiveness of task solution
Efficiency of application handling
Part 12 – data representation
Part 13 – user guidance

As far as we know, there is no DEA software available which discloses its source code for individual adaption of the tool. The open source framework openDEA closes this gap, longing to build a bridge between industrial practitioners and the DEA research community. Therewith, the focus lies on industry-oriented needs in order to draw practitioners’ attention on DEA. For example, it will be possible to connect the openDEA tool to different types of databases and typical commercial software such as SAP.

CHARACTERISTICS OF THE OPEN SOURCE FRAMEWORK
In order to reach these targets, a module-oriented software architecture was developed. Modifications of the following main modules are possible:
Graphical user interface
Model data
Solver
Reporting/visualization

The open source product “Eclipse” (see http://www.eclipse.org) served as development environment. For the most part, the source code in openDEA consists of the well-known programming language Java. A data interface supports text formats as well as spreadsheets, and SQL-databases can be connected quite easily without previously converting them to be compatible with openDEA. Furthermore, the framework is also conceptually designed for connection with Oracle databases.

OpenDEA offers two modes of guidance: the non interactive mode which assumes that the user is skilled in DEA-methodology, and the interactive mode which provides information about basic functions.

Depending on the knowledge of the user and the use case, pre-configured models or manual model construction can be selected. Release 1.0 of openDEA offers basic DEA models, such as CCR, BCC and ADDVRS (see Cooper/Seiford/Tone 2007). At each point in the process while using openDEA, the user is informed about further compulsory settings that are needed. Before a data analysis starts, the user has to confirm the current settings.

As solver, the Java-based open source LP Solver (see http://lpsolve.sourceforge.net/5.5), has been chosen. Its GNU license is community supported and goes along with helpful documentation. Reporting as well as visualization is based on BIRT (Business Intelligence and Reporting Tool). This is an established tool which works on Eclipse (see http://www.eclipse.org/birt) and which can be properly adapted to practitioner-oriented needs. Besides static reports on performance of DMUs, e.g., real time reporting is possible.

OpenDEA is accessible on the internet platform http://www.openDEA.org (an English version will be available soon). The platform is divided in a public and a private area which requires registration in order to get access to it. The public area provides general information about DEA-methodology and recommended literature. In the private area, specific information about openDEA and software download is accessible. Moreover, a pre-structured forum is offered for experience exchange and discussion, especially between DEA users and researchers.
IMPLICATIONS FOR THE FUTURE

There are several possibilities for further development. Some aspects are:

- Enlargement of available DEA models
- Incorporation of real use cases, together with comments on the respective performance measurement process and results
- Implementation of dynamic reporting
- Creation of an interface for uploading varied openDEA software in order to share it with the openDEA community.

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A simulation study on the efficiency evaluation of hierarchical decision making units

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ABSTRACT
In this paper, a Monte Carlo method is conducted for the comparison of in evaluating the relative efficiency scores of fixed upper decision making units (uDMUs) each with additive inputs and outputs and consisting of many sub decision making units (sDMUs). The problem appeared for the first time during the evaluation of efficiency scores of teams in World Cup 2002. In such an evaluation, the players and the teams can be thought as sDMUs and uDMUs, respectively. The team efficiency scores can be evaluated by starting the (individual) player scores or the teams (groups) data. In this study, the cases that uDMUs have the same (fixed, regular) size are considered; and the experiments of the efficiency evaluations having different number of uDMUs and different input-output numbers are repeated 1000 times. A Mann-Whitney U test is conducted on the results obtained. From the simulation studies arranged for the CCR and BCC models of DEA, it is obtained parallel results to each others. 

Keywords: Data Envelopment Analysis, Group Efficiency, Simulation, Hierarchy.

INTRODUCTION
For the first time, the concept of economical efficiency measure is discussed by Debreu [1]. Farrell’s work [2] strengthened the concept. In the third quarter of foregoing century, Charnes, Cooper and Rhodes [3] showed that the relative efficiency measurement of decision making units (DMU) can be evaluated by the models of operational research. Because the models proposed and called as CCR and the similar models envelope the DMUs whose performance will be determined, the new method of analysis is named as Data Envelopment Analysis (DEA). Up to now, the studies in this field have increased exponentially. The works of Emrouznejad et al [4], and Cook et al. [5] became the most comprehensive bibliographies listing the studies in the DEA field. In addition, Gattoufi et al. [6] gave an extensive taxonomy of studies done.

In this method which was spread gradually, the DMUs may be sportive teams or the players of teams as they can be banks, hospitals and educational institutions. The problem that comes in question appeared while DEA applied to the football teams. In football and basketball the players can be regarded as individual DMUs (i.e., sDMUs), and the teams themselves can also be seen as upper decision making units (i.e,uDMUs) consisting of these individuals.

The relative efficiency of this kind of uDMUs among which there is a hierarchical structure can be evaluated in two ways:

The efficiency of each sDMUs is evaluated. Then the average of these individual efficiency values becomes the efficiency score of the uDMUs.

The raw input and output data of variables for uDMU’s are obtained by summing the individual raw input and output data of variables. The efficiency scores for uDMUs can be evaluated by starting the summed data.

Since in basketball statistics, statistics for each player is published, so the first way seems very fitting. Unfortunately, in football, the systematic and detailed statistics for the teams and the players are not recorded except FIFA World Cup Statistics. In football, the teams efficiency scores can be evaluated in two above mentioned ways. The problem arose meanwhile such a work is done. In the official site of FIFA World Cup [7], both statistics for players and statistics for teams as sum of statistics for players are given. In the question, it is also aimed to search which DEA models are convenient while DEA can be modelled in football by using the raw data. Somewhat in this fashion it is confronted with the above mentioned dilemma. It is reached lower efficiency scores with respect to the second way when the evaluations are done in the above mentioned first way. In this work, a Monte Carlo study is planned for whether or not the case can be generalized to fixed or variable sized hierarchical DMUs.

In the second section of work, the structure of fixed and variable sized DMUs will be defined, in the third section, DEA and simulation
studies related to DEA, and in the fourth section the first problem originating this work, and in the fifth section the conducted Monte Carlo experiment, and in the sixth and last section the results and discussions will be given.

FIXED SIZED HIERARCHICAL DECISION MAKING STRUCTURE

Some DMUs whose efficiencies are evaluated by DEA may be in a more complicated structure than the structure of banks, schools or of their branches which can be qualified primitively. They may consist of the second or the more sublevels of sDMUs. Teams, hospitals and universities can be considered under this structure. As for the example of universities, academic staff, departments and faculties can be qualified as ordered sDMUs and finally each of universities can be seen as an uDMU. A sDMU can also be considered as an individual DMU and uDMU can also be considered as a group DMU. The commercial holdings which are put in working order with sub and branch organizations in many different areas can be described as irregular hierarchical DMUs while the DMU structure of a team, a hospital or a university exhibits a regular hierarchical DMU structure. In this study, the uDMUs which may consist of a fixed sized hierarchical structure.

The structure of fixed and variable sized hierarchical uDMUs which is also given schematically below can be described in that way. Let each of \( n \) uDMUs consist of \( l_{nk} \) sDMUs.

In the case in which each uDMUs has the same number of sDMUs this structure is called as fixed sized regular hierarchical DMU structure while each DMU consist of different number of sDMUs is called as variable sized hierarchical DMU structure.

**Figure 1: A hierarchical DMU structure.**

DATA ENVELOPMENT ANALYSIS AND SIMULATION STUDIES

Data Envelopment Analysis (DEA) is a nonparametric method based on mathematical programming. In efficiency evaluation, DEA takes multiple inputs and multiple outputs into consideration simultaneously. It forms a piecewise and linear efficiency bound. DEA qualifies DMUs over efficiency bound as efficient, and DMUs under efficiency bound as inefficient. Valuating multiple inputs and multiple outputs simultaneously, DEA objectively determines the weights that make the relative efficiency scores of DMUs maximum.

The first original work in this area was done by Charnes, Cooper and Rhodes [3]. The model they developed is named as CCR model with supposition constant returns to scale. Banker, Charnes and Cooper [8] developed the BCC model with supposition variable returns to scale by adding a new constraint to this model.

At the outset, DEA was considered for the relative efficiency calculation of not-for-profit DMUs. But it is applied gradually in a very broad spectrum such as for-profit institutions, banks, hospitals, agricultural farms, and not-for-profit public and private institutions, schools, universities and libraries. (Gattoufi et al. [5, 6], Banker et al. [9], Shawna et al. [10]). DEA forms a piecewise or unpiecewise linear efficient bound from the best of DMUs. It assigns a technical efficiency score (1 or 100) for each of decision making units over the efficient bound. The decision making units
under this bound are called as technically inefficient DMUs and it is assigned efficiency scores to these DMUs according to their distances to efficient DMUs. The CCR and BCC models can be given by model equations [1] and [2], respectively.

**CCR Model**

\[
\begin{align*}
\max w_o & = \sum_{i=1}^{s} u_i y_{o_i} \\
\sum_{i=1}^{m} v_i x_{s_i} & = 1 \\
\sum_{j=1}^{n} u_i y_{ij} - \sum_{j=1}^{n} v_i x_{ij} & \leq 0 \\
v_i & \geq 0, u_i \geq 0 \\
j = 1, 2, \ldots, n \\
i = 1, 2, \ldots, m \\
r = 1, 2, \ldots, s \\
\end{align*}
\]

where the subscript \( o \) denotes the DMU whose efficiency will be evaluated, and \( x_s, y_i, W_s, n, m \) and \( s \) denote inputs, outputs, efficiency value of the corresponding unit, the number of DMUs, the number of inputs and the number of outputs respectively and the variable \( u_o \) symbolizes the return to scale.

Most of the simulation studies conducted on the DEA are on the estimation parameters of production frontier. Gong and Sickles [11], in which they compared the Aigner’s estimation method, the stochastic frontier model and the DEA conducted a simulation study considering three inputs and one output case. Banker, Gadh and Gorr [12], in which they compared the BCC model of DEA and the method of corrected ordinary least squares for estimating the parameters of production frontier, designed a simulation experiment including two inputs and one output case and having 25, 50, 100 and 200 DMUs. There are also some other simulation studies on DEA: the studies of McMullen and Frazier [13] related on the DEA and production frontier, Desai, Ratick and Schinnar [14] on the DEA and stochastic variation in input-output variables, Chaparro and Jimeney [15], Lothgren [16], Lothgren [17], Haas and Murphy [18], Hughes and Yaisawarng [19], Bojanic, Caudill and Ford [20], Banker, Chang and Cooper [21], and Suoyeshi [22] on the theory of DEA, and Bardhan, Cooper and Kumbhakar [23] on the DEA and regression analysis, and Worry [24], and Bifulco and Breitschneider [25] on the DEA and education are given among others. In the literature, it is not encountered any study on the efficiency of a hierarchical uDMU in the DEA.

**BCC Model**

\[
\begin{align*}
\max w_o & = \sum_{i=1}^{s} u_i y_{o_i} + u_0 \\
\sum_{i=1}^{m} v_i x_{s_i} & = 1 \\
\sum_{j=1}^{n} u_i y_{ij} - \sum_{j=1}^{n} v_i x_{ij} + u_0 & \leq 0 \\
v_i & \geq 0, u_i \geq 0 \text{ ve } u_0 \text{ free} \\
j = 1, 2, \ldots, n \\
i = 1, 2, \ldots, m \\
r = 1, 2, \ldots, s \\
\end{align*}
\]

**FIRST PROBLEM**

This study arose the performance evaluation of FIFA World Cup teams and the search of their goodness-of-ordering after championship. As mentioned before, in the official site of FIFA, the separate and detailed statistics for both the teams and the players participating in the championship are given a place. In this problem, each player can be considered as a sDMU and each team can be considered as an uDMU. The teams consist of the players and there are only 11 players of each team in the playing field when the matches hold. The statistics for the teams are obtained by summing the statistics for players. When we keep all of these cases in mind we meet a fixed sized regular hierarchical structure. The efficiency scores of teams can be found by two methods mentioned in introduction part. In the following, the efficiency evaluation will be given as an example for the point of attack view in football. The team efficiency in football is not the subject of this study. In an exact team efficiency evaluation, the defense and goalkeeper efficiency should be taken separately as an addition. For this reason, the given example should be examined merely from the subject of this study.

There are totally 32 teams participating in 2002 FIFA World Cup. Since there were no statistics of players of four teams we have taken 28 teams for the examination.

Output variables for teams and players are given below.

Team outputs are TMP: Team Matches Played, TGF: Team goals, TA: Team Assists, TSOG: Team Shots on Goal, TP: Team Penalties and TFK: Team Free Kicks.

Since 2002 FIFA World Cup is in elimination order, the number of matches are not equal and not the same. In relation to this, the other
statistics are not also in a directly comparable structure. For this reason, it will be meaningful to proportion these statistics to the number of TMP. The other outputs are proportioned to TMP and the new output variables are obtained:

\( y_1 \): TMP,
\( y_2 \): TGF/TMP,
\( y_3 \): TA/TMP,
\( y_4 \): TSOG/TMP,
\( y_5 \): TP/TMP,
\( y_6 \): TFK/TMP.

Outputs for players which are parallel to outputs for teams are MP: Matches Played, G: Overall number of goals, A: Assists, SOG: Shots on Goal, PG: Penalty Goals and FK: Free Kicks. Similar to the transforms of team output variables, the variables for players are defined as follows:

\( y_1 \): MP,
\( y_2 \): Gr=G/MP,
\( y_3 \): Ar=A/MP,
\( y_4 \): SOGr=SOG/MP, \( y_5 \): PGr=PG/MP,
\( y_6 \): FKr=FK/MP.

Table 2: The Efficiency Scores of Teams due to Attack

<table>
<thead>
<tr>
<th>Takımlar</th>
<th>CCR Model</th>
<th>BCC Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uDMU appr.</td>
<td>sDMU appr.</td>
</tr>
<tr>
<td>ARG</td>
<td>0.867</td>
<td>0.448</td>
</tr>
<tr>
<td>BEL</td>
<td>0.941</td>
<td>0.668</td>
</tr>
<tr>
<td>BRA</td>
<td>1</td>
<td>0.727</td>
</tr>
<tr>
<td>CMR</td>
<td>0.891</td>
<td>0.449</td>
</tr>
<tr>
<td>CRC</td>
<td>0.744</td>
<td>0.575</td>
</tr>
<tr>
<td>CRO</td>
<td>0.512</td>
<td>0.625</td>
</tr>
<tr>
<td>ENG</td>
<td>0.650</td>
<td>0.365</td>
</tr>
<tr>
<td>ESP</td>
<td>1</td>
<td>0.591</td>
</tr>
<tr>
<td>GER</td>
<td>0.830</td>
<td>0.423</td>
</tr>
<tr>
<td>IRL</td>
<td>1</td>
<td>0.607</td>
</tr>
<tr>
<td>ITA</td>
<td>0.632</td>
<td>0.381</td>
</tr>
<tr>
<td>JPN</td>
<td>0.814</td>
<td>0.280</td>
</tr>
<tr>
<td>KOR</td>
<td>0.727</td>
<td>0.514</td>
</tr>
<tr>
<td>MEX</td>
<td>0.543</td>
<td>0.291</td>
</tr>
</tbody>
</table>

As observed from Table 2, the efficiency scores of teams due to attack by the uDMU approach (appr. in table) is greater than or equal to the sDMU’s. At the same time, under the two approaches the efficiency scores of teams obtained by the BCC model are greater than or equal to the CCR model’s.

Later, under the CCR and BCC models it is applied a Mann-Whitney U test for controlling whether or not the efficiency scores evaluated by the uDMU and sDMU approaches are different. Since the sample size \( n_1, n_2 = 28 \) is sufficiently large, the normal distribution approximation is used in place of Mann-Whitney U test and found the test statistic value \( z_h = 4.172 \) \( (p \text{ val} = 0.00000314) \).

This means that the hypothesis \( H_0 \) is rejected. Consequently, there is a difference between the efficiency scores obtained by uDMU and sDMU approaches for the CCR model.

The test statistic value for the BCC model is evaluated as \( z_h = 4.581 \) \( (p \text{ val} = 0.000000453) \). This value points out the hypothesis \( H_0 \) is rejected. Hence, there is a difference between the efficiency scores found by uDMU and sDMU approaches for the BCC model.

From these results, in the evaluation of efficiency scores of DMUs having a regular hierarchical structure, it will become more meaningful in respect of DEA reason to use the summed data under the uDMU approach. However, it is required an enough number of simulation trials in order to generalize the obtained results.
A SIMULATION STUDY

In order to test the validity of efficiency score evaluation for fixed sized DMUs, a simulation experiment is devised by the program MATLAB 7.0. The efficiency scores of uDMUs consisting of fixed sized sDMUs are computed for two approaches mentioned. In the trials the number of sDMUs, uDMUs and input-output variables are changed. The raw data of input and output variables for sDMUs are derived from Uniform Distribution on (0, 1000). For the CCR and BCC models thirty variant cases of inputs and outputs are considered and for each case one-thousand trials and hypothesis testing are realized. The overall results for the CCR and BCC models are shown in Table 3.

As an example, from Table 3 the detail of a fixed sized case for the CCR model (trial #10) is given in Table 4. In this trial, the number of uDMUs is 18, the number of sDMUs is 90 (i.e., the integer randomly chosen from [3, 18] is 5, and so the number of all sDMUs in each uDMU is 5, and hence the total number of sDMUs is 90), the number of inputs and outputs are 1 and 5, respectively. For this trial, at the outset, the values of one input and five outputs for 90 sDMUs are drawn randomly from Uniform Distribution. The efficiency evaluations for each sDMUs are computed by the CCR model and the efficiency scores for each uDMUs are obtained by finding their averages (the sDMU approach). Then, summing the data of input and output variables of all sDMUs for the same uDMU, it is obtained the data of input and and output variables for the uDMU. Later, the efficiency values for eighteen uDMUs are computed by the CCR model (the uDMU approach). These two procedures are repeated one-thousand times. The equality of efficiency scores for the uDMUs gotten by two different methods are controlled by the Mann-Whitney U Test at the significance level α = 0.05. For the one-thousand trials, it is counted the number of cases rejecting the hypothesis which claims there is no difference between the unit and group efficiencies. For this sample, in the whole of 1000 trials, it is reached to the result that the efficiency scores obtained by two separate evaluation methods are different in respect of the average p value (significance level) 0.0000014.

When the whole simulation results for the CCR and the BCC models are compared (Table 3), it is observed a parallelism between them. Theoretically, in the BCC model application, it is expected that many more numbers of DMUs are efficient than that of the CCR model applications, as seen also in Table 3. There are 30 separate cases located in Table 3 in respect of the numbers of uDMUs, sDMUs and input-output variables for the CCR and the BCC models. In high frequencies of 1000 (usually in the whole of 1000) repetions, the hypothesis is rejected that claims there is statistically no difference between the efficiency values obtained by the sDMU and uDMU approaches. In the trials, the number of rejections is increased in the cases in which the number of DMUs exceeds three times the number of input+output frequencies and otherwise it is decreased.
CONCLUSION AND DISCUSSION

Under the DEA models, the efficiency scores for fixed sized regular hierarchical uDMUs can be evaluated by the sDMU and the uDMU approaches. The efficiency scores obtained by the aforementioned approaches are statistically different than each others. The efficiency scores gotten by the uDMU approach are higher than that of the sDMU approach. In the case in which the whole of the individual efficiency scores are efficient (i.e., one), the uDMU for this group will also be efficient. However, in the trial conducted by the uDMU approach having the same inputs, outputs and other elements, one or more uDMUs are resulted as efficient. In this approach, in the case in which any uDMU concluded as efficient, all of the sDMUs contained in many it are also automatically respected as efficient whereas in the sDMU approach it is seen that most of group individuals are inefficient. In the

first of two evaluation approaches there is an underestimation (the sDMU approach) and in the second there is an overestimation (the uDMU approach). A new algorithm for the calculation of more balanced efficiency scores will be the subject of our subsequent paper.

Another new finding is that the uDMU approach has yielded a lot more numbers of efficient DMUs than that of the sDMU approach which is similar to that of the BCC models which have yielded higher numbers of more efficient DMUs as efficient than the CCR models.

The general validity of these cases are seen in the simulation study conducted. The concluded results confirm all of these cases when the number of uDMUs becomes larger than three times the number of inputs+outputs.
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Deriving the DEA frontier for supply-chain efficiency

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ABSTRACT
Data envelopment analysis (DEA) deals with measurements of relative efficiency of DMUs regarding multiple-inputs vs. multiple-outputs. Recently DEA has been extended to examine the efficiency of supply-chain operations. Due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs, as in the standard DEA approach, does not necessarily yield a frontier projection. This paper introduces the production possibility sets for supplier and manufacturer in supply-chain operations.

Keywords: Data Envelopment Analysis, Supply Chain Operation

INTRODUCTION
Data envelopment analysis (DEA) is an approach for measuring the relative efficiency of peer decision making units (DMUs) (See Charnes et al. (1978)). This efficiency approach cannot be applied directly to the problem of evaluating the efficiency supply-chains. Many researchers applied standard DEA models to measure the performance of supply-chain members. For example, Weber and Desai (1996) employed DEA to construct an index of relative supplier performance. Easton et al. (2002) suggested a DEA model to compare the purchasing efficiency of firms in the petroleum industry. Chen et al. (2006) and Liang et al. (2006) developed several DEA-based approaches for characterizing and measuring supply-chain efficiency when intermediate measures are incorporated into the performance evaluation. In this paper, a method for determining the DEA frontier points for inefficient supply-chain members has been proposed. Due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs, as in the standard DEA approach, does not necessarily yield a frontier projection. This paper introduces the production possibility sets for supplier and manufacturer in supply-chain operations.

PROBLEM STATEMENT
Consider a two-stage supply-chain, e.g. supplier-manufacturer supply-chain as shown in figure 1. Suppose we have n homogeneous supply-chain operations. Each supply-chain observation is considered to be a DMU. It is assumed that each supplier $S_j$ in $DMU_j$, $j = 1, \cdots, n$ has $m$ inputs $x_i$, $i = 1, \cdots, m$ and $s$ outputs $y_{rj}$, $r = 1, \cdots, s$. These $s$ outputs can become the inputs to the manufacturer $M_j$. The manufacturer $M_j$ has its own inputs $z_{dj}$, $d = 1, \cdots, D$. The final outputs from manufacturer are $q_j$, $l = 1, \cdots, L$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Supplier-manufacturer supply-chain}
\end{figure}

AXIOMATIC FOUNDATION
Let $T_1$ be the production possibility set of technology under consideration for the supplier $S$. We postulate the following:

$P1$. Feasibility of observed data. $(x_j, y_j) \in T_1$ for any $j = 1, 2, \cdots, n$.

$P2$. Unbounded ray. $(x, y) \in T_1$ implies $\lambda (x, y) \in T_1$ for any $\lambda \geq 0$. 


\( P3 \)- Convexity. Let \((x', y') \in T_1\) and \((x'', y'') \in T_1\). Then, for any \(\lambda \in [0, 1]\) the unit
\[ \lambda(x', y') + (1 - \lambda)(x'', y'') \in T_1. \]

P4- Free disposability. \((x, y) \in T_1\), \(x' \geq x\) and 
y' \leq y, implies \((x', y') \in T_1. \)

\( P5 \)- Minimal extrapolation. For each \( T' \) satisfying in the axioms \( P1-P4 \), we have
\( T_1 \subseteq T'. \)

Now, an Algebraic representation of the PPS of the technology \( T_1 \), satisfying the axioms \( P1-P5 \), is given.

**Theorem 1** - The PPS \( T_1 \), which satisfies the axioms \( P1-P5 \), is defined as
\[
T_1 = \{(x, y) : x \geq \sum_{j=1}^n \lambda_j x_j, \quad y \leq \sum_{j=1}^n \lambda_j y_j, \quad \lambda_j \geq 0, \ j = 1, 2, \ldots, n \}
\]

**Proof** - The proof is clear.

Also, let \( T_2 \) be the production possibility set of technology under consideration for the manufacturer M. Again, to determine the technology of the manufacturer M, we postulate the following:

\( P1' \)- Feasibility of observed data. (\( y_j, z_j, q_j \)) \( \in T_2 \) for any \( j = 1, 2, \ldots, n \).

\( P2' \)- Unbounded ray. \((y, z, q) \in T_2 \) implies 
\[ \lambda(z, q) \in T_2 \] for any \( \lambda \geq 0 \).

\( P3' \)- Convexity. Let \((y', z', q') \in T_2 \) and 
\((y'', z'', q'') \in T_2 \). Then, for any \( \lambda \in [0, 1] \) the unit
\[ \lambda(y', z', q') + (1 - \lambda)(y'', z'', q'') \in T_2. \]

\( P4' \)- Free disposability. \((y, z, q) \in T_2\), \( y' \geq y, \ z' \geq z \) and \( q' \leq q \), implies 
\((y', z', q') \in T_2 \).

\( P5' \)- Minimal extrapolation. For each \( T' \) satisfying in the axioms \( P1-P4 \), we have 
\( T_2 \subseteq T' \).

**Theorem 2** - The PPS \( T_2 \), which satisfies the axioms \( P1'-P5' \), is defined as
\[
T_2 = \{(y, z, q) : y \geq \sum_{j=1}^n \lambda_j y_j, \quad z \geq \sum_{j=1}^n \lambda_j z_j, \quad q \leq \sum_{j=1}^n \lambda_j q_j, \quad \lambda_j \geq 0, \ j = 1, 2, \ldots, n \}
\]

**Proof** - The proof is clear.

**THE PROPOSED MODEL**

In applying the model described herein, attention is paid to additive model. Consider the assessment of DMU in additive form. Taking into account Theorems 1 and 2 and considering the intermediate measure \( y_0 \), this is obtained as the optimal value of the following model:

\[
\begin{align*}
\text{Min} & \quad s^{(1)} + s^{(2)} + s^{(3)} + s^{(4)} + s^{(5)} \\
\text{s.t.} & \quad \sum_{j=1}^n \lambda_j x_j + s^{(1)} = x_0 \quad (1-1) \\
& \quad \sum_{j=1}^n \lambda_j y_j - s^{(2)} = y_0 \quad (1-2) \\
& \quad \sum_{j=1}^n \mu_j y_j + s^{(3)} = y_0 \quad (1-3) \\
& \quad \sum_{j=1}^n \mu_j z_j + s^{(4)} = z_0 \quad (1-4) \\
& \quad \sum_{j=1}^n \mu_j q_j - s^{(5)} = q_0 \quad (1-5) \\
& \quad \lambda_j, \mu_j \geq 0, \ j = 1, 2, \ldots, n.
\end{align*}
\]

Obviously, this problem is feasible and the optimal objective value to this problem is bounded. The rationality for selecting different multipliers for suppliers and manufacturers is that we compare the assessed member of supply-chain with the other same kind members.

**Definition 1** - Supplier \( o \) is said to be additive efficient if and only if \( s^{(1)} + s^{(2)} = 0 \).

**Definition 2** - Supplier \( o \) is said to be additive efficient if and only if \( s^{(3)} + s^{(4)} + s^{(5)} = 0 \).

For an inefficient supplier \( S_o(x_o, y_o) \), we have
\[ x_o = \sum_{j=1}^{n} \lambda_j x_j - s^{(1)} \]
\[ y_o = \sum_{j=1}^{n} \lambda_j y_j + s^{(2)} \]

On the other hand, for an inefficient manufacturer \( M_o(y_o, z_o, q_o) \), we have
\[ y_o = \sum_{j=1}^{n} \mu_j y_j + s^{(2)} \]
\[ z_o = \sum_{j=1}^{n} \mu_j z_j - s^{(3)} \]
\[ q_o = \sum_{j=1}^{n} \mu_j q_j + s^{(4)} \]

The supplier \( S_o \) and manufacturer \( M_o \) can be improved and become efficient by deleting the input excess and augmenting the output shortfalls. We point out that the intermediate measure \( y_o \) may be increased or decreased to make the overall system as efficient. These operations are called supply-chain projection.

**NUMERICAL EXAMPLE**

To illustrate the proposed approach consider a simple example involving ten supply-chains taken from Chen et al. (2006). The suppliers use three inputs \( x_1, x_2 \) and \( x_3 \) to produce two outputs \( y_1, y_2 \). On the other hand, the manufacturer uses \( y_1, y_2 \) and \( z_1 \) to generate \( q_1, q_2 \). The data are summarized in Table 1. The results from model 1 are listed in Table 2. As the table indicates, only one supply chain, 6, is efficient in aggregate sense. Clearly, a DMU may be efficient in supplier or manufacturer only, such as in the case for members 5 and 9. The projection points are listed in Table 3.

**CONCLUSIONS**

Recently DEA has been extended to examine the efficiency of supply-chain operations. Due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs, as in the standard DEA approach, does not necessarily yield to a frontier projection. The current paper introduced the technologies used in supply-chain operations. A method for determining the DEA frontier points for inefficient supplier and manufacturers in supply-chain operations has been faced.

**REFERENCES**


**ACKNOWLEDGMENTS**

This research has been founded by Grant-in-Aid for scientific research, Islamic Azad university of Rasht, to whom I owe thanks.
Table 1: Data set

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Table 2: Efficiency of suppliers and manufacturers

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Table 3: Projection points

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The minimal allocated cost and maximal allocated benefit

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ABSTRACT
This Paper deals with computing the minimal and maximal shares of individuals or organizations based on different criteria. Suppose that, players are selfish. Each player makes any possible efforts to bring about his or her ideal condition. In this paper a new scheme to compute the minimal cost ratio and the maximal benefit is offered. This scheme also avoids zero weight. Here ultimate allocated is achieved for each player with suitable coalition within several defined coalition.

Keywords: DEA, Game Theory, Assurance Region Method

INTRODUCTION
Let us suppose n players each have m criteria for evaluating their competency or ability, which is represented by a positive score for each criterion. As with usual classroom examination, the higher score for a criterion is, the better player is judged to perform as regard to that criterion. Each person k has a right to choose two sets of nonnegative weights \( w_i^k \) to the criteria that are most preferable to the player. Using the weight \( w_i^k \), the relative scores of player k to the total score are defined as follows:

\[
\frac{\sum_{j=1}^{n} w_i^k x_{ij}}{\sum_{j=1}^{n} (\sum_{i=1}^{m} x_{ij})}
\]

(1)

The denominator represents the total score of all players as measured by player k weight selection, while the numerator indicates player k self-evaluation using the same weight selection. We assume that the weighted scores are transferable. Player k wishes to maximize this ratio by selecting the most preferable weights, thus resulting in the following fractional program.

\[
\max_{\alpha} \frac{\sum_{i=1}^{m} w_i^k x_{\alpha}}{\sum_{j=1}^{n} (\sum_{i=1}^{m} x_{ij})}
\]

(2)

s.t. \( w_i^k \geq 0 \) (\( \forall i \))

Player k aims to maximize his relative value as measured by the ratio. This arbitrary weight selection is the fundamental concept underlying DEA initiated by Charnes [1]. We reformulate the problem, without losing generality. We normalize the data set X so that
it is row-wise normalized, i.e., \( \sum_{j=1}^{m} x_{ij} = 1 \) \((\forall i)\). We divide the row \((x_{i1},\ldots, x_{im})\) by the row-sum \( \sum_{j=1}^{m} x_{ij} \) for \( i = 1,\ldots, m \). Thus, using the Charnes-Cooper transformation scheme, the fractional program (2) can be expressed using a linear program follows:

\[
c(k) = \max \sum_{i=1}^{m} w_{i}^{*} x_{i}
\]

\[
s.t. \sum_{i=1}^{m} w_{i}^{*} = 1 \quad (3)
\]

\[
w_{i}^{*} \geq 0 \quad \forall i
\]

Apparently, the optimal solution is given by assigning 1 to \( w_{i(k)} \) for the criterion \( i(k) \) such that \( x_{i(k)} = \max \{ x_{i} \mid i = 1,\ldots, m \} \) and assigning 0 to the weight of remaining criteria. We denote this optimal value by \( c(k) \).

\[
c(k) = x_{i(k)} \quad k = 1,\ldots, n \end{equation}

(4)

The \( c(k) \) indicates the highest relative score for player \( k \) which is obtained by the optimal weight selecting behavior. The optimal weight \( w_{i(k)}^{*} \) may differ from one player to another.

\[
\sum_{k=1}^{n} c(k) \geq 1 \quad \text{Theorem 1.}
\]

**Proof.** Let the optimal weight for player \( k \) be \( w_{i(k)}^{*} = (w_{i(k)1},\ldots, w_{i(k)m}) = 1 \) and \( w_{i(k)}^{*} = 0 \) \((\forall i \neq i(k))\). Then we have

\[
\sum_{k=1}^{n} c(k) = \sum_{k=1}^{n} \sum_{i=1}^{m} w_{i(k)}^{*} x_{i} = \sum_{i=1}^{m} \sum_{k=1}^{n} w_{i(k)}^{*} \geq \sum_{k=1}^{n} x_{i(k)} = 1
\]

The inequality above follows from \( x_{i(k)} \geq x_{i(k)} \) and the last equality follows from the row-wise normalization.

This theorem assert that, if each player sticks to his egoistic sense of value and insists on getting the portion of the benefit as designated by \( c(k) \), the sum of shares usually exceeds 1 and hence \( c(k) \) cannot fulfill the role of division of the benefit. If eventually the sum of \( c(k) \) turns out to be 1, all players will agree to accept the division \( c(k) \), since this is obtained by the players most preferable weight selection. The latter case will occur when all players have the same and common optimal weight selection, we have the following theorem.

**Theorem 2.** The equality \( \sum_{k=1}^{n} c(k) = 1 \) holds if and only if our data satisfies the condition \( x_{i(k)} = x_{i(k)} = \ldots = x_{i(k)} \), \( \forall k = 1,\ldots, n \).

That is, each player has the same score with respect to the \( m \) criteria.

**Proof.** The (if) part can be seen as follows. Since \( c(k) = x_{i(k)} \) for all \( k \), we have:

\[
\sum_{k=1}^{n} c(k) = \sum_{k=1}^{n} x_{i(k)} = 1
\]

The (only if) part can be proved as follows. Suppose \( x_{i_1} > x_{i_2} \) then there must be column \( h \neq 1 \) such that \( x_{i_2} < x_{i_3} \), other wise the second row sum cannot attain 1. Thus we have \( c(1) = x_{i_1} \), \( c(h) = x_{i_2} \), \( x_{i_3} \) and \( c(j) = x_{i_2} \) \((\forall j \neq 1,h)\). Hence it holds that

\[
\sum_{k=1}^{n} c(k) \geq \sum_{j\neq i_1,h} x_{i_2} + x_{i_2} \geq \sum_{k=1}^{n} x_{i(k)} = 1
\]

This leads to a contradiction. Therefore player1 must have the same score in all criteria. The same relation must hold for the other players.

In the above case, only one criterion is needed for describing the game and the division proportional to this score is a fair division. However, such situation might occur only in rare instances. In the majority of cases, we have \( \sum_{k=1}^{n} c(k) \geq 1 \).

2.2. Coalition with Additive Property

Let the coalition \( S \) be a subset of player set \( N = \{1,\ldots, n\} \). The record for coalition \( S \) is defined by \( x_{i}(S) = \sum_{j=1}^{m} x_{ij} \) \((i = 1,\ldots, m)\) (6)

These coalitions aim to maximize the out comes \( c(S) \).

\[
c(S) = \max \sum_{i=1}^{m} w_{i} x_{i}(S)
\]

\[
s.t. \sum_{i=1}^{m} w_{i} = 1 
\]

\[
w_{i} \geq 0 \quad \forall i
\]

The \( c(S) \) with \( c() = 0 \), defines a characteristic function of the coalition \( S \). Thus this game is represented by \((N,c)\).

**Definition 1.** A function \( f \) is called sub – additive if for any \( S \subseteq N \) and \( T \subseteq N \) with \( S \cap T = \emptyset \) the following statement holds:

\[
f(S \cup T) \leq f(S) + f(T)
\]

**Definition2.** A function \( f \) called super – additive if for any \( S \subseteq N \) and \( T \subseteq N \) with \( S \cap T = \emptyset \) the following statement holds:

\[
f(S \cup T) \geq f(S) + f(T)
\]

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Theorem 4. $c(N) = 1$. 

Proof. 

**A DEA MINIMUM GAME**

The opposite side of the game can be constructed by $(N,d)$ as follows:

$$d(k) = \min \sum_{i=1}^{m} w^k_i x_q$$

$$\text{s.t.} \sum_{i=1}^{m} w^k_i = 1 \quad (9)$$

$$w^k_i \geq 0 \quad \forall i$$

The optimal value $d(k)$ assures the minimum division that player $k$ can expect from the game

Theorem 5. $\sum_{k=1}^{m} d(k) \leq 1$ \quad (10)

Analogously to the max game, for the coalition $S \subset N$, we define

$$d(S) = \min \sum_{i=1}^{m} w_i x_i(S)$$

$$\text{s.t.} \sum_{i=1}^{m} w_i = 1 \quad (11)$$

$$w_i \geq 0 \quad \forall i$$

Theorem 6. The min game $(N,d)$ is super – additive we have $d(S \cup T) \geq d(S) + d(T)$ for each $S, T \subset N$ with $S \cap T = \phi$.

Proof. 

By renumbering the indexes, we have $S = \{ 1,\ldots,h \}, T = \{ h+1,\ldots,k \}$ and $S \cup T = \{ 1,\ldots,k \}$. For these sets it holds that

$$d(S \cup T) = \min \sum_{j=1}^{h} x_j \geq \min \sum_{j=1}^{h} x_j + \min \sum_{j=h+1}^{k} x_j$$

$$\quad = d(S) + d(T)$$

Thus these game start from $d(k)$>0, $k = 1,\ldots,n$ and enlarges the gains by the coalition until the grand coalition $N$ with $d(N)=1$ is reached.

Theorem 7.

d(S) + c(N \setminus S) = 1 \quad \forall S \subset N

Proof. 

By renumbering the indexes, we can assume that $S = \{ 1,\ldots,h \}, N = \{ 1,\ldots,n \}$ and $N \setminus S = \{ h+1,\ldots,n \}$. For these sets, it holds that

$$d(S) + c(N \setminus S) = \min \sum_{j=1}^{h} x_j + \max \sum_{j=h+1}^{n} x_j$$

$$\quad = \min (\sum_{j=1}^{h} x_j - \sum_{j=h+1}^{n} x_j) + \max \sum_{j=h+1}^{n} x_j$$

$$\quad = \min (1- \sum_{j=h+1}^{n} x_j) + \max \sum_{j=h+1}^{n} x_j$$

$$\quad = 1 - \max \sum_{j=h+1}^{n} x_j = 1$$

EXTENSIONS

In this section, we extend the basic model to maximal allocated benefit and minimal allocated cost and discuss the zero weight.

**Maximal Allocated Benefit**

Suppose that there are $s$ criteria for representing benefits. Let $y_{ij} (i = 1,\ldots,s)$ be the benefits of player $j (j = 1,\ldots,n)$ where $u (u_1,\ldots,u_s)$ is the virtual weights for benefits. Analogous to the expression (1) we define the relative score of player $j$ to the total scores as:

$$\sum_{i=1}^{s} u_i y_{ij} \quad (12)$$

Player $j$ wishes to maximize his benefits. We can express this situation by linear program below:

$$\max \sum_{i=1}^{s} u_i y_{ij}$$

$$\text{s.t.} \sum_{i=1}^{s} u_i (\sum_{j=1}^{n} y_{ij}) = 1 \quad (13)$$

$$\sum_{i=1}^{s} u_i y_{ij} \geq 0 \quad (j = 1,\ldots,n)$$

$$u_i \geq 0 \quad \forall i$$

The weights of benefits are nonnegative. A characteristic function of the coalition $S$ is defined by the linear program below:

$$c(S) = \max \sum_{i=1}^{s} u_i \sum_{j=1}^{n} y_{ij}$$

$$\text{s.t.} \sum_{i=1}^{s} u_i (\sum_{j=1}^{n} y_{ij}) = 1 \quad (14)$$

$$\sum_{i=1}^{s} u_i y_{ij} \geq 0 \quad (j = 1,\ldots,n)$$

$$u_i \geq 0 \quad \forall i$$
In the program (14), the benefits of all players are nonnegative. Since the constraints of program (14) are the same for all coalitions, we have the following theorem.

**Theorem 8.** The maximal allocated benefits game satisfies a sub-additive property.

**Proof.** For any \( S \subset N \) and \( T \subset N \) with \( S \cap T = \emptyset \), we have:

\[
c(S \cup T) = \max \sum_{i=1}^{m} u_i \sum_{j \in S} y_{ij} = \max \left( \sum_{i=1}^{m} u_i \sum_{j \in S} y_{ij} + \sum_{j \in T} y_{ij} \right)
\]

\[
\leq \max \sum_{i=1}^{m} u_i \sum_{j \in S} y_{ij} + \max \sum_{i=1}^{m} u_i \sum_{j \in T} y_{ij}
\]

\[
\leq c(S) + c(T)
\]

**Minimal Allocated Cost**

Suppose that there are \( m \) criteria for representing costs. Let \( x_j (i = 1, \ldots, m) \) be the costs of player \( j \) \((j = 1, \ldots, n)\) where \( v \) (\( v_1, \ldots, v_m \)) is the virtual weights for costs. Player \( j \) wishes to minimize his costs then we have:

\[
\min \sum_{i=1}^{m} v_i x_i
\]

\[
s.t. \sum_{i=1}^{m} v_i x_i = 1 \quad (15)
\]

\[
\sum_{i=1}^{m} v_i x_i \geq 0 \quad (j = 1, \ldots, n)
\]

\[
v_i \geq 0 \quad \forall i
\]

The weights of costs are nonnegative. A characteristic function of the coalition \( S \) is defined by the linear program below:

\[
d(S) = \min \sum_{i=1}^{m} v_i \sum_{j \in S} x_{ij}
\]

\[
s.t. \sum_{i=1}^{m} v_i \sum_{j \in S} x_{ij} = 1 \quad (16)
\]

\[
\sum_{i=1}^{m} v_i x_{ij} \geq 0 \quad (j = 1, \ldots, n)
\]

\[
v_i \geq 0 \quad \forall i
\]

In the program (16), the costs of all players are nonnegative. Minimal allocated costs game satisfies a super–additive property.

**Theorem 9.** The maximal allocated benefit game \((N, c)\) and min game \((N, d)\) are dual games, for any \( S \subset N \), we have \( d(S) + c(N \setminus S) = 1 \).

Proof.
Min \( \sum_{i,j} v_{ij} x_{ij} \)

s.t. \( \sum_{j=1}^{m} v_{ij} \sum_{j=1}^{n} x_{ij} = 1 \quad (19) \)

\( \sum_{j=1}^{m} v_{ij} x_{ij} \geq 0 \quad (j = 1, \ldots, n) \)

\( L_i \leq \frac{v_i}{u_i} \leq U_i, (i = 2, \ldots, m) \)

\( v_i \geq 0 \quad \forall i \)

The (14), (16) are modified in (20), (21) respectively.

c(S) = Max \( \sum_{i=1}^{s} u_i \sum_{j=1}^{n} y_{ij} \)

s.t. \( \sum_{j=1}^{s} u_i \sum_{j=1}^{n} y_{ij} = 1 \)

\( \sum_{j=1}^{s} u_i y_{ij} \geq 0 \quad (i = 1, \ldots, n) \quad (20) \)

\( L_i \leq \frac{u_i}{y_{ij}} \leq U_i, (i = 2, \ldots, s) \)

\( u_i \geq 0 \quad \forall i \)

d(S) = Min \( \sum_{i=1}^{s} v_i \sum_{j=1}^{n} x_{ij} \)

s.t. \( \sum_{j=1}^{s} v_i \sum_{j=1}^{n} x_{ij} = 1 \)

\( \sum_{j=1}^{s} v_i x_{ij} \geq 0 \quad (i = 1, \ldots, n) \quad (21) \)

\( L_i \leq \frac{v_i}{x_{ij}} \leq U_i, (i = 2, \ldots, m) \)

\( v_i \geq 0 \quad \forall i \)

The Best Coalition

In program (14),(16), we presented a scheme for computing maximal allocated benefits, minimal allocated cost for coalitions. Also we can compute maximal allocated benefits and minimal allocated costs for members of coalition, using programs (13),(15). These values can determine the players expected percentages of the total benefit, cost in the game. Each player can increase benefit allocation and decrease cost allocation, establishing coalition. In other words the possible ultimate benefit allocated to the coalition can increase and minimal cost allocated can decrease once the best circumstance is provided. There is a question, how can player \( j \) establish coalition? Now, knowing this it’s easy for the player to examine which other players, she/he can establish coalition with so that he can reach the ultimate benefit ratio and minimal cost ratio. Each player can establish coalition in different ways, chance coalition, coalition concerning players of minimal allocated benefit (cost), coalition concerning players of maximal allocated benefit (cost), coalition with the player enjoying the ultimate benefit and the players with the minimal allocated benefit (cost), coalition with the player enjoying the minimal benefit (cost) and the players with maximal allocated benefit (cost). Clearly, coalition with the player having the ultimate allocated benefit would be better than the others. Coalition with the player having the minimal allocated benefit would be poorer in comparison to others. Now, it’s easy to understand that a player with the minimal benefit ratio establish a coalition with the player who has allocated the ultimate benefit for him/her self and player with the maximal benefit ratio establish a coalition with the player who has allocated the minimal benefit for him/her self. Having established the coalition the player cost ratio would be less or unchanged. Clearly, player with the minimal cost ratio establish a coalition with the player who has allocated the minimal cost for him/herself, and player with the maximal cost ratio establish a coalition with the player who has allocated the minimal cost for himself. These results represented by the example below.

4. NUMERICAL RESULTS

There are 10 players in this game. Each player uses 2 cost criteria and 4 benefit criteria. We compute maximal allocated benefit and the minimal allocated cost for each player. Table (2) shows these results.

Table 1: Cost and benefit criteria

<table>
<thead>
<tr>
<th>Player</th>
<th>( x_u )</th>
<th>( x_y )</th>
<th>( y_u )</th>
<th>( y_y )</th>
<th>( y_u )</th>
<th>( y_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>20</td>
<td>800</td>
<td>200</td>
<td>350</td>
<td>340</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>18</td>
<td>900</td>
<td>160</td>
<td>320</td>
<td>470</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>22</td>
<td>1000</td>
<td>175</td>
<td>395</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
<td>30</td>
<td>950</td>
<td>185</td>
<td>290</td>
<td>510</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>17</td>
<td>960</td>
<td>186</td>
<td>280</td>
<td>480</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>24</td>
<td>870</td>
<td>210</td>
<td>360</td>
<td>370</td>
</tr>
<tr>
<td>7</td>
<td>65</td>
<td>26</td>
<td>780</td>
<td>165</td>
<td>300</td>
<td>440</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>32</td>
<td>670</td>
<td>150</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>29</td>
<td>810</td>
<td>170</td>
<td>410</td>
<td>510</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>16</td>
<td>910</td>
<td>190</td>
<td>420</td>
<td>390</td>
</tr>
</tbody>
</table>

We now apply this approach to the data above.

Table 2: Maximal allocated benefit of obtained total benefit, minimal allocated cost of total game cost

<table>
<thead>
<tr>
<th>Player</th>
<th>Maximal allocated benefit</th>
<th>Minimal allocated cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.17%</td>
<td>6.71%</td>
</tr>
<tr>
<td>2</td>
<td>10.66%</td>
<td>7.69%</td>
</tr>
<tr>
<td>3</td>
<td>11.56%</td>
<td>9.40%</td>
</tr>
<tr>
<td>4</td>
<td>11.56%</td>
<td>12.82%</td>
</tr>
<tr>
<td>5</td>
<td>11.10%</td>
<td>7.26%</td>
</tr>
<tr>
<td>6</td>
<td>11.73%</td>
<td>7.3%</td>
</tr>
<tr>
<td>7</td>
<td>9.98%</td>
<td>8.72%</td>
</tr>
<tr>
<td>8</td>
<td>11.25%</td>
<td>10.7%</td>
</tr>
<tr>
<td>9</td>
<td>11.63%</td>
<td>6.71%</td>
</tr>
<tr>
<td>10</td>
<td>11.91%</td>
<td>6.84%</td>
</tr>
</tbody>
</table>
7th player has the least maximal allocated benefit and the 10th player has the highest maximal allocated benefit. Also, the 4th player has the highest maximal allocated cost and first and 9th players have the least minimal allocated cost.

Table (3) shows modified results. In this table, 8th player has the least maximal allocated benefit and the 3rd has the highest maximal allocated benefit. Also, the 4th has the highest minimal allocated cost and 1st has the least minimal allocated cost.

Table 3: Modified allocated benefits and costs (we set constraints on the ratio \( u_{i\neq j} \))

<table>
<thead>
<tr>
<th>Player ( j )</th>
<th>Modified maximal allocated benefit</th>
<th>Modified minimal allocated cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.22%</td>
<td>6.80%</td>
</tr>
<tr>
<td>2</td>
<td>10.33%</td>
<td>9.13%</td>
</tr>
<tr>
<td>3</td>
<td>11.12%</td>
<td>10.53%</td>
</tr>
<tr>
<td>4</td>
<td>10.90%</td>
<td>14.47%</td>
</tr>
<tr>
<td>5</td>
<td>10.78%</td>
<td>11.34%</td>
</tr>
<tr>
<td>6</td>
<td>9.91%</td>
<td>7.52%</td>
</tr>
<tr>
<td>7</td>
<td>9.22%</td>
<td>8.84%</td>
</tr>
<tr>
<td>8</td>
<td>9.11%</td>
<td>10.24%</td>
</tr>
<tr>
<td>9</td>
<td>10.20%</td>
<td>6.98%</td>
</tr>
<tr>
<td>10</td>
<td>10.52%</td>
<td>12.41%</td>
</tr>
</tbody>
</table>

Table 4 (a)

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Maximal allocated benefits for coalition</th>
<th>Members of coalition</th>
<th>Maximal allocated benefits of total game benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>31.77%</td>
<td>10 8 7</td>
<td>12.25% 11.94% 10.51%</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>33.76%</td>
<td>10 9 8</td>
<td>11.92% 12.44% 13.56%</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>31.52%</td>
<td>9 7 2</td>
<td>11.47% 11.21% 10.51%</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>31.55%</td>
<td>7 6 10</td>
<td>11.47% 12.27% 11.73%</td>
</tr>
<tr>
<td>( S_5 )</td>
<td>29.94%</td>
<td>2 7 10</td>
<td>12.29% 10.12% 10.13%</td>
</tr>
</tbody>
</table>

Table 4 (b)

Table 5 (a)

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Modified maximal allocated benefits for coalition</th>
<th>Members of coalition</th>
<th>Modified maximal allocated benefits of total game benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1' )</td>
<td>30.09%</td>
<td>3 9 8</td>
<td>11.25% 10.22% 9.13%</td>
</tr>
<tr>
<td>( S_2' )</td>
<td>32.69%</td>
<td>3 4 5</td>
<td>11.26% 10.94% 10.83%</td>
</tr>
<tr>
<td>( S_3' )</td>
<td>27.33%</td>
<td>1 7 8</td>
<td>9.30% 9.29% 9.11%</td>
</tr>
<tr>
<td>( S_4' )</td>
<td>30.71%</td>
<td>3 4 8</td>
<td>11.17% 10.90% 9.21%</td>
</tr>
<tr>
<td>( S_5' )</td>
<td>29.18%</td>
<td>3 7 8</td>
<td>11.33% 9.27% 9.11%</td>
</tr>
</tbody>
</table>

Table 5 (b)

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Modified minimal allocated cost for coalition</th>
<th>Members of coalition</th>
<th>Modified minimal allocated cost of total game cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_6' )</td>
<td>30.59%</td>
<td>1 2 4</td>
<td>6.76% 9.13% 14.46%</td>
</tr>
<tr>
<td>( S_7' )</td>
<td>21.30%</td>
<td>1 6 9</td>
<td>6.62% 7.44% 6.98%</td>
</tr>
<tr>
<td>( S_8' )</td>
<td>38.21%</td>
<td>4 5 10</td>
<td>14.14% 11.33% 12.40%</td>
</tr>
<tr>
<td>( S_9' )</td>
<td>33.87%</td>
<td>1 4 10</td>
<td>6.66% 14.33% 12.40%</td>
</tr>
<tr>
<td>Coalition</td>
<td>Modified minimal allocated cost for coalition</td>
<td>Members of coalition</td>
<td>Modified minimal allocated cost of total game cost</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------</td>
<td>----------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>$S_1'$</td>
<td></td>
<td>1 4 9</td>
<td>6.79% 14.01% 6.98%</td>
</tr>
</tbody>
</table>

Also, consider in Table 5(a), $S_1'$ is chance coalition, $S_2'$ is coalition concerning players of minimal allocated benefit, $S_3'$ is coalition concerning players of maximal allocated benefit, $S_4'$ is coalition with the player enjoying the minimal benefit and $S_5'$ the players with maximal allocated benefit. In Table 5(b), $S_6'$ is chance coalition $S_7'$ is coalition concerning players of minimal allocated cost, $S_8'$ is concerning players of maximal allocated, $S_9'$ is coalition with the players enjoying the minimal cost and the players with maximal allocated cost and $S_{10}'$ is coalition with the player enjoying the maximal cost and the players with minimal allocated cost.

Table (5) shows the same results, establish the coalition the player benefit ratio are increased or unchanged. Also, establish the coalition the player cost ratio is decreased or unchanged. 8th player with the least benefit ratio has the most benefit in $S_1'$. 3th player with the maximal benefit ratio has the most benefit in $S_2'$. 4th player with the most cost ratio has the least cost in $S_3'$. 1st player with the minimal cost ratio has the least cost in $S_4'$.

**CONCLUSION**

In this paper, we have studied the common weight issues that connect the game solution with arbitrary weight selection behavior of the players. Regarding this subject, we have proposed a method for compute maximal allocated benefit and minimal allocated costs. We have introduced coalitions and the ways for finding the best coalitions. In this sense, we avoided occurrence zero weight by assurance region method. Furthermore a numerical example, have been calculated with proposed ways, has been considered.

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**REFERENCES**


**Data Envelopment Analysis with bounded outputs: An investigation in the Integer-valued**

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**ABSTRACT**

Integer-valued data envelopment analysis with alternative returns to scale technology has been introduced and developed recently by Kuosmanen and Kazemi Matin (2009). The proportionality assumption of their introduced “natural augmentability” axiom in constant and non-decreasing returns to scale technologies makes it possible to achieve feasible decision making units (DMUs) of arbitrary large size. However, in real life applications there are many occasions in which it is not possible to achieve such production plans and some of input and output variables are bounded above. In this paper we aim to extend the axiomatic foundation of Integer-DEA models for including bounded output variables. Some model variants are achieved by introducing new axiom of “boundedness” over the selected output variables. A mixed integer linear programming (MILP) formulation is also introduced for computing efficiency scores in the associated production set.  

**Keywords:** DEA, Integer DEA, Efficiency, Mixed Integer Linear Programming, Bounded Outputs

**INTRODUCTION**

Data envelopment analysis (DEA) is a mathematical programming approach for evaluating performance of decision making units (DMUs) that convert multiple inputs into multiple outputs. Although conventional DEA models assume real valued inputs and outputs but there are many occasions in which some inputs and/or outputs must only take integer values. Integer-valued DEA is proposed and developed recently by Kuosmanen and Kazemi Matin (2009) and also is extended to more classical technologies in Kazemi Matin and Kuosmanen (2009). In these papers, it is shown that in activity analysis of the units with integer valued data, some axiomatic and also model refinements should take into account. They also introduce some MILP formulations for computing the efficiency scores under alternative returns to scale (RTS) axioms.  

Under the constant returns to scale (CRS) assumption, Kuosmanen and Kazemi Matin’s minimum extrapolation theorem builds upon the axioms of natural disposability, natural convexity and natural constant returns to scale.  

As pointed out correctly in Podinovski (2004), using CRS models requires full proportionality assumptions between all inputs and outputs. This makes it possible to achieve feasible DMUs of arbitrary large size in the discrete production set of Integer-DEA. However, in real life applications this may not be always possible to achieve such production plans and some of input and output variables could take specific bounded values.  

For example passengers of airlines or full professors of university departments in their associated system could be considered as outputs which in improving the performance measure of system units we can increase their level as much as possible to get efficient points. But in the both examples we may introduce an output level in the computed target points which seems very far or even impossible to achieve by inefficient under evaluation airline or university department. It is because of the number of passengers in an airline or the number of full professors in a specific subject area is bounded above. Another example which also motivated this investigation is the analysing of the performance measurement of 42 departments of an Iranian university. In this evaluation which is reported by Kuosmanen and Kazemi Matin (2009) and also Kazemi Matin and Kuosmane (2009), manager satisfaction is used as one of the output variables in each department and just takes
integer values between 1 and 4. If we use the (modified) output oriented Farrell measure under CRS technology to improve the performance of a university department, we may compute a target point that suggests an increase in the level of this output to an integer number greater than 4 which is meaningless. So, we need to impose some kinds of boundedness in the underlying production possibility set (PPS) to overcome this deficiency.

In this paper we aim to extend the axiomatic foundation of Integer-DEA models for including bounded output variables. The model variants are achieved by introducing new axiom of “boundedness” over the selected output variables.

The rest of the paper is in the following order. The next section gives a brief review of the refined axioms for Integer-DEA theory with CRS technologies. The new axiom of “output boundedness” is then introduced and the associated DEA production sets that satisfy the fundamental minimum extrapolation principle (Banker et al. 1984) is given. This followed by an extension of this axiomatic approach to include hybrid setting, where both output variables with specific upper bounded and output variables without such limitation are coexist. We then introduce an adapted version of the Farrell output efficiency measure and show how the efficiency score of the units can be computed by solving a MILP model.

**BACKGROUND**

In the classic DEA model by Charnes et al. (1978) each observed DMU is characterized by a pair of non-negative input and output vectors \((x_j, y_j) \in R^{m\times s}\), \(j \in J = \{1,...,n\}\). The notations \(X\) and \(Y\) are also used for inputs and outputs matrixes respectively. It is assumed that the underlying production possibility set denoted by \(T = \{(x,y)| x \in R^m, y \in R^s \}\) under CRS technology contains the observations and satisfies the following axioms:

(A1) **Free (strong) disposability**

(A2) **Constant returns to scale**

(A3) **Convexity**

According to the minimum extrapolation principle (Banker et al. 1984), the DEA production possibility set (PPS) is the intersection of all subsets of \(R^{m\times s}\) that contains the observed units and satisfies the maintained axioms. Under the above assumptions (A1)-(A3), the minimum extrapolation PPS can be written as:

\[
T_{DEA} = \left\{ (x,y) \bigg| x \geq \sum_{j=1}^{n} x_j \lambda_j, y \leq \sum_{j=1}^{n} y_j \lambda_j, \lambda \geq 0 \right\}.
\]

In DEA literature numerous of axiomatic approaches have been presented (Emrouznejad et al. 2008). Models of weak disposability (e.g. Kuosmanen 2005, 2009) and congestion (e.g. Cherchye et al. 2001) introduced by a relaxation of (A1). Models of selective proportionality and hybrid returns to scale (e.g Podivovski 2004, 2005) are the results of relaxation of (A2) and (A3).

Relaxation of (A2) itself leads to models of variable and non-increasing (decreasing) returns to scale (e.g. Seiford and Thrall 1990). Free disposable hull (Deprins et al. 1984) and free replicable hull models (Tulkens 1990) are also achieved by relaxation of (A3). Models of integer-valued DEA (Kuosmanen and Kazemi Matin 2009, Kazemi Matin and Kuosmanen 2009) are also result of restricting and refining classical DEA axioms for integer data environments. Since we aimed to deal with the case of bounded outputs in integer environment, let us first give a brief review of the Integer-DEA axioms in the case of CRS technologies.

Following the notations in Kuosmanen and Kazemi Matin (2009) we can write these axioms as:

(B1) **Natural disposability**: If \((x,y) \in T\), \((u,v) \in Z^{m\times s}\) and \(y \geq v\) then \((x+u,y-v) \in T\).

(B2) **Natural convexity**: If \((x,y), (x', y') \in T\), \((\tilde{x},\tilde{y}) = (\lambda(x,y) + (1-\lambda)(x', y'))\) and \((\tilde{x},\tilde{y}) \in Z^{m\times s}\) then \((\tilde{x},\tilde{y}) \in T\).

(B3) **Natural constant returns to scale**: If \((x,y) \in T\) and \(\exists \alpha \geq 0; (\alpha x, \alpha y) \in Z^{m\times s}\) then \((\alpha x, \alpha y) \in T\).

Kazemi Matin and Kuosmanen (2009) showed that \(T^{CRS}_{DEA} = T_{DEA} \cap Z^{m\times s}\) is the minimal extrapolation technology under the maintained set of axioms (B1) – (B3).

\(^{15}\) For \(0 \leq \alpha \leq 1\) and \(\alpha \geq 1\) this axiom is called “natural divisibility” and “natural augmentability” respectively.
They also introduced a modified version of Farrell (1954) radial measure in efficiency evaluation of the units with both real and integer valued data. An empirical study on 42 university departments is also used in their paper to illustrate the importance of integer structure of input/output variables and the differences resulting from alternative model formulations.

*Natural constant returns to scale*, (B3), is one of the main axioms in the definition of Integer DEA PPS in CRS environment. However in applications, there are many cases that the above axiom may not be satisfied for some variables. For instance, consider the same application of 42 university departments in Kazemi Matin and Kuosmanen (2009). As we discussed in the previous section, one of the output variables is level of manager satisfaction ($y_4$) in each department. However there is a problem if we use output-oriented Integer-DEA model in this evaluation. The level of manager satisfaction is a bounded variable (1 to maximum 4); even if all inputs are increased to infinity then the maximum value that $y_4$ can gain is 4. Therefore for dealing with this situation we have to amend the B3 axiom to satisfy some kind of “boundedness” condition; the subject which we will discuss in the next sections of this paper.

For simplicity of the presentation, in the first step we focus on the case where all outputs are bounded integer; the extensions for including selective output bounded variables are straightforward and will present afterward.

**MODEL DEVELOPMENT**

Here we introduced the following axiom for bounding outputs where $k = (k_1, ..., k_n)$ is an integer-valued vector.

(B4) *Outputs boundedness*: If $(x, y) \in \mathcal{T}$ then $y \leq k$.

It is easy to see that (B4) contradicts axioms (B3). Therefore, direct applying bounded outputs in some Integer-DEA models like models with constant or non-decreasing returns to scale technology (Kazemi Matin and Kuosmanen 2009) are not axiomatically sound and needs more investigations. To deal with this case in a systematic fashion, we need to introduce an alternative set of axioms. To do this end, we note that the following refinement of axioms (B3) and (B4) results both.

(B'3-1) *Outputs bounded scale*: If $(x, y) \in \mathcal{T}$, $\alpha \geq 0$ and $(\alpha x, \alpha y) \in \mathcal{Z}_{s}^{w+s}$ then $(\alpha x, \alpha y) \in \mathcal{T}$ where $\gamma_r = \min \{ \alpha y_r, k_r \}$.

Axiom (B'3-1) is a refinement of *natural constant returns to scale* axiom (B3) with this different that all the scaled outputs is bounded above by the predefined integer-valued scalar $k_r$.

**Main Results**

Equipped with the new set of axioms, now we can introduce associated Integer-DEA production set as follows.

$$\mathcal{T}_{\text{Integer-DEA}}^{CRS} = \left\{ (x, y) \in \mathcal{Z}_{s}^{w+s} \mid x \geq X_\alpha, y \leq Y_\lambda, y \leq k, \lambda \geq 0 \right\}$$

In fact $\mathcal{T}_{\text{Integer-DEA}}^{CRS} = \mathcal{T}_{\text{DEA}}^{CRS} \cap \left\{ (x, y) \mid y \leq k \right\}$.

The following theorem shows the minimal extrapolation interpretation for this reference technology under our adapted set of axioms.

**Theorem 1.** $\mathcal{T}_{\text{Integer-DEA}}^{CRS}$ is the minimal extrapolation set which contains the observations and satisfies the maintained axioms (B1), (B2) and (B'3-1).

**Extensions to the Hybrid Case**

Now we consider more general setting when only some of the (integer-valued) outputs satisfy the boundedness axiom (B'3-1). We partition the output vector of any feasible activity as $y = (y^{NB}, y^B)'$ where $y^B$ denotes output variables with the specific upper bound and $y^{NB}$ shows the other output variables. For this hybrid case we use the following axiom to impose the boundedness on the selected outputs.

(B'3-2) *Partial outputs bounded scale*: If $(x, y^{NB}, y^B) \in \mathcal{T}$, $\alpha \geq 0$ and $(\alpha x, \alpha y^{NB}, \alpha y^B) \in \mathcal{Z}_{s}^{w+s}$ then $(\alpha x, \alpha y^{NB}, \alpha y^B) \in \mathcal{T}$ where $\gamma_r = \min \{ \alpha y^B_r, k_r \}$.

A DEA PPS that can be used for the hybrid setting could be stated as follows.

$$\mathcal{T}_{\text{Integer-DEA}}^{CRS} = \left\{ (x, y^{NB}, y^B) \in \mathcal{Z}_{s}^{w+s} \mid x \geq X_\alpha, y^B \leq Y_\lambda, y^B \leq k, \lambda \geq 0 \right\}$$
By means of the following theorem, the minimal extrapolation property is also deduced for the PPS of hybrid case.

**Theorem 2.** $T_{\text{HB–IDEA}}^{\text{CRS}}$ is the minimal extrapolation set which contains the observed units and satisfies the maintained axioms (B1), (B2) and (B’3-2).

**Efficiency Measurement**

Based on the theorem 2, the modified version of Farrell output radial efficiency measure of DMU$_o$ (by Kuosmanen and Kazemi Matin, 2009) in the bounded outputs technology $T_{\text{HB–IDEA}}^{\text{CRS}}$ is obtained by solving the following MILP.

\[
\begin{align*}
\max_{\phi, \lambda} & \quad \phi \\
\text{s.t.} & \quad x_o \geq X\lambda, \\
& \quad \phi y_o^B \leq y \leq Y\lambda, \\
& \quad \phi y_o^B \leq k, \\
& \quad y \in Z^s_+ \\
& \quad \lambda \geq 0
\end{align*}
\]

The above model evaluates the radial output oriented efficiency of DMU$_o$ and identifies its radial projection in the discrete PPS as $(x_o, y)$.

As in the case of classical radial DEA models, the computed radial projection of the bounded outputs technology $T_{\text{HB–IDEA}}^{\text{CRS}}$ may not be fully efficient in the Pareto sense. This may be improved in the second optimization stage by eliminating any input excesses and output shortfalls.

**CONCLUSIONS**

In this paper we introduced the notion of boundedness on the subset of output variables in an Integer DEA model. In an axiomatic approach, based on the new introduced axiom of “outputs bounded scale”, the associated minimal extrapolation PPS is constructed. A MILP formulation similar to Integer DEA models is suggested for computing output efficiency scores of the units.

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DEA models for decision making support in negotiation process

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ABSTRACT
In this paper, an approach is proposed on evaluation of agreements on transnational projects during the negotiation process. One can show that the negotiation process can be represented as the behavior of decision making units (countries) in the multidimensional space of economic indicators using DEA models. In this case, the goals that can be reached by units as a result of accomplishment of joint project can be determined as points in the multidimensional space. Optimal directions toward these goals and cones of possible directions can be found with the help of Analytic Hierarchy Process (AHP). Our approach is illustrated on the real-life data taken from open international sources.

Keywords: Data Envelopment Analysis, Analytic Hierarchy Process, Negotiation process

INTRODUCTION
The problem can be formulated as follows. Two or more countries are in the negotiation process on the accommodation of a joint transnational project. Assume, for example, that countries discuss a port construction, investment in the construction and share-holdings of the port in the future. At first sight, shares could be divided according to the investment in the port construction. However, such method is not quite suitable from the point of view of countries purposes. The point is that one country may collect large revenue from the port in the future, while the revenue of the other country would be low. However, the port construction would be of great strategic importance for the second country. As it follows from this example the division of shares only on the basis of the future revenue may be not quite suitable.

METHODOLOGY
Now we dwell on our approach in detail.

At first stage of our approach production possibility set \( T \) is constructed on the basis of socio-economic indicators of countries using DEA models (Banker et al., 1984; Cooper et al., 2000).

Consider a set of \( n \) observations of actual production units \((X_j, Y_j), j = 1, \ldots , n\), where the vector of outputs \( Y_j = (y_{1j}, \ldots , y_{nj}) \geq 0, j = 1, \ldots , n\), is produced from the vector of inputs \( X_j = (x_{1j}, \ldots , x_{mj}) \geq 0\). The production possibility set \( T \) is the set \( \{(X, Y) | the \ outputs \ Y \geq 0 \ can \ be \ produced \ from \ the \ inputs \ X \geq 0 \} \).

The production possibility set \( T_B \) for BCC (Banker, Charnes, Cooper) model can be written in the form (Banker et al., 1984)

\[
T_B = \left\{ (X,Y) \mid \sum_{j=1}^{n} X_j \lambda_j \leq X, \sum_{j=1}^{n} Y_j \lambda_j \geq Y, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1,\ldots,n \right\}. \tag{1}
\]

It was shown in the DEA scientific literature (see, Krivonozhko et al., 2009) that the BCC model can approximate any DEA model from a large family of DEA models. For this reason, we consider the BCC model as a basic model in our exposition.

Krivonozhko et al. (2009) proposed a generalized DEA model that, on the one hand, covers any model from a large family of DEA models, on the other hand, this model enables one to construct step-by-step any model from a large family of the DEA model by incorporating artificial units and rays in the space of inputs and outputs in the BCC model, which makes the process of model construction visible and more understandable.

The production possibility set of this model is written in the form

\[
\]
\[ T_c = \left\{ (X,Y) \left| X \geq \sum_{j=1}^{n} X_j \lambda_j + \sum_{i=1}^{m} D_i \mu_i + \sum_{k=1}^{l} A_k \rho_k, \right. \right. \\
Y \leq \sum_{j=1}^{n} Y j \lambda_j + \sum_{i=1}^{m} G_i \mu_i + \sum_{k=1}^{l} B_k \rho_k, \left. \right. \right. \\
\left. \sum_{j=1}^{n} \lambda_j + \sum_{i=1}^{m} \mu_i = 1, \right. \right. \right. \right. \\
\lambda_j \geq 0, j=1,...,n, \mu_i \geq 0, i=I, \rho_k \geq 0, k=J \} \tag{2} \]

where \((D_i,G_i), \ i \in I, \ I\) is a set of artificial production units, \((A_k,B_k), \ k \in J, \ J\) is a set of vectors (rays) added to the model.

Parametric optimization methods developed by our group (Krivonozhko et al., 2004; Volodin et al., 2004) enables ones to visualize the multidimensional space with the help of construction of intersections of the frontier with a two-dimensional plane or a three-dimensional affine subspace. By choosing two or three different directions of the affine subspace our package “FrontierVision” can construct various sections passing through any unit and cutting the frontier. Moreover, this package allows one to insert artificial units and rays in the model just on the screen of the computer by clicking the mouse with cursor located at a point on the specific position. In this way we can transform the frontier in accordance with general model (2). This possibility is very important for our approach.

There exists a variety of ways to compute efficiency score in DEA models depending on the spheres of model applications (Cooper et al., 2000). For our purpose we use the following efficiency score

\[ \mu = \frac{f(Z_i)}{f(A)} \tag{3} \]

where \(f(Z_i)\) and \(f(A)\) are the values of potential function \(f(\cdot)\) at starting position \(Z_i\) of a country and a final position \(A\) of this country in the multidimensional space of indicators, respectively.

At first, we consider the potential function \(f(\cdot)\) as a linear one

\[ f(Z) = \sum_{i=1}^{m} a_i z_i \tag{4} \]

Afterwards, we describe how this function can be specified.

At the second stage of our approach every country (party concerned) determines its goal that this country will reach in a given number of years. Assume, for example, that these are \(t_1\) and \(t_2\) for two given countries. From mathematical point of view this means that two points \(A\) and \(B\) are determined in the multidimensional space of indicators.

At the third stage, vectors of optimal movements towards the goals \(A\) and \(B\) are computed with the help of the analytic hierarchy process (AHP) (Saaty, 1980). The AHP method finds the relative ranks of the decision alternatives. The ranks of the decision alternatives are given by elements of the normalized principal eigenvector of a preference matrix consisting of the pairwise comparisons between alternatives. We use the AHP to compute vectors of movements towards goals \(A(B)\), at least for some vicinity of unit \(Z_i(Z_j)\). Indeed, while moving to goals \(A(B)\), indicators of unit \(Z_i(Z_j)\) should be changed in accordance with some weights or ranks that can be found by AHP.

It is quite natural to believe that the value of potential function is increased while unit \(Z_i(Z_j)\) is moving along vector \(a(b)\) towards goal \(A(B)\). Else, why do we need to move towards goal \(A(B)\)? Hence, we can formulate the following result.

**Assertion 1.** Vector \(a(b)\) determined by AHP is a gradient of potential function in some vicinity of goal \(A(B)\).

Thus, a hyperplane perpendicular to the vector \(a(b)\) at point \(A(B)\) will be equipotential surface of potential function \(f(\cdot)\). At forth stage, we compute efficiency scores for every unit \(Z_i\) and \(Z_j\) in the following manner

\[ \mu_i = \frac{f_i(Z_i)}{f_i(A)} \times 100\% = \frac{\sum_{j=1}^{m} a_i z_j}{\sum_{j=1}^{m} a_i A_j} \times 100\%, \tag{5} \]

\[ \mu_j = \frac{f_j(Z_j)}{f_j(B)} \times 100\% = \frac{\sum_{j=1}^{m} b_i z_j}{\sum_{j=1}^{m} b_i B_j} \times 100\% \]

where \(a = (a_1,\ldots,a_m), \ b = (b_1,\ldots,b_m)\), \(A = (A_1,\ldots,A_m), \ B = (B_1,\ldots,B_m)\).

Now, the parties concerned can compare incrementations of efficiency scores as a result of the accomplishment of the joint project

\[ \Delta \mu_i = 100\% - \mu_i \tag{6} \]

\[ \Delta \mu_j = 100\% - \mu_j \]

If incrementations of efficiency scores (6) satisfy the parties concerned, then the negotiation process is completed. Else, the negotiation process continues. The parties consider modified conditions. As a result, goals \(A\) and \(B\) in multidimensional space may be moved or/and these goals may be reached in \((t_1 + \tau_1)\) years for unit \(A\) and in \((t_2 + \tau_2)\) years for unit \(B\).
Then incrementations of the efficiency scores (6) can be modified as follows

$$\Delta \mu_i' = \Delta \mu_i \frac{t_i}{(t_i + \tau_i)},$$
$$\Delta \mu_j' = \Delta \mu_j \frac{t_j}{(t_j + \tau_j)}.$$  (7)

Now the parties concerned can compare incrementations of efficiency scores on formulas (7). Again, if incrementations satisfy the parties, the negotiation process is completed, else the negotiation process continues under modified initial conditions.

Figure 1 shows the intersection of the production possibility set with two-dimensional plane determined by two output indicators. Methods for construction of such sections are described by Krivonozhko et al. (2004). Units $Z_1$ and $Z_2$ designate initial positions of the two countries under consideration. Points $A$ and $B$ represent the goals for these two countries. Vectors $a$ and $b$ determined as gradients of potential functions are, in a sense, ideal directions. The real-life trajectories will be deviated from these ideal directions.

![Figure 1: Initial positions of countries and their goals](image)

**Theorem 1.** There exist polyhedral cones in the multidimensional space that cannot be described by relations (8).

At the sixth stage, we will modify the potential function. Consider cones $C_i = -C_i^*$ and $C_j = -C_j^*$ outgoing from the vertices $A$ and $B$. These cones cut some regions $S_1$ and $S_2$ on the boundary of $T$, see Figure 1. Units $Z_1$ and $Z_2$ are aimed at these regions at first steps of their development.

Now, let unit $Z_i'(Z_j')$ belongs to cone $C_i'$. Take direction $c_i$ towards goal $A(B)$. This direction may deviate from direction $a(b)$, however this direction is a quite good for movements to goal $A(B)$, i.e. $c_i \in C_i(C_i^*)$.

Therefore any direction outgoing from point $A(B)$ and having acute angle with any vector from cone $C_i'(C_i^*)$ gives us the direction of increasing potential function value. All such directions form negative polar cone $C_i^*(C_i^*)$,

$$C_i^* = \{w'c \geq 0, c \in C_i\},$$
$$C_j^* = \{v'c \geq 0, c \in C_j\}. $$  (9)

The boundaries of cones $C_i^*$ and $C_j^*$ represent equipotential surfaces of potential functions $\tilde{f}_i(Z), \tilde{f}_j(Z)$. Now, they are convex functions. Figure 2 shows equipotential surfaces of linear function for points $A_i, A_j, A_m$ and a modified convex function for points $B, B_i, B_j$, respectively.

![Figure 2: Modifications of potential functions](image)

Now, we consider how to calculate the value of convex potential function. Solve optimization problem

$$\min \alpha$$
$$Z_\alpha \in (A - \alpha A) + C^*.$$  (10)
Then the value of functions $\tilde{f}_i(Z_i)$ and $\tilde{f}_2(Z_i)$ for arbitrary point $Z_i$ is determined according to the following formulas

$$
\tilde{f}_i(Z_i) = (A - \alpha^* a)^T a,
\tilde{f}_2(Z_i) = (B - \alpha^* b)^T b,
$$

where $\alpha^*$ is an optimal value of $\alpha$ in problem (10).

Figure 3 explains the main idea of formulas (10) and (11).

Such point $(A - \alpha^* a)$ or $(B - \alpha^* b)$ is searched on the line going through point $A$ ($B$), which has the same potential value with point $Z_i$. For this purpose, optimization problem of the form (10) is solved. Then the value of function for point $(A - \alpha^* a)^T$ is calculated as in linear case.

A dashed line in Figure 3 shows equipotential surface for linear function. As it follows from Figure 3, points $Z_i, Z_i, (A - \alpha^* a)$ have the same potential relative to a linear function. However, position of unit $(A - \alpha^* a)$ is much better than units $Z_i$ and $Z_i$ if one considers the goal to reach point $A$. Therefore a modified potential function evaluates unit’s position much better.

The seventh stage. Directions $a$ and $b$, that were determined as optimal at the final steps of units movement to the goals, may be not optimal at the initial steps of units development. Therefore, experts using AHP find tactical directions of unit’s development. These directions are designated as $d_i$ for unit $Z_i$ and $g_i$ for unit $Z_i$, respectively, see Figure 4. It is reasonable to choose these vectors so that they aim at the interior of the regions $S_i$ and $S_i$, respectively. These regions are formed by intersections of cones $\tilde{C}_i$ and $\tilde{C}_i$ with the frontier, see Figure 4.

The position of the frontier changes with the course of time since positions of all units change. A new frontier is designated by dashed line in the Figure 4. Let us find new directions so that these directions belong to cones $\tilde{C}_i$ and $\tilde{C}_i$, respectively. These directions are designated by directions $d_i$ and $g_i$, respectively in Figure 4.

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Some ulterior effects in the DEA models

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ABSTRACT
Applications of the DEA models show that inadequate results may arise in some cases: a) too many efficient units may appear in some DEA models; b) a DEA model may show an inefficient unit from the point of view of experts as an efficient one. The purpose of this paper is to reveal the causes of such results. Next, we suggest methods for improving DEA models adequacy.

Keywords: Data Envelopment Analysis, some ulterior effects

MAIN RESULT
Consider a set of \( n \) observations of actual production units \((X_j, Y_j), j = 1, \ldots, n\), where the vector of outputs \( Y_j = (y_{j1}, \ldots, y_{jm}) \geq 0\). \( j = 1, \ldots, n \), is produced from the vector of inputs \( X_j = (x_{j1}, \ldots, x_{jm}) \geq 0\). The production possibility set \( T \) is the set \( \{ (X, Y) \mid \text{the outputs } Y \geq 0 \text{ can be produced from the inputs } X \geq 0 \} \).

The production possibility set \( T_B \) for BCC (Banker, Charnes, Cooper) model can be written in the form (Banker et al., 1984)

\[
T_B = \left\{ (X, Y) \mid \sum_{j=1}^{m} \lambda_j x_{j1} \leq X, \sum_{j=1}^{m} \lambda_j y_{j1} \geq Y, \sum_{j=1}^{m} \lambda_j a_j = 1, \lambda_j \geq 0, j = 1, \ldots, n \right\}.
\]

(1)

It was shown in the DEA scientific literature (see, Krivonozhko et al., 2009) that the BCC model can approximate any DEA model from a large family of DEA models. For this reason, we consider the BCC model as a basic model in our exposition.

An elegant and subtle approach was proposed in the DEA area to deal with the problems of DEA models inadequacies. This approach is based on incorporating domination cones (Yu, 1974) in DEA models. A number of outstanding papers were devoted to substantiation, development and applications of domination cones to DEA models (Brockett et al., 1997; Charnes et al., 1989; Charnes et al., 1990; Thompson et al., 1997; Wei et al., 2008; Yu et al., 1996). Cones are usually determined in the dual space of multipliers.

It is rather difficult, however, for a manager (decision making person) to determine cones in the multipliers space that is dual to the space of inputs and outputs where a production possibility set is constructed (Cooper et al., 2000). For this very reason only two particular DEA models with cones are widely used in practice at present: the assurance region model and the cone-ratio model (Cooper et al., 2000). Allen and Thanassoulis (2004) proposed a method for improving DEA models adequacy for the special case of a single input/multi-output CCR model. They introduced the notion “anchor points” and showed how to use these points in order to construct unobserved units and to improve envelopment in DEA models.

Bougnol and Dula (2009) gave a strong definition of anchor points and presented algorithms for discovering such points. However, their algorithms may produce units that are just usual efficient units (vertices) in DEA models.

Edvardsen et al. (2000) suggested an empirical witty method for discovering “suspicious” units; they call them “exterior units”. However, their method cannot discover all suspicious units.

The main idea of incorporating domination cones in DEA models is to reduce the domain of multipliers. For this purpose, additional constraints on multipliers are incorporated in the DEA models.

In the assurance region method, constraints on the multipliers are added to the CCR (Charnes, Cooper, and Rhodes) model in the following manner; see Charnes et al. (1990),

\[
v_i l_{ir} \leq v_i \leq v_i u_{ir}, \quad (i = 2, \ldots, m), \]

\[
u_i l_{ir} \leq u_i \leq u_i U_{ir}, \quad (r = 2, \ldots, s),
\]

(2)
where $l_{ij}, u_{ij}, l_{uv}, U_{uv}$ are given low and upper bounds on the ratios of multipliers. However, relations (2) cannot describe all possible polyhedral cones in space $E^m_E$.

**Assertion 1.** There exist polyhedral cones in multidimensional space $E^m_E$ that cannot be described by relations (2).

The cone-ratio model is written in the form

$$\text{max} (u_i Y_i - u_e)$$

Subject to

$$v^T X_v = 1,$$
$$-v^T X_v + u^T Y_u - u_e \leq 0, \quad j = 1, \ldots, n,$$
$$u \in U, \quad v \in V,$$

where variables $v \in E^n_E, u \in E^n, u_e \in E^1$ and $U, V$ are given polyhedral cones.

In practice, polyhedral cones $U$ and $V$ are constructed as follows: a) some excellent units are chosen from the point of view of experts; b) averages of the optimal multipliers $u^*_i, v^*_i$ are computed for every excellent unit $i \in \tilde{E}$.

Vectors $u^*_i, v^*_i, i \in \tilde{E}$ form polyhedral cones $U$ and $V$.

**Fig. 1: Transformation of the frontier in the output subspace in the cone-ratio method**

Cone $U$ and $V$ reduce the feasible domain of multipliers, while the feasible domains of outputs, see Fig. 1, and inputs, see Fig. 2, are expanding.

Now, we make an attempt to reveal the causes of inadequacies in DEA models.

**Hypothesis.** Let the cone-ratio method allows one to reduce the number of "suspicious" production units, i.e. the units that have 100% efficiency score, however, these units should be inefficient from the point of view of experts.

Production possibility set $T_B(1)$ is a convex polyhedral set. According to the classical theorems of Goldman (1956) and Motzkien (1936) any convex polyhedral set can be represented as a vector sum of convex combination of vertices and the non-negative linear combination of vectors (rays).

**Definition 1.** We call an efficient unit terminal unit if an infinite edge is outgoing from this unit.

**Definition 2.** We call a face of set $T_B$ a terminal face if this face contains an infinite edge.

Then the following assertion can be proved.

**Theorem 1.** If some production units become inefficient as a result of inserting cones in the BCC (CCR) model (3), then it is necessary that there exist terminal production units among such inefficient units and/or some such units belong to terminal faces.

The following optimization models enable us to find terminal units or units belonging to terminal faces among all set $EF$ of efficient units.

For this purpose two types of models are solved for every efficient unit $q \in EF$.

**Problem $P_i(q)$** ($i = 1, \ldots, m$)

$$\text{max} J_u = \eta$$
$$\sum_{j=1}^n X_j \lambda_j \leq X_q + \tau d_i, \quad \sum_{j=1}^n Y_j \lambda_j \geq \eta Y_q,$$
$$\sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, \quad \tau \geq 0,$$

where $d_i = (0, \ldots, 1, \ldots, 0) \in E^n$, the unity is in $i$-th position.
Theorem 2. If in problem (4) optimal value \( J_{\lambda} = 1 \), then unit \((X_q, Y_q)\) is a terminal one or belongs to a terminal face.

The following models determine infinite edges emanating along direction \( g_i \), where \( g_i = (0, \ldots, 1, \ldots, 0) \in E^i \), the unity is in \( i \)-th position.

Problem \( R_i(q) \) \((i = 1, \ldots, r)\)

\[
\begin{align*}
\min J_{\lambda} &= 0 \\
\sum_{j=1}^{n} X_j \lambda_j &\leq \theta X_q,
\end{align*}
\]

\[
\sum_{j=1}^{n} Y_j \lambda_j \geq Y_q - \tau g_i,
\]

\[
\sum_{j=1}^{n} \lambda_j = 1,
\]

\( \lambda_j \geq 0, \tau \geq 0. \) \((5)\)

Theorem 3. If in problem (5) optimal value \( J_{\lambda} = 1 \), then unit \((X_q, Y_q)\) is a terminal one or belongs to a terminal face.

However, the cone-ratio model (3) cannot help in every case where suspicious units appear in the DEA models. Figure 3 depicts a three-dimensional BCC model. Points A-F (efficient units) determine the production possibility set \( T_B \). Point \( B \) is a terminal unit. However, it is impossible to transform the frontier with the help of cones \( U \) and \( V \) in such a way that terminal point \( B \) would be inefficient, see Fig. 4.

Only simultaneous transformation of the frontier in the space of inputs and outputs enables one to make suspicious unit \( B \) inefficient, see Fig. 5.

Yu et al. (1996) suggested the generalized DEA (GDEA) model that unifies and extends most the well-known DEA models based on using domination cones in their constraint sets. The production possibility set of GDEA model is written in the form

\[
T_{\text{GDEA}} = \left\{ (X, Y) \left\{ \begin{array}{l} X\lambda - X \\
-\bar{X}\lambda + Y \end{array} \right\} \in W^+, \lambda \in K^-, \delta_1 \epsilon^T \lambda + \delta_2 \epsilon^T (-1)^{\delta_3} \lambda_{\alpha+1} = \delta_1, \lambda_{\alpha+1} \geq 0 \right\}, \quad (6)
\]

where \( W \subseteq E^{nr} \) and \( K \subseteq E^n \) are the closed convex cones. \( W^+ \) and \( K^- \) are the negative polar cones of sets \( W \) and \( K \), respectively. Cones are usually determined in the dual space of multipliers. Parameters \( \delta_1, \delta_2 \) and \( \delta_3 \) are binary ones assuming only the values 0 and 1.

It is rather difficult for a manager (expert) to determine cones in the multipliers space that is dual to the space of inputs and outputs where a production possibility set is constructed.

For this very reason it is difficult to use the GDEA model in practice.

Krivonozhko et al. (2009) proposed a model that is more general than the GDEA model, on the one hand, as it covers situations that the GDEA model cannot describe. On the other hand, this model enables one to construct step-by-step any model from a large family of the DEA model by incorporating artificial units.
and rays in the space of inputs and outputs in the BCC model, which makes the process of model construction visible and more understandable.

The production possibility set of this model is written in the form

\[ T_e = \left\{ (X,Y) \mid X \geq \sum_{j=1}^{n} X_j \lambda_j + \sum_{i=1}^{m} D_i \mu_i + \sum_{i=1}^{m} A_i \rho_i, \right. \]

\[ Y \leq \sum_{j=1}^{n} Y_j \lambda_j + \sum_{i=1}^{m} G_i \mu_i + \sum_{i=1}^{m} B_i \rho_i, \sum_{i=1}^{m} \lambda_i + \sum_{i=1}^{m} \mu_i = 1, \]

\[ \lambda_i \geq 0, j=1, \ldots, n, \mu_i \geq 0, i \in I, \rho_i \geq 0, k \in J \} \], \quad (7)

where \((D_i,G_i), i \in I, I\) is a set of artificial production units, \((A_k,B_k), k \in J, J\) is a set of vectors (rays) added to the model.

Figure 6 shows the transformation of the frontier of the two-dimensional BCC model with the help of artificial units and rays.

In addition to problem (4) and (5), we can also discover terminal (suspicious) production units with the help of constructions of two-dimensional and three-dimensional sections of the frontier.

Define three-dimensional affine subspace in space \(E^{mir}\) as

\[ \text{Pl}(X,Y,d_1,d_2,d_3) = (X,Y) + \alpha d_1 + \beta d_2 + \gamma d_3, \] \quad (8)

where \((X,Y) \in T_e\), \(\alpha\), \(\beta\) and \(\gamma\) are any real numbers, directions \(d_1, d_2, d_3 \in E^{mir}\) are not parallel to each other.

Next, define intersections of the frontier with three-dimensional affine subspace

\[ \text{Sec}(X,Y,d_1,d_2,d_3) = \{(X,Y) \mid (X,Y) \in \text{Pl}(X,Y,d_1,d_2,d_3) \cap W_{Eff,T}(X,Y); d_1,d_2,d_3 \in E^{mir}\}, \] \quad (9)

where \(W_{Eff,T}\) is a set of weakly Pareto-efficient points. Krivonozhko et al. (2005) have proved that set \(W_{Eff,T}\) coincides with the boundary of \(T_e\) (1).

By choosing different directions \(d_1, d_2\) and \(d_3\), we can construct various two-dimensional and three-dimensional sections going through point \((X_o,Y_o)\) and cutting the frontier.

Parametric optimization algorithms for construction of sections of the type (9) are described in detail by Krivonozhko et al. (2004) and Volodin et al. (2004).

Moreover, thanks to our package FrontierVision, one can add to the DEA model any artificial units and rays on the computer screen interactively.

**Assertion 2.** There always exists a section (9) that reveals any terminal unit.

However, the specific section may not reveal some terminal units. In the three-dimensional BCC model, see Fig. 3, unit \(B\) is a terminal one. In Fig. 4, unit \(B\) does not look like a terminal one. The section in Fig. 5 reveals this unit as a terminal point.

Generally speaking, a two-dimensional section of the type (9) consists mainly of a number of segments and two rays. The first and the last vertices in the chain of segments are usually terminal units.

**CONCLUSION**

In this paper, we have shown that terminal points may cause inadequate results in the DEA models. Terminal units arise because non-countable (continuous) production possibility set \(T\) is determined on the basis of a finite number of production units, some of these units turn out to be terminal ones. The gap of derivatives may take place at these points. For example, the left-hand side scale elasticity takes infinite value, and the right-hand side scale elasticity takes zero value at some terminal points, see Førsund et al. (2007).

Let us remember that Farrell (1957) introduced artificial units at infinity in order to smooth his model, see also Førsund et al. (2009).

In this paper we propose to incorporate artificial units and rays interactively on the screen of the computer by experts into some BCC model, what makes the DEA models more adequate and adjustable.

Our computational results show that terminal units are responsible for more than 90% inadequacies in the real-life DEA models.

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ABSTRACT

Since 1978 that Data Envelopment Analysis (DEA) was first introduced by Charnes et al. in [1], there has been rapid and continuous growth in this field. In the current paper we investigate one of the most recent and challenging advances, called “predictive DEA”. The presented framework is an extension of the previous study [2] in which “Predictive Operational Benchmarking” (POB™) model was designed based on Supply Chain Operational Reference (SCOR™) model as the basis for competitive benchmarking, as well as Analytic Hierarchy Process (AHP), DEA and Genetic Algorithm (GA). POB™ has been developed to forecast company’s future performance by reviewing and analyzing its past performance metrics in Supply Chain Management (SCM). The current paper aims to improve the forecasting precision of POB™ framework by comparing prediction capability of different mathematical methods, e.g., Neural Networks, Seasonal Exponential Smoothing, and Optimized Moving Average with that of POB™. Through a series of testing it is shown how the new methodology could help Decision-Making-Units (DMUs) improve their prediction accuracy over a long run. Finally we share some of the experiences we had and discuss the results. To the best of our knowledge, this work is among the very few studies that have been carried out on this area and it will serve as an impetus to further research.

Keywords: Decision-Making-Units, Data Envelopment Analysis, Predictive Analytics

INTRODUCTION

Data envelopment analysis is a data oriented approach for measuring the performance and evaluating the relative efficiency of decision-making-units. This analysis identifies a frontier in multiple-input multiple-output settings on which the relative performance of all utilities in the sample can be compared. Building on the ideas of Farrell [3], DEA was first introduced by Charnes et al. in 1978 [1]. Since then many DEA models have been developed such as CCR [1], BCC [4], FDH [5], RAM [6], and slack-based measure (SBM) [7]. The main developments of DEA are documented by Seiford and Thrall [8] and Emrouznejad et al. [9].

In the current paper we investigate one of the most recent and challenging advances in this field, called “predictive DEA”. The presented framework is an extension of the previous study [2] in which “predictive operational benchmarking” model was designed based on supply chain operational reference model as the basis for competitive benchmarking, as well as analytic hierarchy process, data envelopment analysis and genetic algorithm. Off-the-shelf POB™ software was developed to assess supply chain management performance at the aggregated level while covering both operations and business aspects, such as cost of goods sold, operating expenses, gross sales and net profit. In POB™, GA functions as a tool to generate the evolution of a population. Combining with DEA, GA was employed to forecast future performances of 272 companies in South East Asia. It is shown that the proposed framework is an effective and efficient tool to examine and forecast a company’s overall strength and sustainability in the future business setting, primarily in terms of profit and growth. It enhances strategic decision-making for investments with a long term horizon. The current paper aims to validate POB™ capability to predict with prediction capability of different mathematical methods, e.g., Neural Networks (NN), Seasonal Exponential Smoothing (SES), and Optimized Moving Average (OMA).

Through a series of testing, it is shown how the new methodology could help improve the
prediction accuracy of DMUs over a long run compared to the previous study.

**PREDICTIVE ANALYTICS**

Predictive analytics comprises a variety of techniques from statistical analysis and data mining that analyze current and historical facts to make predictions about future events. In business, predictive models can capture relationships among many factors associated with a particular set of conditions, and can discover and exploit hidden patterns in historical data. Basically, these models ensure that the actions taken today will directly achieve the organization’s goals tomorrow. They can help companies optimize existing processes, better understand customer behavior, identify unexpected opportunities, and anticipate problems before they happen by allowing them to move beyond “How are we doing?” to “What does our future look like?”

Despite many advantages of conducting predictive analysis, this practice has not been leveraged optimally at most enterprises. Some common problems with the implementation of predictive analytics include:

1. Getting started: What is the approach? Who should we hire, how do we organize the project and how do we build the environment?
2. Developing a model: for example, issues relating to the preparation of huge amounts of data, training models and statistical application
3. Identifying and applying the right predictive model
4. Ongoing maintenance of models and validations

**Figure 1: Common problems in implementing predictive analytics**

In the current paper a predictive data envelopment analysis model is proposed to integrate performance benchmarking from individual SC measurements in operations and finance, based on which the future performance can be expected with the application of mathematical algorithms. The predictive DEA model has two main objectives: (1) to expect the company’s performance in the dynamic and complex business environment over a long run and (2) to suggest best practices in a proactive manner so as to reduce the risk of poor performance and adapt to the potential growth of the company. In general, these steps should be followed to develop predictive models:

1. Identify the outputs and key metrics that need to be analyzed.
2. Identify the predictors, which are variables that can be measured for individuals or other entities to predict future.
3. Collect data according to the predictors identified.
4. Identify the right predictive model to forecast future trends.
5. Verify the models and tweak them for better accuracy.

**Figure 2: Steps to develop predictive models**

After collecting data, the next step in the predictive model developing is identifying the right predictive model. There are many predictive models that can be applied across industries and domains, based on applicability. Data mining [10] and computational intelligence techniques such as neural networks, fuzzy set, evolutionary algorithms, rough set theory, machine learning, multi-criteria decision aid, etc., emerged as alternative techniques to the conventional statistical and econometric models and techniques that have dominated this field since the 1930s [11] and have paved the road for the increased usage of these techniques in various areas of economics and finance [12-14]. Examples of the utilization of these techniques are the applications of neural network in stocks selection [15] and predicting the S&P 100 index using rough sets [16] and various types of intelligent systems for making trading decisions [17-27]. Other real world applications in the field of finance such as credit cards assessment, country risk evaluation, credit risk assessment, corporate acquisitions [28], business failure prediction, [29-31] prediction of the financial health of the dot.com firms. [32] and bankruptcy prediction [33], customer segmentation [34] are but few examples showing the diversity of the coverage of these new techniques.

In the previous study [2], we explored the prediction capability of genetic algorithm. In the current paper, Prediction capability of POB™ is being validated with prediction capability of neural networks. Furthermore the results obtained from neural networks approach are compared to that of two well known methods, i.e., seasonal exponential smoothing and optimized moving average. In the following we briefly describe these methods.
FORECASTING METHODS

Neural Networks
A neural network is an adaptive system that changes its structure based on external or internal information that flows through the network during the learning phase. Compared with econometric models, which require numerous hypotheses and suffer various other limitations, neural network models are more flexible, able to solve any nonlinear problems, and more suitable for analyzing dynamic environments.

Modern neural networks are non-linear statistical data modeling tools. They consist of different layers which are connected to each other by connection weights. Between the extremities of the input layer and the output layer, are the hidden layers. The nodes in each layer are connected by flexible weights which are adjusted based on the error or bias. This process of changing the connection weights is called training or learning.

To model our problem, we build a Focused Time-Delay Neural Network (FTDNN), which is well suited to data prediction. This network is then trained in the Matlab neural network toolbox using backpropagation, which is the most popular neural network model and is widely applied in various problems. The following figure illustrates a two-layer FTDNN.

Seasonal Exponential Smoothing
Exponential smoothing is a very popular scheme to produce a smoothed data or to give the forecast as a weighted sum of the past observations. Whereas in single moving averages the past observations are weighted equally, exponential smoothing assigns exponentially decreasing weights as the observation get older. In other words, recent observations are given relatively more weight in forecasting than the older observations.

Consider a series of data \( \{a_t; 1 \leq t \leq T\} \) which is a sample of observations of a metric over time period \([1,T]\). We assume the seasonal index to be 1, that is, the number of periods that complete one season; \( S_t \) the smoothed observation at time \( t \); \( b_t \) the trend smoothing at time \( t \); \( L_t \) the seasonal smoothing at time \( t \); \( F_{t+k} \) the forecast at \( k \) periods ahead; \( k \) the number of periods ahead we want to predict; \( \alpha \) the overall smoothing parameter; \( \beta \) the seasonal smoothing parameter; \( \gamma \) the trend smoothing parameter. Here, \( 0 < \alpha, \beta, \gamma < 1 \).

The \( k \)-period ahead forecast is:
\[
F_{t+k} = (S_t + kb_{t-L_t+k}).
\]
Where \( S_t, b_t, L_{t-L_t+k} \) are calculated according to the following formulas:
\[
S_t = \alpha \frac{a_t}{\lambda_{t-1}} + (1- \alpha)(S_{t-1} + b_{t-1});
\]
\[
b_t = \gamma (S_t - S_{t-1}) + (1- \gamma)b_{t-1};
\]
\[
L_t = \beta \frac{a_t}{\lambda_t} + (1- \beta)L_{t-1}.
\]

Optimized Moving Average
In statistics, a moving average is a type of finite impulse response filter used to analyze a set of data points by creating a series of averages of different subsets of the full data set. A moving average is commonly used with time series data to smooth out short-term fluctuations and highlight longer-term trends or cycles. In optimized moving average, we have:
\[
f(t) = \frac{1}{n} \sum_{i=1}^{n} y(t-i).
\]

RESULT AND DISCUSSION
To test and verify the prediction capability of the POB™ model, three forecasting methods, neural networks, seasonal exponential smoothing, and optimized moving average, have been applied. Based on available data we select monthly sales of 40 products from different industries. Using real data enables us for checking our predictions with real results.

The next step is to compare the results obtained from POB™ to that is neural networks, seasonal exponential smoothing, and Optimized Moving Average. To conduct these two analyses (SES and OMA), we use iForcaster™ software which has been developed based on the iCognitive and SIMTech joint R&D project. iForcaster™ is a one-stop forecasting platform covering from data importing, configuration, forecasting, results comparison to report generation. iForcaster™ engine is designed to incorporate a number of algorithms to do forecast including SES and OMA which we used in our study.

To identify the prediction accuracy, Relative Root Mean Squared Error (RRMSE) is calculated. RRMSE measures relative dispersion of the forecast from actual. Denote
by $y_1, y_2, ..., y_n$ the actual values, and by $\hat{y}_1, \hat{y}_2, ..., \hat{y}_n$ the corresponding forecasted values.

Based on RRMSE results for Neural Network and POB™ shows similar results and SES and OMA have relatively low accuracy levels. This somehow validates proposed framework which could help decision makers make more precise decisions.

The results confirm the validity of the POB™ model and by using POB™ companies can take proactive measures to adapt their capacities and avoid incurring dramatic losses. The predictive DEA model can be applied to align capacity with potential growth so as to mitigate risks of uncertainty and to sustain profitable growth in the turbulent business environment. With forecasted performance improving, capacity should be ramped up to capture the maximum profit and to increase the market share. On the other hand, with forecasted performance declining, measures should be taken for companies to get prepared; for instance, to reengineer the organization structure, to focus more on R&D, to look for new business streams or markets. Consequently, strategic decision-making for investments with a long term horizon will be enhanced.

The current study is a research work and prototype on predictive DEA using Genetic Algorithms. This model needs to be further tested and validated using different mathematical models like Fuzzy Logic, Rough Set Theory, Multi agent models etc. Prediction is always challenging and right prediction is even more challenging, this paper is trying to predict a right model for prediction as the title suggests but not sure about outcome. This model should also be tested against other predictive models, which will be followed by extensive use of the tool in industry practices, including regional annual benchmarking studies and strategic benchmarking for a particular industry sector for predicting all SCOR™ performance metrics. Furthermore, the model employs mainly strategic level metrics, which require inputs of lower level measurements. Hence, it also allows the company to investigate root causes of its market positioning towards its lower level operations. The bi-directional analysis empowers the company to discover detailed problems and re-engineer processes along the supply chain, in order to achieve maximum benefits with minimum efforts.

REFERENCES


Ranking of IDEA inefficient DMUs in DEA

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ABSTRACT

Inefficient frontier can be obtained by Inverted Data Envelopment Analysis (IDEA) models, which contain some of inefficient DMUs and by IDEA models the IDEA inefficiency of these Decision Making Units (DMUs) are the same. To appreciate IDEA inefficient DMUs, it is essential that these inefficient units to be ranked. In this paper, we introduce ranking idea of inefficient DMUs and propose a method for ranking IDEA inefficient DMUs using IDEA model. By an example we illustrate the proposed method.

Keywords: DEA, Inverted DEA, Inefficient DMUs, Ranking

INTRODUCTION

The conventional Data Envelopment Analysis (DEA) measures the relative efficiency of a set of Decision Making Units (DMUs), but Inverted DEA (IDEA) measures the relative inefficiency of them. By considering that the DEA model evaluates each DMU, optimistically [2], the IDEA model [4, 5] evaluates each DMU, pessimistically. But by considering that the objective function DEA model and IDEA are not comparable by the reason of restricted conditions, so we can trigger the same restricted conditions for these two models and propose a model which called bounded DEA [5].

Inspire of always is not necessary to recognize the efficient units, it is essential sometimes to appreciate inefficient DMUs. For example, all governments try to understand the level of poverty and make shorter its distance from the level of welfare, these conditions exist in modern societies and it makes shorter hierarchies and causes social justice. So, in order to appreciate the poverty level, we should recognize the people who are at this level. In reality these people are inefficient units. For appreciating inefficient unity we use IDEA model, since the inefficient units are not the same, it is necessary that these inefficient units to be ranked. There are so many examples such as the uneducated and the poor which would be called inefficient units. When DMUs are evaluated by IDEA models, it is possible there are more than one unit lie on the inefficient frontier.

In this paper, we introduce the inefficient ranking and propose a method for ranking IDEA inefficient DMUs using IDEA model.

The paper unfolds as follows. Section 2 provides a brief of DEA and IDEA models. Section 3 proposes inefficient ranking method. Example and its results are shown in Section 4. Section 5 contains conclusions.

BASIC CONCEPTS

We assume that there are \( n \) DMUs to be evaluated, indexed by \( j (j = 1, \ldots, n) \) and each DMU is assumed to produce \( s \) different outputs from \( m \) different inputs. Let the observed input and output vectors of DMU \( j \) be \( X_j = (x_{j1}, \ldots, x_{jm}) \) and \( Y_j = (y_{j1}, \ldots, y_{js}) \), respectively. And all components of vectors \( X_j \) and \( Y_j \) for all DMUs are non-negative and each DMU has at least one strictly positive input and output. The production possibility set \( T_c \) is defined as

\[
T_c = \left\{ (X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j, \quad Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n \right\}.
\]

The weights \( v_i, (i = 1, \ldots, m) \) and \( u_r, (r = 1, \ldots, s) \) that maximize the ratio of weighted sum of outputs and weighted sum of inputs, for observed DMU, \( \alpha \in \{1, 2, \ldots, n\} \), on \( T_c \) are found through the following model [2]:

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\[ \theta_0^* = \max \sum_{i=1}^{m} u_i y_{i0} \]
\[ \text{s.t.} \sum_{i=1}^{m} v_j x_{i0} = 1 \]
\[ \sum_{j=1}^{n} u_j y_{j0} - \sum_{i=1}^{m} v_j x_{ij} \leq 0, \]
\[ j = 1, \ldots, n, \]
\[ u_j, v_j \geq 0, i = 1, \ldots, m, \]
\[ r = 1, \ldots, s. \]

\( u_r, (r = 1, \ldots, s) \) and \( v_i, (i = 1, \ldots, m) \) are output and input weights assigned to the \( r \)th output and the \( i \)th input, respectively. They must be determining as decision variables. If there exists a set of positive weight that makes \( \theta_0^* = 1 \), then DMU \( o \) is referred to be CCR efficient [2]; otherwise, we call it to be CCR inefficient. IDEA model which has been proposed in [5] evaluates DMUs from pessimistic viewpoint and in contrast to DEA model which evaluates DMU \( o \) from the optimistic viewpoint. IDEA model is formulated as follows [5]:

\[ \eta_o^* = \min \sum_{i=1}^{m} u_i y_{i0} \]
\[ \text{s.t.} \sum_{i=1}^{m} v_j x_{i0} = 1 \]
\[ \sum_{j=1}^{n} u_j y_{j0} - \sum_{i=1}^{m} v_j x_{ij} \geq 0, \]
\[ j = 1, \ldots, n, \]
\[ u_j, v_j \geq 0, i = 1, \ldots, m, \]
\[ r = 1, \ldots, s. \]

Definition 1: DMU \( o \) is IDEA inefficient if \( \eta_o^* = 1 \) and there exist at least one optimal \((v^*, u^*)\) with \( v^* > 0 \) and \( u^* > 0 \), otherwise DMU \( o \) is non-IDEA inefficient. The models (1) and (3) determine efficiency and inefficiency frontiers, respectively for \( n \) DMU. In the next section we rank DMUs, which are on inefficient frontier.

RANKING OF IDEA INEFFICIENT DMUS

Using DEA evaluations some DMUs lie on efficient frontier, and some other lie on inefficient frontier. For instance in Figure 1, DMUs A, B, C, D and F lie on efficient frontier, while DMUs H and G lie on inefficient frontier, and these DMUs aren’t same. To verify IDEA inefficient DMUs, we can rank IDEA inefficient DMUs.

![Figure 1: IDEA efficient frontier and IDE inefficient frontier](image)

Similar to supper efficiency methods [1,6] by omitting \((X_o, Y_o)\) from \( T_e \), we define the production possibility set \( T_e' \) as
\[ T_e' = \{(X, Y) | X \leq \sum_{j=1, j \neq o}^{n} \lambda_j X_j, \]
\[ Y \geq \sum_{j=1, j \neq o}^{n} \lambda_j Y_j, \lambda_j \geq 0, j = 1, \ldots, n, j \neq o \}. \]

To obtain the rank score of inefficient DMU, say DMU \( o \), we propose the following model on \( T_e' \):
\[ \psi_o^* = \min \sum_{i=1}^{m} u_i y_{i0} \]
\[ \text{s.t.} \sum_{i=1}^{m} v_j x_{i0} = 1 \]
\[ \sum_{j=1}^{n} u_j y_{j0} - \sum_{i=1}^{m} v_j x_{ij} \geq 0, \]
\[ j = 1, \ldots, n, \]
\[ u_j, v_j \geq 0, i = 1, \ldots, m, \]
\[ r = 1, \ldots, s. \]

By solving model (4) for extreme IDEA inefficient DMUs, the ranking score of them are obtained. Note that if DMU \( o \) be a non-extreme IDEA inefficient DMU, then the ranking score of it cannot determine by model (4).

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NUMERICAL EXAMPLE

Consider 10 DMUs with one input, I, and two outputs, O1 and O2. The data set is taken from Entani et al. [2] and has been shown in Table 1.

<table>
<thead>
<tr>
<th>DMU</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>O1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>O2</td>
<td>8</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>7</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

The relative inefficiency of DMUs are calculated by model (3), and the results are recorded in the 2ed and 4th rows of Table 2. Using definition 1, we find that DMU_A, DMU_B, DMU_F and DMU_J lie on inefficient frontier and are IDEA inefficient. These DMUs aren’t same. Using proposed ranking model, model (4), the ranking results of IDEA inefficient DMUs has been proposed in Table 3.

<table>
<thead>
<tr>
<th>DMU</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>η^*_A</td>
<td>1</td>
<td>1</td>
<td>1.23</td>
<td>1.13</td>
<td>1.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMU</td>
<td>E</td>
<td>G</td>
<td>H</td>
<td>I</td>
<td>J</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>η^*_A</td>
<td>1</td>
<td>1.75</td>
<td>1.1</td>
<td>1.2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is evident that the bigger optimal value of model (4), is the lower the order ranking.

<table>
<thead>
<tr>
<th>IDEA Ineff. DMUs</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ranking score</td>
<td>0.500008</td>
<td>0.761904</td>
<td>0.947368</td>
<td>0.500007</td>
<td></td>
</tr>
</tbody>
</table>

The order ranking of IDEA inefficient DMUs is as,

\[ DMU_J > DMU_A > DMU_B > DMU_F. \]

CONCLUSION

In this paper, we introduced the inefficient ranking and proposed a method for ranking IDEA inefficient DMUs using IDEA models. If DMU, be a non-extreme IDEA inefficient DMU, then its ranking score cannot determine by proposed model. The ranking of extreme and non-extreme IDEA inefficient DMUs could be studied in the future research.

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A method to determine the best game crosses efficiency for DMUs with VRS

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ABSTRACT
Cross efficiency links Decision Making Units (DMUs) performance together. The main idea of cross efficiency is to use Data Envelopment Analysis (DEA) in a peer evaluation. Several methods have been proposed to obtain cross efficiency. These algorithms need a lot of computation efforts. The current paper provides a model to compute the best game cross efficiency for DMUs with Variable Returns to Scale (VRS) by solving only one model. The proposed method can be used to determine the cross efficiency of DMUs with Constant Returns to Scale (CRS). Finally, the proposed model is applied to a numerical example.

Keywords: Data Envelopment Analysis (DEA), Cross Efficiency, Game Cross Efficiency

INTRODUCTION
Data Envelopment Analysis (DEA), introduced by Charnes et al. in 1978 [1], is a mathematical programming based approach that evaluates the efficiency of an organization or, in general, a Decision Making Unit (DMU) relative to a set of comparable organizations. DEA considers multiple inputs and outputs simultaneously, requiring neither a priori weights nor a functional form for input/output relationships. The cross efficiency score for a DMU is obtained by computing that DMU’s set of n scores, and then averaging those scores. The main idea of cross efficiency is to use DEA in a peer evaluation. See for example the Sexton et al. [5] and Doyle and Green [3]. Cross efficiency is used to differentiate between good and poor performers. As mentioned by Doyle and Green [3], cross efficiency is a process with the concept of peer appraisal, as opposed to self appraisal implied by simple efficiency. As demonstrated in Doyle and Green [3], because of the non-uniqueness of the DEA optimal weights, the methods of aggressive and benevolent were proposed. Depending on which of the alternative optimal solution to the DEA linear programs is used, it may be possible to improve a DMU’s performance rating, but generally only by worsening the rating of others. Aggressive (benevolent) model not only maximize the efficiency of a particular DMU under evaluation, but also minimize (maximize) the average efficiency of other DMUs.

As pointed out by Liang et al. [4], the game cross efficiency model was represented. Liang et al. [4] presented a procedure under CRS assumption for determining the best average game cross efficiency for DMUs. This procedure converges after some iteration. When the number of DMUs or the number of iterations of this procedure increased, computation efforts increase as well. This paper proposes a method to compute the best game cross efficiency under Variable Returns to Scale (VRS), which reduces the number of computations efforts. The proposed method can be used to DMUs with Constant Returns to Scale (CRS). Then we demonstrate proposed model with a numerical example.

The rest of current paper is organized as follows. Section 2 provides a background of DEA and DEA cross efficiency. Section 3 proposes the new game cross efficiency model under VRS assumption. Section 4 discusses an example that applies the proposed method. Finally, section 5 presents concluding remarks.

DEA CROSS EFFICIENCY
Let us suppose a production technology transforming a series of input vectors \(x_{ij} (i = 1,2,...,m)\) into the following output vectors \(y_{rj} (r = 1,2,...,s)\) where the subscript \(j\). \((j = 1,2,...,n)\) refers to a set of observed production processes – e.g. country, regional or local economies. The efficiency rating for
any given DMU \( d \) is computed using the BCC model that was rendered by Banker et al. [2]:

\[
\text{max} \quad \theta_d = \sum_{r=1}^{s} \mu_{r, y_{rd}} - u_d
\]

\[
\text{s.t.} \quad \sum_{i=1}^{m} \omega_i x_{ij} - \sum_{r=1}^{s} \mu_{r, y_{ij}} + u_j \geq 0,
\]

\[
j = 1, 2, \ldots, n,
\]

\[
\sum_{i=1}^{m} \omega_i x_{id} = 1,
\]

\[
\omega_i \geq 0, \quad i = 1, 2, \ldots, m,
\]

\[
\mu_i \geq 0, \quad r = 1, 2, \ldots, s,
\]

\[
u_d \text{ free}.
\]

We obtain a set of optimal weights for each DMU \( d = 1, \ldots, n \) under evaluation. The \( d \) cross efficiency for any DMU \( j = 1, \ldots, n \), using the optimal set is then calculated as:

\[
E_d = \frac{\sum_{r=1}^{s} \mu_{r, y_{rd}} - u_d}{\sum_{i=1}^{m} \omega_i x_{ij}},
\]

\[
d, j = 1, 2, \ldots, n.
\]

The average of all \( E_d \) \( (d = 1, \ldots, n) \) is a new efficiency measure for DMU \( j = 1, \ldots, n \) and are used as the cross efficiency score [4]:

\[
\bar{E}_j = \frac{1}{n} \sum_{d=1}^{n} E_d, \quad j = 1, 2, \ldots, n.
\]

As noted in Sexton et al. [5], the optimal weights obtained from model (2) may not be unique. One remedy suggested is to introduce a secondary objective function to resolve ambiguity. Doyle and Green [3] introduced the aggressive and benevolent model. One version of their model seeks to find a multiplier bundle that maximizes the average of the efficiency ratio of the other \( n - 1 \) DMUs with the constraint that the ratio for DMU \( d \) stays at or above its predetermined optimal level. Specially, cross efficiency provides for a measure of efficiency that not only the best multiplier bundle for DMU \( d \) under evaluation, but also the best bundles for all other DMUs. Liang et al. [4] proposed the game cross efficiency model. In their model, rather than using the ideal score for DMU \( d \), they strive to use a score which will actually be representative of its final measure of performance. They defined the game cross efficiency for DMU \( j \) relative to DMU \( d \) as [4]:

\[
\alpha_{dj} = \frac{\sum_{r=1}^{s} \mu_{r, y_{dj}}}{\sum_{r=1}^{m} \omega_{r, y_{dj}}},
\]

\[
d = 1, 2, \ldots, n,
\]

where \( \mu_{r, y_{dj}} \) and \( \omega_{r, y_{dj}} \) are optimal weights in the following model, (5). To calculate the game \( d \) cross efficiency defined in (4) Liang et al. [4] proposed the following mathematical programming for each DMU \( j \):

\[
\text{max} \quad \sum_{r=1}^{s} \mu_{r, y_{rf}}
\]

\[
\text{s.t.} \quad \sum_{i=1}^{m} \omega_i x_{ij} - \sum_{r=1}^{s} \mu_{r, y_{ij}} \geq 0,
\]

\[
l = 1, 2, \ldots, n,
\]

\[
\sum_{i=1}^{m} \omega_i x_{ij} = 1.
\]

\[
\alpha_d \sum_{i=1}^{m} \omega_{i, y_{id}} \leq 0,
\]

\[
\omega_{i, y_{ij}} \geq 0, \quad i = 1, 2, \ldots, m,
\]

\[
\mu_{r, y_{ij}} \geq 0, \quad r = 1, 2, \ldots, s,
\]

where \( \alpha_d \leq 1 \) is a parameter. Liang et al. [4] also rendered an algorithm for deriving average game cross efficiency score. Model (5) maximizes the efficiency of DMU \( j \) under condition that the ratio efficiency of DMU \( d \) isn’t less than its original average cross efficiency. Then the average game cross efficiency for each DMU \( j \) was defined as:

\[
\alpha_j = \frac{1}{n} \sum_{d=1}^{n} \sum_{r=1}^{s} \omega_{r, y_{dj}} \mu_{r, y_{dj}}.
\]

Liang et al. [4] proposed an iterative algorithm to derive optimal average game cross efficiency under CRS assumption. This algorithm is repeated for every \( d \), and it takes some iteration to get the best average game cross efficiency. Thus, this process includes a large number of computation efforts.

**THE PROPOSED MODEL TO OBTAIN THE BEST CROSS EFFICIENCY**

This section presents a model to reduce the computation efforts of the proposed method by Liang et al. [4]. That model maximizes the efficiency of DMU \( j \) under the condition that the difference between ratio efficiency of DMU \( d \) and \( \alpha_d \) isn’t less than zero, where \( \alpha_d \) is average cross efficiency in its first usage. In our model we want to find the optimal weights for each DMU \( j \) and also minimize the difference between simple efficiency of DMU \( j \).
and $\alpha_{jd}(d = 1, \ldots, n)$. For this purpose we add a constraint, the difference between simple efficiency and cross efficiency is less than $\alpha$, to the CCR model and also add the variable $\alpha$ with negative sign to the objective function. We consider the following mathematical programming problem for each $DMU_j$:

$$\max \sum_{r=1}^{s} \mu_{jr} y_{jr} - u_{d}^{d} - \alpha$$

s.t. $\sum_{r=1}^{s} \mu_{jr} y_{jr} - \sum_{i=1}^{m} \alpha_{ij} x_{id} - u_{d}^{d} \leq 0$,

$$l = 1, 2, \ldots, n,$$

$$\sum_{i=1}^{m} \alpha_{ij} x_{id} = 1,$$

$$\theta_d = \sum_{r=1}^{s} \mu_{jr}^{d^*} y_{nr} - u_{d}^{d^*} \leq \alpha,$$

$$\sum_{i=1}^{m} \alpha_{ij}^{d^*} x_{id}$$

$$\alpha_{ij} \geq 0, \quad i = 1, 2, \ldots, m,$$

$$\mu_{jr} \geq 0, \quad r = 1, 2, \ldots, s,$$

$$u_{d}^{d} \text{ free},$$

(7)

where $\theta_{d}$ is the simple efficiency of $DMU_j$.

For each $DMU_j$ this model is solve $n$ times, once for each $d = 1, \ldots, n$. Let $\mu_{jr}^{d^*}$ and $\alpha_{ij}^{d^*}$ be optimal solution to model (7), thus the game cross efficiency is defined as:

$$\alpha_{jd} = \frac{\sum_{r=1}^{s} \mu_{jr}^{d^*} y_{nr} - u_{d}^{d^*}}{\sum_{i=1}^{m} \alpha_{ij}^{d^*} x_{id}}$$

$$d = 1, 2, \ldots, n.$$  

(8)

Therefore, the average game cross efficiency for each $DMU_j$ is defined as:

$$\alpha_{jd} = \frac{1}{n} \sum_{d=1}^{n} \frac{\sum_{r=1}^{s} \mu_{jr}^{d^*} y_{nr} - u_{d}^{d^*}}{\sum_{i=1}^{m} \alpha_{ij}^{d^*} x_{id}},$$

$$j = 1, 2, \ldots, n.$$  

(9)

Obviously, this solution differs from the best average game cross efficiency which is the result of Liang et al’s algorithms, [4].

**NUMERICAL EXAMPLE**

To obtain the game cross efficiency according to proposed method, we consider five DMUs, with three inputs and two outputs.

The inputs and outputs of DMUs are given in Table 1. To obtain the best cross efficiency, we evaluate DMUs using BCC model. Then using the obtained simple efficiencies, the best cross efficiencies are obtained by the proposed method. The results game cross efficiencies are shown in second row of the Table 2.

**Table 1: Inputs and outputs of DMUs**

<table>
<thead>
<tr>
<th>DMUs</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$y_1$</th>
<th>$y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>DMU2</td>
<td>5</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>DMU3</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>DMU4</td>
<td>5</td>
<td>9</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>DMU5</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

**Table 2: Game crosses efficiency of DMUs**

<table>
<thead>
<tr>
<th>DMUs</th>
<th>$\alpha_{DMU1}$</th>
<th>$\alpha_{DMU2}$</th>
<th>$\alpha_{DMU3}$</th>
<th>$\alpha_{DMU4}$</th>
<th>$\alpha_{DMU5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross</td>
<td>0.60</td>
<td>0.98</td>
<td>0.99</td>
<td>0.81</td>
<td>0.72</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

This paper proposed a new DEA game cross efficiency model to evaluate the best cross efficiency of DMUs under assumption f VRS. The proposed method can be applied to the best cross efficiency under CRS assumption. For each DMU, a multiplier bundle is determined that minimizes the difference between simple efficiency and game cross efficiency. In the approach that proposed by Liang et al. [4], a large number of iterations should be done and take a lot of computation efforts, while the proposed model doesn’t have this problem and multiplier bundle were obtained by solving only one model.

**REFERENCES**


Comparison between data envelopment analysis and multiple objective linear programming models

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ABSTRACT
The concept of efficiency plays an important role both in Data Envelopment Analysis (DEA) and Multiple Objective Linear Programming (MOLP). Despite this and other apparent similarities, not many papers have been written on the subject of combining DEA and MOLP, and researchers in these two fields have generally not paid much attention to research performed in the other field. This paper will establish that structurally a DEA model to identify efficient units is equivalent to a linear model to generate efficient solutions in the MOLP problems. Also, we show that the Additive model can be used for identifying the efficient units in the Production Possibility Set (PPS) where the possibility assumption is not satisfied. Our point is illustrated with a numerical example.

Keywords: Data envelopment analysis; multiple objective linear programming; linear programming; Additive model

INTRODUCTION
Data envelopment analysis (DEA) was originally proposed by Charnes, Cooper and Rhodes (1978) as a mathematical programming technique to evaluate the relative efficiency of decision making units (DMUs), specifically when the multiplicity of inputs and outputs associated with the DMUs is considered. Although Charnes and Cooper have played a significant role in the development of DEA and MOLP, they have not attempted to tie these two fields together. Yet there exist only a few cross references to articles that have compared DEA and MOLP. One of the first attempts to combine the DEA and MOLP approaches was presented by Golany (1988). Golany [4] first proposed an interactive model combining both DEA and MOLP where the DM will allocate a set of level of inputs as resources and be able to select the most preferred set of level of outputs from alternative points on the efficient frontier. Korhonen [6] discussed the use of the reference direction approach originally proposed by Korhonen and Laakso [1986] for searching the efficient frontier in multiple objective programming and then he showed how these considerations can be utilized in DEA models. Joro et al. [5] considered similarities and dissimilarities between DEA and MOLP formulations from a mathematical standpoint. They showed that mathematically the original DEA models and the reference point approach proposed by Wierzbicki (1980) to solving MOLP problems use quite similar formulations. Wong et al. [10] established an equivalence model between DEA and MOLP, and showed how a DEA problem can be solved interactively without any prior judgment by transforming it into an MOLP formulation.

The aim of this paper is to establish that the Additive model to identify efficient units is equivalent to a linear model to generate efficient solutions in the MOLP problems. This provides an interesting theoretical aspect, because despite the complexity of DEA for searching the efficient frontier, it can be presented as a special case of the existing methods for characterizing the efficient set of a MOLP problem.

This paper is organized as follows. After brief introductions of the DEA techniques and basic DEA models, the equivalence model between DEA and MOLP is shown. Also, we discuss briefly that the Additive model can be used for identifying the efficient units in the Production Possibility Set (PPS) where the possibility assumption is not satisfied. There are many occasions in which inputs and/or outputs do not satisfy the possibility assumption. The current paper addresses efficient DMUs whose...
inputs and outputs are restricted to be as a convex combination. Our point is then illustrated with a numerical example. Concluding remarks are provided in the last section.

**ASIC DATA ENVELOPMENT ANALYSIS MODELS**

Assume that there are n DMUs, each using m inputs to produce s outputs. Let $X \in \mathbb{R}^{m \times n}$, $Y \in \mathbb{R}^{s \times n}$ be the matrices containing the observed input and output measures for the DMUs. We denote by $x_j$ (the jth column of $X$) the vector of inputs consumed by DMU, and by $y_o$ the quantity of output i consumed by DMU. A similar notation is used for outputs. Also, we assume that all the inputs and outputs are nonnegative and at least one input and one output of each DMU is strictly positive. We call a vector of a pair of the semipositive input and output an activity, and express it by the notation $(x, y)$. The PPS is defined as the set of feasible activities and denoted by $T$.

$$T = \{ (x, y) | y \text{ can be produced by } x \}$$

We postulate the following properties of $T$:

1. The observed activities $(x_j, y_j)$ (j=1,...,n) belong to $T$.
2. If an activity $(x, y)$ belongs to $T$, then the activity $(tx, ty)$ belongs to $T$ for any $t>0$. This property is called the constant returns to scale property.
3. For any activity $(x, y) \in T$, any semipositive activity $(\bar{x}, \bar{y})$ with $\bar{x} \geq x$ and $\bar{y} \leq y$ is included in $T$.
4. $T$ is closed and convex.

We can define the PPS $T_e$ satisfying (1) to (4) by

$$T_e = \{ (x, y) | x \geq X\lambda, y \geq Y\lambda, \lambda \geq 0 \}$$

$T_e$ is built on the assumption of constant returns to scale of activities. $T_e$ is built on the assumption of variable returns to scale of activities, which means postulate 2 is omitted. By considering postulates (1), (3) and (4), $T_v$ can be defined as follows:

$$T_v = \{ (x, y) | x \geq X\lambda, y \leq Y\lambda, e\lambda = 1, \lambda \geq 0 \}$$

Now, we define the PPS $T'$ satisfying (1) and (4) by

$$T' = \{ (x, y) | x = X\lambda, y = Y\lambda, e\lambda = 1, \lambda \geq 0 \}$$

For evaluating the efficiency of DMU, $(o \in \{1,...,n\})$, we can use the input-oriented model (2.1). Model (2.2) is the output-oriented model:

$$\text{Min } \theta$$

s.t. $X\lambda \leq \theta x_o$

$$Y\lambda \geq y_o$$

$$\lambda \in \Lambda$$

and

$$\text{Max } \theta$$

s.t. $X\lambda \leq x_o$

$$Y\lambda \geq \phi y_o$$

$$\lambda \in \Lambda$$

The above-mentioned models are called CCR models if

$$\Lambda = \{ \lambda | \lambda \geq 0 \}$$

and BCC models if

$$\Lambda = \{ \lambda | \lambda \geq 0, e\lambda = 1 \}$$

where $e$ is a row vector with all elements equal to 1.

The preceding models required us to distinguish between input-oriented and output-oriented models. Now, we combine both orientations in a single model, called the Additive model. There are several types of Additive models, out of which we select the following:

$$\text{Max } eS^+ + eS^-$$

s.t. $Y\lambda - S^+ = y_o$

$$X\lambda + S^- = x_o$$

$$e\lambda = 1$$

$$\lambda \geq 0, S^+ \geq 0, S^- \geq 0$$

A variant is an Additive model which omits the condition $e\lambda = 1$. Note that the foregoing Additive model has the same PPS as the BCC model.

**Theorem 1.** Let $(\lambda^*, S_{-}^*, S^*)$ be an optimal solution of (2.4). DMU is efficient by the Additive model (denoted by ADD-efficient) if and only if $S^* = 0$ and $S_{-}^* = 0$. This means that DMU is ADD-efficient if and only if it is BCC-efficient (For more details, see [3]).

**MULTIPLE OBJECTIVE LINEAR PROGRAMMING**

Consider the following MOLP problem:
Max $Y\lambda$
Max $-X\lambda$ \hspace{1cm} (3.1)
s.t. $\lambda \in \Lambda = \{ \lambda \mid A\lambda = b, \lambda \geq 0 \}$

where $\lambda \in \mathbb{R}^n$, $b \in \mathbb{R}^k$, and the constraint matrix $A \in \mathbb{R}^{kn}$ is of full row rank.

Unfortunately, the idea of "maximization", when applied to a vector, is insufficiently robust. There would be no problem if in each MOLP a point exists in $\Lambda$ that simultaneously maximizes all objectives. Since this is rarely the case, the more generalized solution concept of efficiency is adopted.

**Definition 1.** In (3.1), $\hat{\lambda} \in \Lambda$ is efficient iff there does not exist another $\lambda \in \Lambda$ such that $Y\lambda \geq Y\hat{\lambda}$, $X\lambda \leq X\hat{\lambda}$ and $(Y\lambda, X\lambda) \neq (Y\hat{\lambda}, X\hat{\lambda})$.

**Definition 2.** In (3.1), $\hat{\lambda} \in \Lambda$ is weakly efficient iff there does not exist another $\lambda \in \Lambda$ such that $Y\lambda_0 > Y\hat{\lambda}_0$ and $X\lambda_0 < X\hat{\lambda}_0$.

The set of all efficient solutions is called the efficient set (denoted by $E$).

Specifically, in the MOLP literature (See, e.g., Steuer [1986]), the concept of efficiency is used to refer to the solutions in the decision variable space $\mathbb{R}^n$ (set $\Lambda$) and the concept of dominance is used to refer to the efficient solutions in the criterion space $\mathbb{R}^{mn}$ (set $T'$, where $T' = \{(x, y) \mid x = X\lambda, y = Y\lambda, \lambda \in \Lambda \}$).

**Theorem 2.** Let $\hat{\lambda} \in \Lambda$ and $C = \begin{bmatrix} Y_1, \ldots, Y_n \\ -X_1, \ldots, -X_n \end{bmatrix}$ be the $(m+s) \times n$ criterion matrix (matrix of objective function coefficients) whose rows are the gradients of the $(m+s)$ objective functions and $D$ be an $n \times n$ diagonal matrix with

\[ d_{ij} = \begin{cases} 1 & \text{if } \lambda_j = 0 \\ 0 & \text{otherwise} \end{cases} \]

Then, $\lambda \in E$ $\iff$ the system

\[ Cu \geq 0, Cu' \neq 0, Du \geq 0, Au = 0 \]

has no solution $u' \in \mathbb{R}^n$.

**Proof.** $\Rightarrow$ Suppose $u'$ satisfies the system. Let $\hat{\lambda} = \lambda + \alpha u', \alpha u' > 0$. Then, there exists an $\alpha \in [0, \hat{\alpha}]$, $\hat{\lambda} \in \Lambda$. But $C\hat{\lambda} - C\lambda = \alpha Cu' \geq 0$ and $\alpha Cu' \neq 0$. This means that

\[
\begin{bmatrix} \hat{\lambda}_1 \\ \vdots \\ \hat{\lambda}_n \end{bmatrix} = \begin{bmatrix} Y_1, \ldots, Y_n \\ -X_1, \ldots, -X_n \end{bmatrix} \begin{bmatrix} \hat{\lambda}_1 \\ \vdots \\ \hat{\lambda}_n \end{bmatrix}
\]

That is, $Y\hat{\lambda} \geq Y\lambda, X\hat{\lambda} \leq X\lambda$ and $(Y\lambda, X\lambda) \neq (Y\hat{\lambda}, X\hat{\lambda})$. The last relation implies that $\hat{\lambda}$ is not efficient.

\[ \Leftarrow \text{ Suppose the system is inconsistent and let } \lambda \in \Lambda. \text{ Further assume that } u' = \lambda - \lambda'. \text{ Consequently, } Au' = 0 \text{ and } Du' \geq 0. \text{ Hence, it is not true that } Cu' \geq 0, Cu' \neq 0. \text{ Thus, } C\lambda \geq C\lambda', C\lambda \neq C\lambda'. \text{ do not hold. Since } \lambda \in \Lambda \text{ was arbitrary, } \lambda' \text{ is efficient.} \]

**Theorem 3.** Let $\lambda' \in \Lambda$ and $C, D$ be defined as above. Then, $\lambda'$ is efficient iff there exist $\Pi = (u', v') \in \mathbb{R}^{mn}$, $y^1 \in \mathbb{R}^n$ and $y^2 \in \mathbb{R}^k$ such that

\[ C'\Pi + D'y^3 + A'y^4 = 0 \]
\[ \Pi > 0, y^3 \geq 0 \]

**Proof.** By Tucker’s Theorem of the Alternative and Theorem 2 above.

**Theorem 4.** $\lambda' \in \Lambda$ is efficient $\iff$ there exists a

\[ \Pi \in \Lambda' = \{ \Pi \in \mathbb{R}^{mn} \mid \Pi > 0 \} \]

such that $\lambda'$ maximizes the weighted-sums (composite) linear programming (LP) problem

\[ \text{Max } \{ \Pi' C \lambda' \mid \lambda \in \Lambda \}
\]

\[ \text{Max } \{ \Pi' (u'Y - v'X) \lambda' \mid \lambda \in \Lambda \}
\]

**Proof.** $\Rightarrow$ Let $\lambda'$ be an arbitrary efficient point. Since $\lambda'$ is efficient, by Theorem 3 there exist $\Pi \in \mathbb{R}^{mn}$, $y^1 \in \mathbb{R}^n$ and $y^2 \in \mathbb{R}^k$ such that the system

\[ C'\Pi + D'y^3 + A'y^4 = 0 \]
\[ \Pi > 0, y^3 \geq 0 \]

is consistent. But, by Tucker’s Theorem of the Alternative, the system

\[ (\Pi' C)u' \geq 0, (\Pi' C)u' \neq 0, Du' \geq 0, Au' = 0 \]

has no solution. Now, for any $\lambda \in \Lambda, D(\lambda - \lambda') \geq 0$ and $A(\lambda - \lambda') = 0$. Hence, it is not true that $\Pi' C(\lambda - \lambda') \geq 0$ and $\Pi' C(\lambda - \lambda') \neq 0$. 


This implies that $\Pi' C\lambda \geq 0$ (since $\Pi' C\lambda$ is a scalar).

$\iff$ Assume $\lambda^* \in \Lambda$ is not efficient. Then there exists $\lambda \in \Lambda$ such that $C\lambda \geq C\lambda^*$, $C\lambda \neq C\lambda^*$. Since $\Pi > 0$, this implies that $\Pi' C\lambda \geq \Pi' C\lambda^*$, which contradicts the maximality of $\lambda^*$.

**COMPARISON BETWEEN DEA AND MOLP MODELS**

In this section, the theoretical considerations of combining MOLP and DEA are presented, and the equivalence between DEA and MOLP models will be shown.

We now discuss methods to find an efficient solution in MOLP problems. The subproblem test of Theorem (5) can be used for determining the efficiency of a given extreme point of $\Lambda$.

**Theorem 5.** Let $\lambda_0 \in \Lambda$ be an extreme point. Consider the following linear program:

\[
\begin{align*}
\text{Max} & \quad eS^+ + eS^- \\
\text{s.t.} & \quad C\lambda - IS = C\lambda_0 \\
& \quad A\lambda = b \\
& \quad 0 \leq \lambda \in \mathbb{R}^n \\
& \quad 0 \leq S \in \mathbb{R}^{m\times n}
\end{align*}
\]

Then, (i) $\lambda_0 \in E$ iff (4.1) has an optimal objective function value of zero, (ii) if (4.1) has a positive-unbounded objective function value, $E = \emptyset$, and (iii) if $C\lambda^* - C\lambda_0 = IS^*$ with $C\lambda^* - C\lambda_0 = IS^*$ implies that $\lambda^*$ and $\lambda_0$ are efficient.

**Proof.** (i) $\lambda_0 \in E$ is an efficient extreme point iff there does not exist another $\lambda \in \Lambda$ such that $C\lambda_0 \leq C\lambda_0$ and $C\lambda \neq C\lambda_0$. That is, there does not exist another $S = C\lambda - C\lambda_0 \geq 0$. This implies that $\lambda_0 \in E$ iff the optimal objective function value of (4.1) is zero.

(ii) The dual of (4.1) is

\[
\begin{align*}
\text{Max} & \quad (C\lambda_0)^t p + b^t y \\
\text{s.t.} & \quad C^t p + A^t y \geq 0 \\
& \quad -lp + e^t p, y \\
& \quad p, y \text{ free}
\end{align*}
\]

Since the objective function of (4.1) above is unbounded, by the weak duality theorem, the dual constraints in (4.2) must be inconsistent. But the dual constraints are independent of the constants vector $\lambda_0$ and, hence, if they are inconsistent for some $\lambda_0$, they remain inconsistent for any $\lambda_0$. We therefore conclude that (4.1) remains unbounded, even if we change $\lambda_0$, as long as it remains feasible. Then, from (i), $E = \emptyset$.

(iii) suppose $\lambda^* \notin E$. Then, there exists a $\lambda$ such that $C\lambda \leq C\lambda^*$, $C\lambda \neq C\lambda^*$, with $C\lambda - C\lambda_0 = IS^*$ and $C\lambda - C\lambda_0 = IS^*$. We have $IS \leq IS^*$, $IS \neq IS^*$. Since $(\lambda, S^*)$ contradicts the optimality of $(\lambda, S)$, we have $\lambda^* \in E$.

Suppose $C = \left[ \begin{array}{c} Y_1, \ldots, Y_n \\ -X_1, \ldots, -X_n \end{array} \right]$ and $S' = [ S^{+t}, S^{-t} ]$ where $S^{+t} = (s_1^+, \ldots, s_m^+)$ is the output shortfall vector and $S^{-t} = (s_1^-, \ldots, s_m^-)$ is the input excess vector. Problem (4.1) can be equivalently rewritten as follows:

\[
\begin{align*}
\text{Max} & \quad eS^+ + eS^- \\
\text{s.t.} & \quad Y\lambda - S^+ = Y\lambda_0 \\
& \quad X\lambda + S^- = X\lambda_0 \\
& \quad A\lambda = b \\
& \quad \lambda \geq 0
\end{align*}
\]

In the BCC (ADD) model, as was seen above, $\Lambda$ is defined as $\Lambda = \{ [\lambda] : 0 \lambda = 0, e\lambda = 1 \}$. Since $\lambda_0$ is an extreme point of $\Lambda$, the set of the column vectors of $\ell$ corresponding to positive $\lambda_0$ is a linearly independent set by definition. On the other hand, $\lambda_0$ must be the $j$th unit vector in $\mathbb{R}^n$. Consequently, the preceding LP is transformed into:

\[
\begin{align*}
\text{Max} & \quad eS^+ + eS^- \\
\text{s.t.} & \quad Y\lambda - S^+ = y_j \\
& \quad X\lambda + S^- = x_j \\
& \quad e\lambda = 1 \\
& \quad \lambda \geq 0, S^+ \geq 0, S^- \geq 0
\end{align*}
\]

The above model is an Additive model in evaluating the efficiency of DMU. Therefore, the subproblem test of Theorem (5) to generate efficient solutions in the MOLP problems can be transformed to an Additive model similar to the above model. Now we redefine the efficient units in DEA by the concept of efficiency in the MOLP-literature.
Recall that $\text{DMU}_j$ is ADD-efficient iff $S^* = 0$ and $S^{**} = 0$. This is equivalent to part (i) of Theorem (5). Hence, by Theorem (1), $\text{DMU}_j$ is BCC-efficient iff (4.1) has an optimal objective function value of zero.

All efficient DMUs lie on the efficient frontier, which is defined as a subset of points of set $T'$ satisfying the efficiency condition above. The definition of efficiency and the corresponding definition for weak efficiency can be given in the following equivalent form:

**Definition 3.** A solution $(X^*, Y^*) = (x^*, y^*)$ of DEA models is efficient iff there does not exist another $(x, y) \in T'$ such that $y \geq y^*, x \leq x^*$ and $(x, y) \neq (x^*, y^*)$.

**Definition 4.** A solution $(x^*, y^*) \in T'$ is weakly efficient iff there does not exist another $(x, y) \in T'$ such that $y > y^*$ and $x < x^*$.

As we can see, Definitions 1, 2 and Definitions 3, 4 define efficiency in the same manner. The only difference is that in Definitions 1, 2, the concept efficiency is used in a variable space ($\mathbb{R}^n$), whereas Definitions 3, 4 define efficiency in the criterion space ($\mathbb{R}^{m+s}$) (See $T'$). In spite of a slight terminology difference, the efficient units for DEA models can be characterized by model (4.1) in MOLP.

From the above discussion, we conclude that the Additive model can be used for identifying the efficient units in the PPS where postulates (1) and (4) are satisfied.

**CONCLUDING REMARKS**

The purpose of this paper was to establish that the Additive model to identify efficient units is equivalent to a linear model to generate efficient solutions in the MOLP problems. This provided an interesting theoretical aspect, because despite the complexity of DEA for searching the efficient frontier, it can be presented as a special case of the existing methods for characterizing the efficient set of a MOLP problem. Moreover, we showed that the Additive model can be used for identifying the efficient units in the PPS where the possibility assumption is omitted (See $T'$). In the next section, this fact is illustrated by a numerical example.

**NUMERICAL EXAMPLE**

Now we can utilize the results of previous section. Consider a system of seven DMUs as shown in Fig. 1, whose data is given in Table 1.

<table>
<thead>
<tr>
<th>DMUs</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Output</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 1: Data of the numerical example**

By solving the Additive model for each DMU, we conclude that DMUs A, B, C and E are efficient in the PPS where postulates (1) and (4) are satisfied.

**REFERENCES**


Applying quality-driven, efficiency-adjusted DEA (QE-DEA) in the pursuit of high-efficiency high-quality service units: an input-oriented approach

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ABSTRACT
The Data Envelopment Analysis (DEA) is used extensively for estimating the comparative efficiency between operational units that employ similar production processes. Efficiency optimization, however, is not adequate by itself to ensure the successful operation of an enterprise, because it is related only to the internal production process of a business unit. Therefore, the unit, to be able to achieve its goals, should take into consideration external factors, such as customer satisfaction, which reflect the quality of the services provided. In this context, the current paper strongly supports the view that synchronous assessment of both efficiency and quality is necessary. Using the Q-DEA model of Sherman and Zhu (2006a), we develop a Quality-driven, Efficiency-adjusted DEA model (QE-DEA) which seeks to provide a road map for high-quality and high-efficiency operation of every service unit of the sample.

Keywords: Data Envelopment Analysis (DEA); Efficiency; Quality; Non-discretionary variables

INTRODUCTION
Attaining technical efficiency and providing high-quality services, which are assessed solely by the end-users after the receipt of service (Parasuraman, et al16, 1988; Zeithaml17, 1988), are considered to be ideal concepts for modern enterprises. Regardless of its importance, quality is frequently omitted from performance analyses (Callen, 1991) as if the operational units should focus solely on efficiency optimization. If this was adopted, especially by enterprises operating in mature markets where marginal growth rates and intense competition dominate (Hayes, 2008; Anderson & Fornell, 1994), the results could be disastrous as in those markets; consumers are regarded as ‘scarce sources’. The satisfaction of the consumers, which is derived from the delivered service quality standards, would secure mid-to-long-term viability of an enterprise.

To this extent, the scope of the present paper is the development of a geometrical and algebraic model, based on issues raised by Sherman and Zhu (2006a), to estimate the efficiency of all the operational units under evaluation, even those that are categorized as low-quality – high-efficiency. In the following section, we analyze the proposed Quality-driven, Efficiency-adjusted DEA model (QE-DEA), first in the case where all inputs are controlled by the SU and, second, under the assumption that some of the inputs are non-discretionary. By running this model in conjunction with DEA, we can develop a road map for achieving high-quality – high-efficiency for every operational unit. It has to be stressed that, here, we have assumed a trade-off between quality and efficiency. To be more precise, improvement on services’ quality standards entails need for additional resources engagement, given the outputs’ level, and also, efficiency decline. Conclusions are presented in the last section of the paper.

QUALITY-DRIVEN, EFFICIENCY-ADJUSTED DEA (QE-DEA)
To start with, quality (q) and efficiency (e) are quoted on the x-axis and the y-axis, respectively, of the Cartesian coordinate system, while the quality-efficiency bundle determines the geometrical position of a Service Unit (SU). The chart is separated into four segments: 1) high-quality – high-efficiency (HQ-HE); 2) low-quality – high-efficiency (LQ-HE); 3) low-quality – low-efficiency (LQ-LE); and 4) high-quality – low-efficiency (HQ-LE) (Figure 1). In addition,
two cut-off levels are depicted with the first referring to efficiency and the second referring to quality. The feasible area of these two elements is determined by the interval (0.2, 1]. The selection of the feasible area of efficiency scores can be determined from the work of Paradi et al. (2004), who demonstrated that faulty input and output data entries or missing values are likely to occur when the estimated efficiency score of an SU is 0.2 or less. Therefore, the results and the data should be cross-checked. The feasible area of the quality scores, which is derived from fieldwork research, is expressed by the conversion of the five-point Likert scale response format questionnaires into percentages.

In Figure 1, SUs which get solely quality score equal to 0.80 or greater and efficiency score equal to unity are considered HQ and HE. Unlike the Q-DEA model, in analysis with the QE-DEA model, no SUs are removed from the evaluation process. Especially for the LQ-HE operational units, a boost in their quality score is the primary emphasis. As a result of the inverse relationship between quality and efficiency (De Bruijn, 2007; Sherman & Zhu, 2006a; Sherman & Zhu, 2006b; Athanassopoulos, 1997; Anderson & Fornell, 1994), increases in quality are expected to reduce efficiency scores. In fact, LQ-HE SUs are supposed to move to the HQ-LE segment. Namely, in Figure 1, SU “A”, specified by the coordinates of the point (q_A, 1) is directed to the point A’ (q_A’, e_A’), holding the same quality-efficiency relative size.

To apply the QE-DEA model, we have developed an algorithm involving two steps:

**Step 1:** Run DEA in order to estimate efficiency scores.

**Step 2:** If the number of LQ-HE SUs is null, then stop.

Otherwise, before adjusting the efficiency score to high quality standards determine the trade-off between quality and efficiency for each LQ-HE SU. Next, compute the inputs of the hypothetical SUs given the outputs (input-oriented approach) and return to Step 1.

A milestone for the application of the QE-DEA methodology is the substitution of the actual LQ-HE SUs (e.g., A), if they exist, with the hypothetical counterparts (e.g., A’). Moreover, it should be noted that the efficiency score assigned to LQ-HE SUs is important for the input adjustment to high-quality standards. The adjustment (increase) of the input levels provided, is estimated rather than determined after the implementation of the second phase in Step 2 of the preceding algorithm, because of the possible variation of the assigned weights. More specifically, the second phase in Step 2 is detached from the DEA methodology. As a consequence, the weights attached to the hypothetical inputs\(^{19}\) \(x^{'}\) are expected to be an approximation of the final inputs, which will be derived after returning to Step 1 and running DEA again. Following a similar process, the efficiency score of the hypothetical (initially LQ-HE) SUs (e.g. e_A’), which is the outcome of the first stage (Step 1 and Step 2) of QE-DEA application, will probably differ from those estimated using DEA at a second stage of analysis (following Step 2 rerun DEA). This deviation is due to efficiency score sensitivity to data (input or output) perturbation. For instance, input modification (e.g. increase), when the output is kept at its current level, for a technically efficiency SU does not necessarily entail downward change of the efficiency score.

Going back to Figure 1, subsequent to the determination of the two straight lines bounded by the points A_o, B_o and A’, B’ in the plane regarding the actual and the hypothetical operational units, respectively, the coordinates of quality (q_A’) and efficiency (e_A’) of the latter unit should be calculated. The quality score of the hypothetical unit is arbitrarily decided to be in the range of 0.8 - 1.0. The efficiency score \(e_A\) is determined after the computation of the distance function between equivalent points of the two straight lines. It should be pointed out that the quality-efficiency symmetry is fixed for the actual and hypothetical units A and A’, respectively, so that the latter SU is derived from the former.

\[
\frac{(A_o B_o)}{(A' B'_o)} = \frac{(q_A - 0.20)(0.20 - 1)}{(q'_A - 0.20)(0.20 - e_A)}
\]  

(1)

Given the distance function formula:

\[
(AB) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\]

and substituting in (1) we take:

\[
\frac{(q_1 - 0.20)^2 + (0.20 - 1)^2}{(q'_1 - 0.20)^2 + (0.20 - e_A)^2}

= \frac{(q_A - 0.20)(0.20 - 1)}{(q'_A - 0.20)(0.20 - e_A)}
\]  

(2)

In general, even if diverse quality and efficiency cut-off points are chosen (cut-off

\(^{19}\) Hypothetical inputs are the resources engaged by the hypothetical units.
points ≠ 0.2), (2) is expressed by the following equation:
\[
\frac{\sqrt{(q_i - q_i')^2 + (e_i - 1)^2}}{\sqrt{(q_i' - q_i)^2 + (e_i - e_i')^2}} = \frac{(q_i - q_i')(e_i - 1)}{(q_i' - q_i)(e_i - e_i')}
\]  
(3)

Equation (4) is the generalized formula [Appendix – Section 1] used to determine the efficiency scores \((e_{i'})\) of the hypothetical SUs:
\[
e_{i'} = e_i + \sqrt{\frac{[q_i - q_i']^2(e_i - 1)}{[q_i' - q_i]^2[e_i - e_i']}}
\]  
(4)

Since the new efficiency score \((e_{i'})\) has been calculated, the inputs of the hypothetical operational units should be adjusted holding the outputs fixed (input orientation).

Efficiency ratio was defined by Cooper et al. (2007) and Sherman & Zhu (2006):
\[
e = \sum_{r=1}^{s} \frac{u_r}{\sum_{i=1}^{m} y_r x_i} = \frac{u_r y_r + u_r y_r + ... + u_r y_r}{v_r x_1 + v_r x_2 + ... + v_r x_m}
\]  
(5)

where: \(e = \) efficiency score
\(y_r = \) amount of output \(r \forall r = 1, ..., s\)
\(u_r = \) weight assigned to output \(r\)
\(x_i = \) amount of input \(i \forall i = 1, ..., m\)
\(v_r = \) weight assigned to input \(i\)

Alternatively, the precedent equation (5) is expressed in matrix form:
\[
e = \frac{\sum_{r=1}^{s} u_r y_r}{\sum_{i=1}^{m} y_r x_i} = \begin{bmatrix} u_1 \\ u_2 \\ ... \\ u_r \\ ... \\ v_1 \\ v_2 \\ ... \\ v_s \\ x_1 \\ ... \\ x_m \end{bmatrix}
\]  
(6)

Assuming technical efficiency prevails, then:
\[
e = \begin{bmatrix} u_1 \\ u_2 \\ ... \\ u_1 \\ ... \\ u_r \\ ... \\ v_1 \\ v_2 \\ ... \\ v_s \\ x_1 \\ ... \\ x_m \end{bmatrix}
\]

Therefore,
\[
[x_1, x_2, ..., x_n] = [y_1, y_2, ..., y_s] [u_1] \left[ \begin{array}{c} u_2 \\ ... \\ u_r \\ ... \\ v_1 \\ v_2 \\ ... \\ v_s \\ x_1 \\ ... \\ x_m \end{array} \right] 
\]
(7)

Functions (5) – (7) are applied for estimating the efficiency scores of actual SUs. In order to form hypothetical operational units, the inputs should be adjusted, given the input orientation of the analysis. In that case, functions (5) – (7) should be altered substantially:
\[
e' = \frac{\sum_{r=1}^{s} u_r y_r}{\sum_{r=1}^{s} v_r x_i'}
\]  
(8)

where \(e' \neq e\) and \(x_i \neq x_i\)

Expressing equation (8) in matrix form and conducting the required calculations [Appendix – Section 2], the input adjustment formula results:
\[
\begin{align*}
x_1' &= \frac{1}{e'} x_1 \\
x_2' &= \frac{1}{e'} x_2 \\
... \\
x_n' &= \frac{1}{e'} x_n
\end{align*}
\]  
(9)

In the above system of equations, \(e'\) is known as far as it is the ordinate of the hypothetical point \(A' (q_{i'}, e_{i'})\), namely, \(e' = e_{i'}\). On the whole, \(e'\) is equal to the ordinate of every estimated HQ-LE hypothetical SU. In the same way, \(x_i' \forall i = 1, ..., m\) is already known as well.

Mainly, \(x_i\) expresses the actual inputs of the LQ-HE SUs.

**QE-DEA AND THE ASSUMPTION OF EXOGENOUSLY-FIXED VARIABLES**

Obviously, the preceding analysis focuses on every input adjustment, holding the output level constant, implicitly assuming that inputs are subject to management control. However, an abundant number of cases exist in the literature (Müñiz et al, 2006; Athanassopoulos, 2004; Worthington & Dollery, 2000) in which exogenously fixed or non-discretionary variables appear. Such variables, e.g., market size and competition, are not controlled by the operational unit, but they indirectly affect its efficiency status.

Assuming at least one exogenously-fixed variable among the inputs, the efficiency determination formula for the hypothetical SUs should be adjusted appropriately.
Particularly, a new equation should be introduced in order to give the efficiency score ($e''$):

$$e'' \sum_{i=1}^{k} x_i = \sum_{i=1}^{k} x_i \cdot (k < m) \quad (10)$$

Rearranging the system of equations (9) by summing up the variables $x_i$ and $x$, individually [Appendix – Section 3] and substituting in (10) results:

$$e'' \sum_{i=1}^{k} x_i = \frac{1}{e'} \sum_{i=1}^{k} x_i$$

Hence,

$$e'' = \frac{\sum_{i=1}^{k} x_i}{\sum_{i=1}^{k} e'}$$

(11)

where

$$e'' > \frac{1}{e'}$$

Unlike the quality-driven, efficiency-adjusted score ($e'$), assuming that all inputs are controllable by the operational unit, the new efficiency score ($e''$) should be multiplied by a subset of inputs $x_i$ ($i = 1, \ldots, k$ $k < m$), solely by the controllable variables. Accordingly, the new input levels ($x_i \forall i = 1, \ldots, k$) will result equation (12), while the exogenously-fixed input levels remain the same.

$$x_i = e'' x_i \quad (i = 1, \ldots, k) \quad (12)$$

However, the summation of inputs ($\sum_{i=1}^{k} x_i$) in the form of ratio of ratios’ ($e''$) may return distorted efficiency scores ($e''$), especially when the input data lack homogeneity (e.g., working hours, number of employees, and market size). To deal with such an irregularity, a ratio of summations could be used, where the observed input is divided by the equivalent mean input of the operational units instead of a naïve ratio of the summation of the absolute input values for the observed SU.

$$e'' = \frac{1}{e'} \frac{\sum_{i=1}^{k} x_i}{\sum_{i=1}^{k} x_i}$$

(13)

where: $x_i$ = amount of input $i$

$\bar{x}$ = mean of input values

In the second phase of Step 2, the adjusted input data replace the actual data, and the DEA model is rerun in order to estimate the comparative efficiency score of all SUs. Clearly, quality and efficiency benchmarks will be the HQ-HE operational units for the rest of the units, which are not qualified. If the LQ-HE SUs are neither removed from the evaluation process nor adjusted to meet the quality-efficiency standards, the DEA model will consider them to be benchmarks, leading likewise to a flaw.

CONCLUSIONS

Quality of services provided and efficient operation are dimensions of substantial importance to the viability of every enterprise. The proposed Quality-driven, Efficiency-adjusted DEA model (QE-DEA) seeks to determine the quality-efficiency benchmarks and the optimum combination of the inputs and outputs for the rest of the operational units under evaluation in order to meet HQ-HE criteria. It is proved that emphasis should be placed on the trade-off determination between quality and efficiency scores for each LQ-HE SU, and either all inputs are controllable by the unit or at least one is non-discretionary.

REFERENCES


APPENDIX

Section 1

Equation (3) can be rewritten as:
\[
(q_j - q_k)^2 + (e_j - 1)^2 = (q_j - q_k)^2 (e_j - e_k)^2
\]
Equation (8a) lead to input adjustment formula:
\[
\sum_{i=1}^{n} \chi_i = \frac{1}{e^*} \sum_{i=1}^{n} \chi_i
\]

Section 2

Equation (8b) can be expressed in matrix form:
Figure 1: Trade-off Analysis between Quality and Efficiency in a Planar Cartesian coordinate system.
Performance Management and Measurement with Data Envelopment Analysis

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